INSTITUTE OF DEVELOPING ECONOMIES

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IDE DISCUSSION PAPER No. 888

Competition between heterogenous online and offline firms

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March, 2023

Abstract

We propose a model with heterogenous online and offline firms in an industry. Online firms need no additional fixed costs to export goods; however, offline firms do. Transaction costs between consumers and sellers are needed only in the online market. Consumers expect the quality of products in the online market but know the quality of products sold by offline firms. We find that low-quality firms with moderate productivity choose to be online firms. We show that a unique equilibrium value of the expected quality exists in the online market if transaction costs are sufficiently low, and if transport costs are sufficiently high. Furthermore, numerical analysis shows that online technology improves welfare, and transaction and transport costs have opposite impacts on welfare.

Keywords: Heterogenous firms; online; offline
JEL classification: D04, R12

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1 Introduction

Online sales have increased substantially due to the COVID-19 pandemic. According to East (2022), online grocery orders will decrease and then continue to grow. Online sales are becoming an increasingly important feature of our economy. The purpose of this paper is to examine the function of online technology under the competition between heterogenous online and offline firms. We can recognize online technology in two ways. First, online technology enables online firms to emerge. Thus, this is very basic technology in the early stage of online sales, such as the Internet with minimum information traffic. Second, online technology can improve online sales. This may be related to faster spread of the Internet, easier access to wi-fi, better digital devices for more consumers and safer mechanism for online sales. In this paper, we use online technology to address the first case. However, we use transaction costs between consumers and firms to address the second case. In other words, we can regard the case without online technology as that in which transaction costs are prohibitibly high. Using the first view on online technology, we compare the cases with and without online firms. We also examine the impact of transaction costs compared with that of transport costs.

We develop our theoretical setting considering the following three important features of online sales: (1) any online firms are accessible to all regions, (2) online sales depend on the devices consumers use to order products, and (3) imperfect product quality information is one characteristic of online sales. More precisely, the first feature means that online firms can save additional fixed costs for entering other regions. In our setting, online firms can sell products in a remote region without additional fixed costs, whereas offline firms need to spend additional fixed costs to export their products. To address the second feature, we introduce transaction costs between firms and consumers, which are separate from transport costs. A popular assumption is that transport costs are negligible for trade within a region. We assume that, unlike transport costs, transaction costs emerge from the transactions within a region and between regions in the online market. In our setting, transport costs for both online and offline firms are introduced; however, transaction costs are only for online firms. The third feature is a characteristic of online sales that provides a clear difference between online and offline firms. Rudolph (2016) found that at least 30% of all products ordered online are returned. This stems from imperfect information in the online market. For simplicity, we assume that imperfect information on product quality emerges only in online market, but not in offline market. Chen, Hu, and Li (2017) examined the imperfect infromation of online sales under oligopolies for industrial organization. As a basis, our paper uses Melitz (2003) under monopolistic competition to easily address the first feature. Other characteristics of the online market, such as search and matching are neglected. Better search and matching in the online market may decrease transaction costs in home and remote markets compared with the offline market. However, the monetary and opportunity costs for returning products are also required. Thus, we introduce transaction costs.

To consider the above mentioned features, we develop a two-region model. We introduce firm productivity and product quality to clarify the differences between online and offline firms. For simplicity, we assume Pareto distributions for quality and productivity. Following Johnson (2012), we assume that no relationship exists between production cost and product quality. Furthermore, we assume that the level of an expected quality in online market is shared in the economy. Transaction costs are iceberg costs as transport costs. In our setting, all firms sending products to a remote region need transport costs as Hortaçsu, Martínez-Jerez and Douglas (2009) showed that distance is a deterrent to trade in online sales. Although Lendle et al. (2016) empirically demonstrated that transport costs in the online market are lower than those in the offline market, we assume that transport costs are the same between online and offline firms. We further assume that online firms and offline firms selling in home market share the same fixed costs, and additional fixed costs for offline exporting firms are the same as those for the sales in home market. We also assume that there is no production inputs mobile between regions.

Our model provides three main results. First, firms with very low productivity exit the market, and firms with low quality and moderate productivity choose to be online firms. In the remaining domain, firms with high quality and/or productivity choose to be offline exporting firms. Second, the expected quality of online firms has a unique equilibrium value if transaction costs are sufficiently low and transport costs are sufficiently high. Third, the expected quality of online firms decreases with a decrease in transport costs. The expected quality has the same relationship with transaction costs if transaction or transport costs are sufficiently high. Furthermore, numerical analysis provides that online technology utilization improves welfare, and transaction and transport costs provide opposite impacts on welfare and the ex-ante probability of the successful entry of online firms.

The remainder of this paper is organized as follows. In Section 2, we construct a model. In Section 3, we determine firm entry, exit and status. Section 4 characterizes the equilibrium with and without online technology. Section 5 focuses on the symmetric setting to examine the impacts of online technology, transport costs, and transaction costs. Section 6 concludes the paper.

2 The model

2.1 Basic setup

We consider the economy to have two regions and total population L. Two regions are symmetric except for online transactions. Each individual inelastically supplies one unit of labor, which is the only production factor. The economy has agricultural and manufacturing sectors. Manufactured products are horizontally differentiated. Firms use increasing returns to scale technology in the Dixit-Stiglitz monopolistically competitive market. The agricultural sector produces a homogenous good with constant returns to scale technology in the perfectly competitive market. One unit of labor can produce one unit of an agricultural good, which is traded freely across regions. Without loss of generality, the agricultural good is chosen as the numéraire, which implies the price of agricultural good, p_A, and nominal wage rate, w, in each region equals to 1 (i.e., $w = p_A = 1$). Accordingly, the total income of region r is determined by $Y_r = wL/2 = L/2$.

2.2 Consumer behavior

All consumers in the economy share the same Cobb-Douglas utility function. The individual utility function in region r is given as follows:

$$U_r \equiv \frac{1}{(1-\mu)^{1-\mu}\mu^{\mu}} A_r^{1-\mu} \left[\int_{\omega\in\Omega_r} \varphi(\omega)^{\frac{\sigma-1}{\sigma}} m_r(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\mu\sigma}{\sigma-1}}, \mu \in (0,1) \tag{1}$$

where A_r and $m_r(\omega)$ are, respectively, the individual consumption of an agricultural good and variety ω of manufactured goods in region r, $\varphi(\omega)$ is the *product quality index* of variety ω, Ω_r is the set of available varieties in region r, $\sigma > 1$ is the elasticity of substitution between any two varieties, and μ is the consumption share of manufactured goods. The consumer's budget constraint is given as follows:

$$\int_{\omega\in\Omega_r} p_r(\omega) m_r(\omega)\mathrm{d}\omega + A_r = y_r,$$

where $p_r(\omega)$ is the consumer price of variety ω in region r and $y_r = w = 1$ is the individual income in region r.

Utility maximization yields the aggregate demand for variety ω in region r, $q_r(\omega)$, determined by

$$q_r(\omega) = \frac{\mu Y_r}{\mathcal{P}_r}\varphi(\omega)^{\sigma-1}\left[\frac{p_r(\omega)}{\mathcal{P}_r}\right]^{-\sigma}, \tag{2}$$

where Y_r is the aggregate income in region r and $\mathcal{P}_r$ is the price index of the composite manufactured goods in region r given as follows:

$$\mathcal{P}_r \equiv \left[\int_{\omega\in\Omega_r}\varphi(\omega)^{\sigma-1}p_r(\omega)^{1-\sigma}\mathrm{d}\omega\right]^{\frac{1}{1-\sigma}}. \tag{3}$$

Since $\sigma > 1$, (2) implies that the higher the quality, the larger the demand.

The individual indirect utility in region r, V_r, is determined as $V_r = 1/\mathcal{P}_r^{\mu}$.

2.3 Production

Following Melitz and Ottaviano (2008), we consider a static (one-period) model. Firms are identical prior to entry. Each firm faces uncertainty about its productivity level ψ and quality level φ. To start, each firm must make an initial investment. Thus, entry as a firm requires a sunk cost of $\mathcal{F}$ units of labor. Once this cost is paid, firms observe their productivity $\psi \in (0, +\infty)$ and quality $\varphi \in (0, +\infty)$ from the common joint probability density function $h(\psi, \varphi)$, which has positive supports over $(0, +\infty) \times (0, +\infty)$ and has the joint cumulative distribution $H(\psi, \varphi)$. Firm heterogeneity $H_r(\psi, \varphi)$ in region r takes the same form among all regions, such that $H_r(\psi, \varphi) = H(\psi, \varphi)$, $\forall r$. Following Johnson (2012), we assume that the two variables ψ and φ are independent. For simplicity, we assume that the two variables ψ and φ are drawn from the same density function $g(\cdot)$, which implies that $h(\psi, \varphi) = g(\psi)g(\varphi)$ and $H(\psi, \varphi) = G(\psi)G(\varphi)$ hold with the cumulative distribution function $G(\cdot)$. There are $\mathcal{M}_r$ potential firms who draw the lottery, and M_r active firms in region r.

For simplicity, we introduce Pareto distribution function: $g(\psi) = \kappa\psi_{min}^{\kappa}/\psi^{\kappa+1}$, $\psi \geq \psi_{min}$, and $g(\varphi) = \kappa\varphi_{min}^{\kappa}/\varphi^{\kappa+1}$, $\varphi \geq \varphi_{min}$, $\kappa > 0$, where ψ_{min} and φ_{min} are the minimum value of ψ and φ respectively. Therefore, we have $h(\psi, \varphi) = (\kappa\psi_{min}^{\kappa}/\psi^{\kappa+1})\cdot(\kappa\varphi_{min}^{\kappa}/\varphi^{\kappa+1})$, $\psi \geq \psi_{min}$ and $\varphi \geq \varphi_{min}$. Accordingly, we have:

$$\begin{aligned} H(\psi, \varphi) &\equiv \int_{\varphi_{min}}^{\varphi}\int_{\psi_{min}}^{\psi} h(\psi, \varphi)\mathrm{d}\psi\mathrm{d}\varphi \\ &= \left[1 - \left(\frac{\varphi_{min}}{\varphi}\right)^{\kappa}\right]\left[1 - \left(\frac{\psi_{min}}{\psi}\right)^{\kappa}\right]. \end{aligned}$$

Without loss of generality, we choose $\varphi_{min} = \psi_{min} = 1$.

Prior to selling its product, each firm incurs a fixed labor requirement $f > 0$ in production. Furthermore, there are no economies of scope in production. Thus, each firm produces a single variety, and each variety is produced by a single firm. To produce a variety, firm (ψ, φ) needs a marginal requirement of c/ψ units of labor with $c > 0$. Choosing the unit of each variety, we set $c = (\sigma - 1)/\sigma$. Online firms (N-firms) and offline firms (F-firms) may coexist in the manufacturing sector in each region. Specifically, each N-firm can serve consumers in both regions through an integrated online marketplace, whereas an F-firm serves local and foreign consumers separately. Consumers have imperfect information about N-firms' quality φ, but perfect information about their productivity ψ. This is because consumers can identify N-firms' productivity by observing N-firms' prices. Observing an N-firm's price $p(\psi)$, a consumer can deduce its productivity ψ under the markup pricing strategy. However, consumers have perfect information about the F-firm's quality and productivity.

Since quality φ and productivity ψ are independent, all consumers share a common expected value of N-firms' quality $\mathbb{E}\varphi$ in the integrated online marketplace. We assume that consumers in each region have the same rational expectations on firm (ψ, φ) which chooses to be an N-firm, which is also common knowledge for all firms after spending the sunk cost. Thus, both consumers and firms make their optimal decisions based on the same $\mathbb{E}\varphi$. Note that each firm's behavior does not affect the other firms under monopolistic competition. Similarly, we can assume that each firm's behavior has no impact on consumers' choices, although the aggregate behavior of firms as a whole affects each consumer's choice.

Each firm incurs both iceberg transport costs and iceberg transaction costs, as in Samuelson (1954), to sell its goods in the other region. Transport costs are the same for all firms across regions. Specifically, $\tau > 1$ units of goods must be shipped from region r to ensure the delivery of one unit in region $s \neq r$. For simplicity, we assume that the transport costs are zero within a region. Transaction costs depend on the information and communications technology available to consumers, such as better mobile phone and Internet access result in lower transactions costs. Transaction costs are consumer-specific. More precisely, transaction costs between consumers in region r and N-firms in any region are characterized by the iceberg form defined as $\iota_r > 1$.

Profit maximization of N-firms yields the delivered price of goods produced and sold in region r as $p_{rr,N}(\psi, \varphi) = \iota_r/\psi$, and the delivered price of goods produced in region r and sold in region $s \neq r$ as $p_{rs,N}(\psi, \varphi) = \tau\iota_s/\psi$. F-firms and consumers transact directly, which means zero transactions costs in each region. Thus, the delivered prices of the F-firm located in region r are $p_{rr,F}(\psi,\varphi) = 1/\psi$ and $p_{rs,F}(\psi,\varphi) = \tau/\psi$, respectively. Accordingly, the total demand in region s for the variety produced by firm (ψ,φ) in region r is obtained as follows:

$$q_{rs}(\psi,\varphi) = \mu Y_s \frac{\varphi_s^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} p_{rs}^{-\sigma}, \tag{4}$$

where $\varphi_s = \mathbb{E}\varphi$ for N-firm (ψ,φ), which is the quality expected by consumers in region s, and $\varphi_s = \varphi$ for F-firm (ψ,φ).

Using (4), the profit of N-firm (ψ,φ) locating in region r, $\pi_r^N(\psi,\varphi)$, is given by

$$\begin{aligned}
\pi_r^N(\psi,\varphi) &= \frac{\mu Y_r}{\sigma}\frac{\theta_r \psi^{\sigma-1}(\mathbb{E}\varphi)^{\sigma-1}}{\mathcal{P}_r^{1-\sigma}} + \frac{\mu Y_s}{\sigma}\frac{\phi\theta_s \psi^{\sigma-1}(\mathbb{E}\varphi)^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} - f, \\
&= \frac{\mu Y}{\sigma}\left(\frac{\theta_r}{\mathcal{P}_r^{1-\sigma}} + \frac{\phi\theta_s}{\mathcal{P}_s^{1-\sigma}}\right)\psi^{\sigma-1}(\mathbb{E}\varphi)^{\sigma-1} - f,
\end{aligned} \tag{5}$$

where $\phi \equiv \tau^{1-\sigma}$ is trade freeness, $\theta_r \equiv \iota_r^{1-\sigma}$ is transaction freeness, f is the fixed marketing costs for each N-firm, and $Y = Y_1 = Y_2 = L/2$. The profits of F-firm (ψ,φ) locating and selling in region r, $\pi_{rr}^F(\psi,\varphi)$, is given by

$$\pi_{rr}^F(\psi,\varphi) = \frac{\mu Y}{\sigma}\frac{\psi^{\sigma-1}\varphi^{\sigma-1}}{\mathcal{P}_r^{1-\sigma}} - f. \tag{6}$$

The profit of F-firm (ψ,φ) locating in region r and selling in region s, $\pi_{rs}^F(\psi,\varphi)$, is given by

$$\pi_{rs}^F(\psi,\varphi) = \frac{\mu Y}{\sigma}\frac{\phi\psi^{\sigma-1}\varphi^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} - f. \tag{7}$$

That is, each F-firm incurs the fixed marketing costs for selling in each region, but each N-firm incurs the fixed marketing costs only once because of the integrated online marketplace. However, the demand for N-firm depends on consumers' internet access.

We define the condition for the equilibrium profit of firms operating in region r as follows:

$$\pi_r(\psi,\varphi) = \max\left\{\pi_r^N(\psi,\varphi), \pi_{rr}^F(\psi,\varphi), \pi_{rr}^F(\psi,\varphi) + \pi_{rs}^F(\psi,\varphi), 0\right\}.$$

That is, each firm (ψ,φ) has no incentive to deviate from its choice if and only if the firm has a positive profit that is the largest among all possible choices.

2.4 Aggregation

The ex-ante probability of successful entry for N-firms producing in region r, $pe_{r,N}$, is

$$pe_{r,N} \equiv \int\int_{\mathcal{A}_{N,r}} g(\psi)g(\varphi)\mathrm{d}\psi\mathrm{d}\varphi = H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{N,r}}$$

where firm $(\psi,\varphi) \in \mathcal{A}_{N,r}$ chooses to be an N-firm. The ex-ante probability of successful entry for F-firms producing and selling in region r, $pe_{rr,F}$, and F-firms producing in region r and selling in region s, $pe_{rs,F}$, are, respectively, given by

$$pe_{rr,F} \equiv \int\int_{\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}} g(\psi)g(\varphi)\mathrm{d}\psi\mathrm{d}\varphi = H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}}$$
$$pe_{rs,F} \equiv \int\int_{\mathcal{A}_{F,r}} g(\psi)g(\varphi)\mathrm{d}\psi\mathrm{d}\varphi = H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{F,r}}$$

where firm $(\psi,\varphi) \in \mathcal{A}_{F,r}$ chooses to be an F-firm in region r selling in two regions, and where firm $(\psi,\varphi) \in \mathcal{A}_{Fr,r}$ chooses to be an F-firm selling only in home market.

The relationship between the mass of entrants in region r, $\mathcal{M}_r$, and active N-firms in region r, $M_{r,N}$, is $M_{r,N} = pe_{r,N}\mathcal{M}_r$. The relationship between $\mathcal{M}_r$ and active F-firms producing and selling in region r, $M_{rr,F}$, and active F-firms producing in region r and selling in region s, $M_{rs,F}$, are $M_{rr,F} = pe_{rr,F}\mathcal{M}_r$ and $M_{rs,F} = pe_{rs,F}\mathcal{M}_r$, respectively. Accordingly, the total mass of varieties available to consumers in region r, M_r^T, is determined by $M_r^T = M_{rr,F} + M_{sr,F} + M_{r,N} + M_{s,N}$.

The conditional distribution of N-firm (ψ,φ) in region r is given by

$$\nu_{r,N}(\psi,\varphi) = \begin{cases} \dfrac{g(\psi)g(\varphi)}{H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{N,r}}} & \text{if } (\psi,\varphi) \in \mathcal{A}_{N,r}, \\ 0 & \text{otherwise.} \end{cases}$$

The conditional distribution of F-firm (ψ,φ) producing and selling in region r is given by

$$\nu_{rr,F}(\psi,\varphi) = \begin{cases} \dfrac{g(\psi)g(\varphi)}{H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}}} & \text{if } (\psi,\varphi) \in \mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}, \\ 0 & \text{otherwise.} \end{cases}$$

The conditional distribution of F-firm (ψ,φ) producing in region r and selling in region s

is given by

$$\nu_{rs,F}(\psi,\varphi) = \begin{cases} \dfrac{g(\psi)g(\varphi)}{H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{F,r}}} & \text{if } (\psi,\varphi)\in\mathcal{A}_{F,r}, \\ 0 & \text{otherwise.} \end{cases}$$

We define the aggregate productivity level of N-firms in region r as

$$\begin{aligned} \widetilde{\Psi}_{r,N} &= \int_0^\infty \int_0^\infty \psi^{\sigma-1}\nu_{r,N}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi \\ &= \frac{1}{H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{N,r}}} \int\int_{\mathcal{A}_{N,r}} \psi^{\sigma-1}g(\psi)g(\varphi)\mathrm{d}\psi\mathrm{d}\varphi. \end{aligned}$$

We define the aggregate productivity and quality level of F-firms producing and selling in region r as

$$\begin{aligned} \widetilde{\Phi}_{rr,F} &= \int_0^\infty \int_0^\infty \varphi^{\sigma-1}\psi^{\sigma-1}\nu_{rr,F}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi \\ &= \frac{1}{H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}}} \int\int_{\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}} \varphi^{\sigma-1}\psi^{\sigma-1}g(\psi)g(\varphi)\mathrm{d}\psi\mathrm{d}\varphi, \end{aligned}$$

and that of F-firms producing in region r and selling in region s as

$$\begin{aligned} \widetilde{\Phi}_{rs,F} &= \int_0^\infty \int_0^\infty \varphi^{\sigma-1}\psi^{\sigma-1}\nu_{rs,F}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi \\ &= \frac{1}{H(\psi,\varphi)|_{(\psi,\varphi)\in\mathcal{A}_{F,r}}} \int\int_{\mathcal{A}_{F,r}} \varphi^{\sigma-1}\psi^{\sigma-1}g(\psi)g(\varphi)\mathrm{d}\psi\mathrm{d}\varphi. \end{aligned}$$

Accordingly, the price index of the composite manufactured goods in region r is written as

$$\begin{aligned} \mathcal{P}_r^{-(\sigma-1)} = & \int\int_{\mathcal{A}_{N,r}} M_{r,N}\left(\mathbb{E}\varphi\right)^{\sigma-1} p_{rr,N}(\psi,\varphi)^{1-\sigma} \upsilon_{r,N}(\psi,\varphi)\mathrm{d}\varphi\mathrm{d}\psi \\ & + \int\int_{\mathcal{A}_{Fr,r}} M_{rr,F}\varphi^{\sigma-1} p_{rr,F}(\psi,\varphi)^{1-\sigma} \upsilon_{rr,F}(\psi,\varphi)\mathrm{d}\varphi\mathrm{d}\psi \\ & + \int\int_{\mathcal{A}_{N,s}} M_{s,N}\left(\mathbb{E}\varphi\right)^{\sigma-1} p_{sr,N}(\psi,\varphi)^{1-\sigma} \phi\upsilon_{s,N}(\psi,\varphi)\mathrm{d}\varphi\mathrm{d}\psi \\ & + \int\int_{\mathcal{A}_{F,s}} M_{sr,F}\varphi^{\sigma-1} p_{sr,F}(\psi,\varphi)^{1-\sigma} \phi\upsilon_{sr,F}(\psi,\varphi)\mathrm{d}\varphi\mathrm{d}\psi \\ = & M_{r,N}\widetilde{\Psi}_{r,N}\theta_r(\mathbb{E}\varphi)^{\sigma-1} + M_{rr,F}\widetilde{\Phi}_{rr,F} + M_{s,N}\widetilde{\Psi}_{s,N}\phi\theta_r + M_{sr,F}\widetilde{\Phi}_{sr,F}\phi. \end{aligned} \tag{8}$$

The expected revenue and profit of N-firms producing in region r are, respectively, determined as follows:

$$
\begin{aligned}
\bar{r}_{r,N} &= \int\int_{\mathcal{A}_{N,r}} \left(\mu Y \frac{\theta_r \left(\mathbb{E}\varphi\right)^{\sigma-1}}{\mathcal{P}_r^{1-\sigma}} \psi^{\sigma-1} + \mu Y \frac{\phi\theta_s \left(\mathbb{E}\varphi\right)^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} \psi^{\sigma-1} \right) \nu_{r,N}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi \\
&= \mu Y \left(\frac{\theta_r}{\mathcal{P}_r^{1-\sigma}} + \frac{\phi\theta_s}{\mathcal{P}_s^{1-\sigma}} \right) \left(\mathbb{E}\varphi\right)^{\sigma-1} \widetilde{\Psi}_{r,N}, \\
\bar{\pi}_{r,N} &= \int\int_{\mathcal{A}_{N,r}} \left(\frac{\mu Y}{\sigma} \frac{\theta_r \left(\mathbb{E}\varphi\right)^{\sigma-1}}{\mathcal{P}_r^{1-\sigma}} \psi^{\sigma-1} + \frac{\mu Y}{\sigma} \frac{\phi\theta_s \left(\mathbb{E}\varphi\right)^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} \psi^{\sigma-1} - f \right) \nu_{r,N}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi \\
&= \frac{\bar{r}_{r,N}}{\sigma} - f.
\end{aligned}
$$

The expected revenue and profit of F-firms producing and selling in region r are, respectively, determined as follows:

$$
\begin{aligned}
\bar{r}_{rr,F} &= \int\int_{\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}} \left(\mu Y \frac{\varphi^{\sigma-1}\psi^{\sigma-1}}{\mathcal{P}_r^{1-\sigma}} \right) \nu_{rr,F}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi = \frac{\mu Y}{\mathcal{P}_r^{1-\sigma}} \widetilde{\Phi}_{rr}, \\
\bar{\pi}_{rr,F} &= \int\int_{\mathcal{A}_{Fr,r}\cup\mathcal{A}_{F,r}} \left(\frac{\mu Y}{\sigma} \frac{\varphi^{\sigma-1}\psi^{\sigma-1}}{\mathcal{P}_r^{1-\sigma}} - f \right) \nu_{rr,F}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi = \frac{\bar{r}_{rr,F}}{\sigma} - f.
\end{aligned}
$$

The expected revenue and profit of F-firms producing in region r and selling in region s are, respectively, determined as follows:

$$
\begin{aligned}
\bar{r}_{rs,F} &= \int\int_{\mathcal{A}_{F,r}} \left(\mu Y_s \frac{\phi\varphi^{\sigma-1}\psi^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} \right) \upsilon_{rs,F}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi = \frac{\mu Y_s \phi}{\mathcal{P}_s^{1-\sigma}} \widetilde{\Phi}_{rs}, \\
\bar{\pi}_{rs,F} &= \int\int_{\mathcal{A}_{F,r}} \left(\frac{\mu Y_s}{\sigma} \frac{\phi\varphi^{\sigma-1}\psi^{\sigma-1}}{\mathcal{P}_s^{1-\sigma}} - f \right) \upsilon_{rs,F}(\psi,\varphi)\mathrm{d}\psi\mathrm{d}\varphi = \frac{\bar{r}_{rs,F}}{\sigma} - f.
\end{aligned}
$$

The average revenue and profit of active F-firms in region r are, respectively, expressed as

$$
\begin{aligned}
\bar{r}_{r,F} &= \bar{r}_{rr,F} + \frac{pe_{rs,F}}{pe_{rr,F}} \cdot \bar{r}_{rs,F}, \\
\bar{\pi}_{r,F} &= \bar{\pi}_{rr,F} + \frac{pe_{rs,F}}{pe_{rr,F}} \cdot \bar{\pi}_{rs,F}.
\end{aligned}
$$

The free entry condition is expressed as $\mathcal{F} = pe_{rr,F}\bar{\pi}_{rr,F} + pe_{rs,F}\bar{\pi}_{rs,F} + pe_{r,N}\bar{\pi}_{r,N}$.

The expected value of N-firms' quality is defined as follows:

$$
\mathbb{E}\varphi \equiv \sum_r \frac{M_{r,N}}{M_{r,N} + M_{s,N}} \int\int_{\mathcal{A}_{N,r}} \varphi \nu_{r,N} \mathrm{d}\psi\mathrm{d}\varphi.
$$

3 Firm entry, exit and status

We next determine when firm (ψ, φ) exits and when firm (ψ, φ) chooses to be an online firm, offline firm selling in home market, or offline firm selling in both regions.

Using (5) and solving $\pi_r^N(\psi, \varphi) = 0$ yields the zero cutoff profit condition of N-firms locating in region r, Z_r^N, given by

$$Z_r^N \equiv \left\{ (\psi, \varphi) \in \mathbb{R}_+^2 : \psi = \underline{\Psi}_r \right\},$$

where

$$\underline{\Psi}_r \equiv \frac{1}{\mathbb{E}\varphi} \left[\frac{2\sigma f}{\mu L \left(\frac{\theta_r}{\mathcal{P}_r^{1-\sigma}} + \frac{\phi\theta_s}{\mathcal{P}_s^{1-\sigma}} \right)} \right]^{\frac{1}{\sigma-1}}. \tag{9}$$

Using (6) and solving $\pi_{rr}^F(\psi, \varphi) = 0$ yields the zero cutoff profit condition of F-firms locating and selling in region r, Z_{rr}^F, given by

$$Z_{rr}^F \equiv \left\{ (\psi, \varphi) \in \mathbb{R}_+^2 : \varphi^{\sigma-1}\psi^{\sigma-1} = \underline{\Phi}_{rr} \right\},$$

where

$$\underline{\Phi}_{rr} \equiv \frac{2\sigma f}{\mu L \mathcal{P}_r^{\sigma-1}}. \tag{10}$$

Using (7) and solving $\pi_{rs}^F(\psi, \varphi) = 0$ yields the zero cutoff profit condition of F-firms locating in region r and selling in region s, Z_{rs}^F, given by

$$Z_{rs}^F \equiv \left\{ (\psi, \varphi) \in \mathbb{R}_+^2 : \varphi^{\sigma-1}\psi^{\sigma-1} = \underline{\Phi}_{rs} \right\}.$$

where

$$\underline{\Phi}_{rs} \equiv \frac{2\sigma f}{\mu L \phi \mathcal{P}_s^{\sigma-1}}. \tag{11}$$

Using (10) and (11), $\underline{\Phi}_{rr} < \underline{\Phi}_{rs}$ and $\underline{\Phi}_{ss} < \underline{\Phi}_{sr}$ are satisfied if and only if

$$\phi < \frac{\mathcal{P}_r^{\sigma-1}}{\mathcal{P}_s^{\sigma-1}} < \frac{1}{\phi}. \tag{12}$$

Specifically, if $\mathcal{P}_r = \mathcal{P}_s$, then $\underline{\Phi}_{rr} < \underline{\Phi}_{rs}$ and $\underline{\Phi}_{ss} < \underline{\Phi}_{sr}$ hold. We assume that the difference between ι_r and ι_s is sufficiently small to hold (12), which implies $\underline{\Phi}_{rr} < \underline{\Phi}_{rs}$.

Furthermore, using (10) and (11) yields

$$\underline{\Phi}_{rr} = \phi \underline{\Phi}_{sr}. \tag{13}$$

Using (9), (10) and (11) yields

$$\underline{\Psi}_r = \frac{1}{\mathbb{E}\varphi}\left[\frac{\theta_r}{\underline{\Phi}_{rr}} + \frac{\theta_s}{\underline{\Phi}_{rs}}\right]^{-\frac{1}{\sigma-1}} = \frac{1}{\mathbb{E}\varphi}\left[\frac{\theta_r}{\underline{\Phi}_{rr}} + \frac{\phi\theta_s}{\underline{\Phi}_{ss}}\right]^{-\frac{1}{\sigma-1}}. \tag{14}$$

In addition, we can rewrite (5), (6), and (7) as follows:

$$\pi_r^N(\psi,\varphi) = \left(\frac{\psi^{\sigma-1}}{\underline{\Psi}_r^{\sigma-1}} - 1\right) f, \tag{15}$$

$$\pi_{rr}^F(\psi,\varphi) = \left(\frac{\psi^{\sigma-1}\varphi^{\sigma-1}}{\underline{\Phi}_{rr}} - 1\right) f, \tag{16}$$

$$\pi_{rs}^F(\psi,\varphi) = \left(\frac{\psi^{\sigma-1}\varphi^{\sigma-1}}{\underline{\Phi}_{rs}} - 1\right) f. \tag{17}$$

Thus, the profit of N-firms increases with lower productivity in Z_r^N, whereas the profit of F-firms increases with lower productivity and quality in Z_{rr}^F and Z_{rs}^F.

Using (15) and (16) and solving $\pi_r^N(\psi,\varphi) = \pi_{rr}^F(\psi,\varphi)$, we define the iso-profit condition between N-firms locating in region r and F-firms selling only in region r as follows:

$$E_{Nr-Frr} \equiv \left\{(\psi,\varphi) \in \mathbb{R}_+^2 : \varphi = \varphi_{r1}\right\}$$

where $\varphi_{r1} \equiv \underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r$. Thus, quality in E_{Nr-Frr} increases with a rise in the productivity and quality in Z_{rr}^F and a decline in productivity in Z_r^N.

We define the iso-profit condition between N-firms locating in region r and F-firms selling in two regions as follows:

$$E_{Nr-Fr} \equiv \left\{(\psi,\varphi) \in \mathbb{R}_+^2 : \varphi = \varphi_{r2}(\psi)\right\}$$

where

$$\varphi_{r2}(\psi) \equiv \left(\frac{\frac{1}{\psi^{\sigma-1}} + \frac{1}{\underline{\Psi}_r^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}}\right)^{\frac{1}{\sigma-1}}.$$

Thus, productivity and quality have negative relationships in E_{Nr-Fr}. Quality in E_{Nr-Fr} increases with a rise in productivity and quality in Z_{rr}^F and Z_{rs}^F and a decline in productivity in Z_r^N.

As a thought experiment, we consider the case in which the online technology is unavailable in the economy. In Appendix A, we obtain the following results. Firm (ψ, φ) in region r exits the market if and only if

$$1 = \psi_{min}\varphi_{min} < \psi\varphi < \underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}; \tag{18}$$

firm (ψ, φ) in region r chooses to be an F-firm selling in the home market if and only if

$$\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}} < \psi\varphi < \underline{\Phi}_{rs}^{\frac{1}{\sigma-1}}; \tag{19}$$

and firm (ψ, φ) in region r chooses to be an F-firm selling in both markets if and only if

$$\psi\varphi > \underline{\Phi}_{rs}^{\frac{1}{\sigma-1}}. \tag{20}$$

We now turn to the case in which online technology is available. In Appendix A, firm (ψ, φ) in region r exits the market if and only if

$$1 = \psi_{min} < \psi < \underline{\Psi}_r \qquad \text{and} \qquad 1 = \psi_{min}\varphi_{min} < \psi\varphi < \underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}; \tag{21}$$

firm (ψ, φ) in region r chooses to be an online firm if and only if

$$\psi > \underline{\Psi}_r, \qquad \text{and} \qquad \varphi < \min\{\varphi_{r1}, \varphi_{r2}(\psi)\}; \tag{22}$$

and firm (ψ, φ) in region r chooses to be an offline firm selling only in home market if and only if

$$\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}} < \psi\varphi < \underline{\Phi}_{rs}^{\frac{1}{\sigma-1}} \qquad \text{and} \qquad \varphi > \varphi_{r1}; \tag{23}$$

and firm (ψ, φ) in region r chooses to be an offline firm selling in two regions if and only if

$$\psi\varphi > \underline{\Phi}_{rs}^{\frac{1}{\sigma-1}} \qquad \text{and} \qquad \varphi > \varphi_{r2}(\psi). \tag{24}$$

It is readily verified that Z_r^N, Z_{rr}^F, and E_{Nr-Frr} are satisfied at $(\psi, \varphi) = (\underline{\Psi}_r, \varphi_{r1})$, and that Z_r^F, E_{Nr-Frr} and E_{Nr-Fr} are satisfied at $(\psi, \varphi) = (\psi_{r2}, \varphi_{r1})$ where $\psi_{r2} \equiv \underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r/\underline{\Phi}_{rr}^{1/(\sigma-1)}$. Additionally, $\varphi_{r2}(\psi)$ has an asymptote such that

$$\varphi^{\sigma-1} = 1/\left[\underline{\Psi}_r^{\sigma-1}\left(1/\underline{\Phi}_{rr} + 1/\underline{\Phi}_{rs}\right)\right],$$

which means that $\varphi_{r2}(\psi) \neq 1$ for $\forall\psi$ by assuming $1/\left[\underline{\Psi}_r^{\sigma-1}\left(1/\underline{\Phi}_{rr} + 1/\underline{\Phi}_{rs}\right)\right] > 1$, which is rewritten as

$$\underline{\Psi}_r^{\sigma-1} < (1/\underline{\Phi}_{rr} + 1/\underline{\Phi}_{rs})^{-1} = \frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}} \underline{\Phi}_{rs}. \tag{25}$$

Otherwise, there exists $\varphi_{r2}(\psi) = 1$ for $\exists\psi$, which implies that the boundary between the domain for online and offline firms selling in home market becomes short.

For the existence of firms exiting the market in region r, we assume that

$$\underline{\Psi}_r > 1 \tag{26}$$

holds. Otherwise, all firms are active and choose to be either online or offline firms.

For the existence of N-firms in region r, we assume

$$\varphi_{r1} > 1 \Leftrightarrow \underline{\Phi}_{rr}^{1/(\sigma-1)} > \underline{\Psi}_r. \tag{27}$$

Otherwise, firms choose to exit or to be an F-firm. Note that (27) holds if we assume (25).

Under (25) and (26), the choice of firm (ψ, φ) is expressed as in Figure 1. Firms with the lowest quality can survive because of imperfect information. The boundary between online firms and exporting offline firms exists because online firms can access a remote region without additional fixed costs but with transaction costs, and because higher-quality firms do not prefer to disguise their quality. The boundary between online firms and offline firms selling only in home market exists because online firms sell in two market with transaction costs, but consumers know the quality of offline firms selling locally.

Figure 1 is around here.

Finally, we summarize our findings in the following proposition:

Proposition 1 *If the economy is relatively symmetric as (12) holds, firms with very low productivity exit the market if* $\underline{\Psi}_r > 1$ *holds, and firms with low quality and without low productivity choose to be online firms if* $\underline{\Phi}_{rr}^{1/(\sigma-1)} > \underline{\Psi}_r$ *holds. Regarding the remaining quality and productivity, firms with lower (resp. higher) productivity and quality choose to be offline firms selling only in home market (resp. in home and remote markets).*

4 Equilibrium

Following the literatures *à* la Melitz models, we assume that $\kappa > \sigma - 1 > 0$ hold.

4.1 No online technology

We first consider the case without online technology, which includes the active F-firms selling in home market or two markets. We can rewrite (19) under $1 < \psi$ and $1 < \varphi$ as follows: $1 < \varphi < \underline{\Phi}_{rr}^{1/(\sigma-1)}$ and $\underline{\Phi}_{rr}^{1/(\sigma-1)}/\varphi \equiv \psi_{r1}(\varphi) < \psi < \infty$; and $\underline{\Phi}_{rr}^{1/(\sigma-1)} < \varphi < \infty$ and $1 < \psi < \infty$. The ex-ante probability of successful entry for F-firms producing and selling in region r, $pe_{rr,F}$, is obtained as follows:

$$\begin{aligned} pe_{rr,F} &= \int_1^{\underline{\Phi}_{rr}^{1/(\sigma-1)}} \int_{\psi_{r1}(\varphi)}^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi + \int_{\underline{\Phi}_{rr}^{1/(\sigma-1)}}^{\infty} \int_1^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi \\ &= \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \left(1 + \frac{\kappa}{\sigma-1} \log \underline{\Phi}_{rr} \right). \end{aligned}$$

The aggregate productivity and quality level of F-firms producing and selling in home region is obtained as follows:

$$\widetilde{\Phi}_{rr} = \frac{A_{rr}}{pe_{rr,F}} = \frac{\kappa^2 \underline{\Phi}_{rr}}{(\kappa-\sigma+1)^2} \frac{\sigma - 1 + (\kappa-\sigma+1)\log \underline{\Phi}_{rr}}{\sigma - 1 + \kappa \log \underline{\Phi}_{rr}},$$

where A_{rr} is determined by

$$\begin{aligned} A_{rr} &= \int_1^{\underline{\Phi}_{rr}^{1/(\sigma-1)}} \int_{\psi_{r1}(\varphi)}^{\infty} \varphi^{\sigma-1}\psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi + \int_{\underline{\Phi}_{rr}^{1/(\sigma-1)}}^{\infty} \int_1^{\infty} \varphi^{\sigma-1}\psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi \\ &= \frac{\kappa^2}{\kappa-\sigma+1} \underline{\Phi}_{rr}^{(\sigma-\kappa-1)/(\sigma-1)} \left(\frac{1}{\kappa-\sigma+1} + \frac{1}{\sigma-1} \log \underline{\Phi}_{rr} \right). \end{aligned}$$

The expected revenue and profit of F-firms producing in region r are, respectively, determined as follows:

$$\bar{r}_{rr,F} = \sigma f \frac{\widetilde{\Phi}_{rr}}{\underline{\Phi}_{rr}} = \frac{\kappa^2 \sigma f}{(\kappa-\sigma+1)^2} \frac{\sigma - 1 + (\kappa-\sigma+1)\log \underline{\Phi}_{rr}}{\sigma - 1 + \kappa \log \underline{\Phi}_{rr}}.$$

and

$$\bar{\pi}_{rr,F} = \left[\frac{\kappa^2}{(\kappa-\sigma+1)^2} \frac{\sigma - 1 + (\kappa-\sigma+1)\log \underline{\Phi}_{rr}}{\sigma - 1 + \kappa \log \underline{\Phi}_{rr}} - 1 \right] f.$$

Thus, we have

$$\bar{\pi}_{rr,F} \cdot pe_{rr,F} = \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \left[\frac{(\sigma-1)(2\kappa-\sigma+1)}{(\kappa-\sigma+1)^2} + \frac{\kappa}{\kappa-\sigma+1} \log \underline{\Phi}_{rr} \right] f.$$

We focus on F-firms selling in a remote market. The ex-ante probability of successful entry for F-firms producing in region r and selling in region s is determined as follows:

$$\begin{aligned}
pe_{rs,F} &= \int_1^{\underline{\Phi}_{rs}^{1/(\sigma-1)}} \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}/\varphi}^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi + \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}}^{\infty} \int_1^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi \\
&= \underline{\Phi}_{rs}^{-\frac{\kappa}{\sigma-1}} \left(1 + \frac{\kappa}{\sigma-1} \log \underline{\Phi}_{rs}\right).
\end{aligned}$$

The aggregate productivity and quality level of F-firms producing in region r and selling in region s is obtained as follows:

$$\widetilde{\Phi}_{rs} = \frac{A_{rs}}{pe_{rs,F}} = \frac{\kappa^2 \underline{\Phi}_{rs}}{(\kappa-\sigma+1)^2} \frac{\sigma - 1 + (\kappa-\sigma+1)\log \underline{\Phi}_{rs}}{\sigma - 1 + \kappa \log \underline{\Phi}_{rs}},$$

where

$$\begin{aligned}
A_{rs} &= \int_1^{\underline{\Phi}_{rs}^{1/(\sigma-1)}} \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}/\varphi}^{\infty} \varphi^{\sigma-1}\psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi + \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}}^{\infty} \int_1^{\infty} \varphi^{\sigma-1}\psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\psi \mathrm{d}\varphi \\
&= \frac{\kappa^2}{(\kappa-\sigma+1)^2} \underline{\Phi}_{rs}^{\frac{\sigma-\kappa-1}{\sigma-1}} \left[1 + \frac{\kappa-\sigma+1}{\sigma-1} \log \underline{\Phi}_{rs}\right].
\end{aligned}$$

The expected revenue and profit of F-firms producing in region r and selling in region s are, respectively, obtained as follows:

$$\bar{r}_{rs,F} = \sigma f \frac{\widetilde{\Phi}_{rs}}{\underline{\Phi}_{rs}} = \frac{\kappa^2 \sigma f}{(\kappa-\sigma+1)^2} \frac{\sigma - 1 + (\kappa-\sigma+1)\log \underline{\Phi}_{rs}}{\sigma - 1 + \kappa \log \underline{\Phi}_{rs}}.$$

and

$$\bar{\pi}_{rs,F} = \left[\frac{\kappa^2}{(\kappa-\sigma+1)^2} \frac{\sigma - 1 + (\kappa-\sigma+1)\log \underline{\Phi}_{rs}}{\sigma - 1 + \kappa \log \underline{\Phi}_{rs}} - 1\right] f.$$

Thus, we have

$$\bar{\pi}_{rs,F} \cdot pe_{rs,F} = \underline{\Phi}_{rs}^{-\frac{\kappa}{\sigma-1}} \left[\frac{(\sigma-1)(2\kappa-\sigma+1)}{(\kappa-\sigma+1)^2} + \frac{\kappa}{\kappa-\sigma+1} \log \underline{\Phi}_{rs}\right] f.$$

The free entry condition is expressed as

$$\begin{aligned}
\frac{\mathcal{F}}{f} &= \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \left[\frac{(\sigma-1)(2\kappa-\sigma+1)}{(\kappa-\sigma+1)^2} + \frac{\kappa}{\kappa-\sigma+1} \log \underline{\Phi}_{rr}\right] \\
&\quad + \underline{\Phi}_{rs}^{-\frac{\kappa}{\sigma-1}} \left[\frac{(\sigma-1)(2\kappa-\sigma+1)}{(\kappa-\sigma+1)^2} + \frac{\kappa}{\kappa-\sigma+1} \log \underline{\Phi}_{rs}\right].
\end{aligned} \tag{28}$$

It is readily verified that

$$\frac{\partial \bar{\pi}_{rr,F} \cdot pe_{rr,F}}{\partial \underline{\Phi}_{rr}} = -\frac{\kappa^2 \underline{\Phi}_{rr}^{-\frac{\kappa-\sigma+1}{\sigma-1}}}{(\sigma-1)(\kappa-\sigma+1)} \left(\frac{\sigma-1}{\kappa-\sigma+1} + \log \underline{\Phi}_{rr} \right) < 0$$

Similarly, it is readily verified that

$$\frac{\partial \bar{\pi}_{rs,F} \cdot pe_{rs,F}}{\partial \underline{\Phi}_{rs}} < 0.$$

Furthermore, $\lim_{\underline{\Phi}_{rr} \to 1} \bar{\pi}_{rr,F} \cdot pe_{rr,F} = \lim_{\underline{\Phi}_{rs} \to 1} \bar{\pi}_{rs,F} \cdot pe_{rs,F} = \frac{(\sigma-1)(2\kappa-\sigma+1)}{(\kappa-\sigma+1)^2} f$ holds. Using l'hopital rule yields $\lim_{\underline{\Phi}_{rr} \to \infty} \frac{\log \underline{\Phi}_{rr}^{1/(\sigma-1)}}{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}} = \lim_{\underline{\Phi}_{rr} \to \infty} \left(\frac{1}{\sigma-1} \frac{\sigma-1}{\kappa} \frac{1}{\underline{\Phi}_{rr} \underline{\Phi}_{rr}^{\kappa/(\sigma-1)-1}} \right) = 0$. Thus, $\lim_{\underline{\Phi}_{rr} \to \infty} \bar{\pi}_{rr,F} \cdot pe_{rr,F} = \lim_{\underline{\Phi}_{rs} \to \infty} \bar{\pi}_{rs,F} \cdot pe_{rs,F} = 0$. The first and second terms of the RHS of (28) decrease with $\underline{\Phi}_{rr}$ and $\underline{\Phi}_{rs}$ respectively.

4.2 Online technology utilization

We now focus on the case under $\underline{\Psi}_r^{\sigma-1} < \frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}} \underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$. As shown in Appendix C, the ex-ante probability of successful entry for N-firms producing in region r is obtained as follows:

$$pe_{r,N} = \frac{1}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} - \frac{1}{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}} + \frac{1}{\underline{\Psi}_r^{\kappa}} - \frac{\kappa}{\sigma-1} \left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}} \right)^{\frac{\kappa}{\sigma-1}} B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}}, \frac{\kappa}{\sigma-1}, 0 \right], \quad (29)$$

where $B(z, a, b)$ is an incomplete beta function such that

$$\int_0^z t^{a-1} (1-t)^{b-1} \, \mathrm{d}t.$$

Furthermore, we have

$$\begin{aligned} \widetilde{\Psi}_{r,N} \cdot pe_{r,N} &= \frac{\kappa^2}{(\sigma-\kappa-1)^2} \left(\underline{\Phi}_{rs}^{\frac{\sigma-\kappa-1}{\sigma-1}} - \underline{\Phi}_{rr}^{\frac{\sigma-\kappa-1}{\sigma-1}} + \underline{\Psi}_r^{\sigma-\kappa-1} \right) \\ &\quad -\kappa^2 \frac{(\sigma-1)^{-1}}{\kappa-\sigma+1} \left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}} \right)^{-\frac{\sigma-\kappa-1}{\sigma-1}} B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}}, \frac{\kappa-(\sigma-1)}{\sigma-1}, 0 \right] (30) \end{aligned}$$

The ex-ante probability of successful entry for F-firms producing and selling in region

r is obtained as follows:

$$pe_{rr,F} = \frac{\kappa \log \underline{\Psi}_r}{\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}} + \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} - \underline{\Phi}_{rs}^{-\frac{\kappa}{\sigma-1}} + \frac{\kappa}{\sigma-1}\left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa}{\sigma-1}} B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}, \frac{\kappa}{\sigma-1}, 0\right]. \tag{31}$$

Furthermore, we have:

$$\begin{aligned}\widetilde{\Phi}_{rr} \cdot pe_{rr,F} = &\frac{\kappa^2}{\kappa-\sigma+1}\underline{\Phi}_{rr}^{-\frac{\kappa-\sigma+1}{\sigma-1}} \log \underline{\Psi}_r + \frac{\kappa^2}{(\kappa-\sigma+1)^2}\underline{\Phi}_{rr}^{-\frac{\kappa-\sigma+1}{\sigma-1}} - \frac{\kappa^2}{(\sigma-\kappa-1)^2}\underline{\Phi}_{rs}^{-\frac{\kappa-\sigma+1}{\sigma-1}} \\ &+ \frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa-\sigma+1}{\sigma-1}} B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}, \frac{\kappa-(\sigma-1)}{\sigma-1}, 0\right].\end{aligned} \tag{32}$$

The ex-ante probability of successful entry for F-firms producing in region r and selling in region s is obtained as follows:

$$pe_{rs,F} = \frac{\kappa}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} \log \frac{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r}{\underline{\Phi}_{rr}^{1/(\sigma-1)}} + \frac{\kappa}{\sigma-1}\left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}\right)^{\kappa/(\sigma-1)} B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}, \frac{\kappa}{\sigma-1}, 0\right]. \tag{33}$$

Furthermore, we have:

$$\begin{aligned}\widetilde{\Phi}_{rs} \cdot pe_{rs,F} = &\frac{\kappa^2}{\kappa-\sigma+1}\underline{\Phi}_{rs}^{-\frac{\kappa-\sigma+1}{\sigma-1}} \log \frac{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r}{\underline{\Phi}_{rr}^{1/(\sigma-1)}} \\ &+ \frac{(\sigma-1)^{-1}}{\kappa-\sigma+1}\kappa^2\left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa-\sigma+1}{\sigma-1}} B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}, \frac{\kappa-(\sigma-1)}{\sigma-1}, 0\right].\end{aligned} \tag{34}$$

The zero cutoff profit conditions $\pi_{rr,F}(\underline{\Phi}_{rr}^*) = 0$, $\pi_{rs,F}(\underline{\Phi}_{rs}^*) = 0$ and $\pi_{r,N}(\underline{\Psi}_r^*) = 0$ are, respectively, equivalent to $\pi_{rr,F}(\widetilde{\Phi}_{rr}) = fk_{rr,F}$, $\pi_{rs,F}(\widetilde{\Phi}_{rs}) = fk_{rs,F}$ and $\pi_{r,N}(\widetilde{\Psi}_r) = fk_{r,N}$ where $k_{rr,F} \equiv \widetilde{\Phi}_{rr}/\underline{\Phi}_{rr} - 1$, $k_{rs,F} \equiv \widetilde{\Phi}_{rs}/\underline{\Phi}_{rs} - 1$, and $k_{r,N} \equiv \widetilde{\Psi}_r/\underline{\Psi}_r^{\sigma-1} - 1$. Thus, the free entry condition is rewritten as follows:

$$\frac{\widetilde{\Psi}_{r,N} \cdot pe_{r,N}/\underline{\Psi}_r^{\sigma-1} + \widetilde{\Phi}_{rr} \cdot pe_{rr,F}/\underline{\Phi}_{rr} + \widetilde{\Phi}_{rs} \cdot pe_{rs,F}/\underline{\Phi}_{rs} - pe_{r,N} - pe_{rr,F} - pe_{rs,F}}{f} = \frac{\mathcal{F}}{f}.$$

Using (30), (32) and (34), we obtain:

$$
\begin{aligned}
&\frac{\widetilde{\Psi}_{r,N}\cdot pe_{r,N}/\underline{\Psi}_r^{\sigma-1}+\widetilde{\Phi}_{rr}\cdot pe_{rr,F}/\underline{\Phi}_{rr}+\widetilde{\Phi}_{rs}\cdot pe_{rs,F}/\underline{\Phi}_{rs}}{f}\\
= \quad & \frac{\kappa^2}{(\sigma-\kappa-1)^2}\left(\frac{\underline{\Phi}_{rs}^{(\sigma-\kappa-1)/(\sigma-1)}}{\underline{\Psi}_r^{\sigma-1}}-\frac{\underline{\Phi}_{rr}^{\frac{\sigma-\kappa-1}{\sigma-1}}}{\underline{\Psi}_r^{\sigma-1}}+\underline{\Psi}_r^{-\kappa}\right)\\
&+\frac{\kappa^2}{(\kappa-\sigma+1)^2}\underline{\Phi}_{rr}^{\frac{-\kappa}{\sigma-1}}-\frac{\kappa^2}{(\sigma-\kappa-1)^2}\underline{\Phi}_{rs}^{\frac{-\kappa}{\sigma-1}}\\
&+\frac{\kappa^2}{\kappa-\sigma+1}\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)}\log\underline{\Psi}_r+\frac{\kappa^2}{\kappa-\sigma+1}\underline{\Phi}_{rs}^{-\kappa/(\sigma-1)}\log\frac{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r}{\underline{\Phi}_{rr}^{1/(\sigma-1)}}\\
&+\frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}-\frac{1}{\underline{\Psi}_r^{\sigma-1}}\right)\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa-\sigma+1}{\sigma-1}}B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right]
\end{aligned}
$$

Using (29), (31) and (33), we obtain:

$$
\begin{aligned}
-\frac{pe_{r,N}+pe_{rr,F}+pe_{rs,F}}{f} \quad = \quad & -\frac{1}{\underline{\Psi}_r^{\kappa}}-\frac{\kappa}{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}\log\underline{\Psi}_r-\frac{\kappa}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}}\log\frac{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r}{\underline{\Phi}_{rr}^{1/(\sigma-1)}}\\
&-\frac{\kappa}{\sigma-1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa}{\sigma-1}}B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}},\frac{\kappa}{\sigma-1},0\right].
\end{aligned}
$$

Thus, the free entry condition is expressed as

$$
\frac{\mathcal{F}}{f}=\mathbb{A}_r+\mathbb{B}_r+\mathbb{C}_r,
$$

where

$$
\mathbb{A}_r\equiv\left[\frac{\kappa^2}{(\kappa-\sigma+1)^2}-1\right]\underline{\Psi}_r^{-\kappa}+\frac{\kappa^2}{(\sigma-\kappa-1)^2}\left[\left(\frac{\underline{\Phi}_{rs}}{\underline{\Psi}_r^{\sigma-1}}-1\right)\underline{\Phi}_{rs}^{\frac{-\kappa}{\sigma-1}}-\left(\frac{\underline{\Phi}_{rr}}{\underline{\Psi}_r^{\sigma-1}}-1\right)\underline{\Phi}_{rr}^{\frac{-\kappa}{\sigma-1}}\right], \tag{35}
$$

$$
\mathbb{B}_r\equiv\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)}\log\underline{\Psi}_r+\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\underline{\Phi}_{rs}^{-\kappa/(\sigma-1)}\log\frac{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r}{\underline{\Phi}_{rr}^{1/(\sigma-1)}}, \tag{36}
$$

$$
\begin{aligned}
\mathbb{C}_r\equiv\ & \frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}-\frac{1}{\underline{\Psi}_r^{\sigma-1}}\right)\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa-\sigma+1}{\sigma-1}}B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right]\\
&-\frac{\kappa}{\sigma-1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\kappa/(\sigma-1)}B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}},\frac{\kappa}{\sigma-1},0\right].
\end{aligned} \tag{37}
$$

Furthermore, the part of $\mathbb{E}\varphi$ is expressed as

$$\int\int_{\mathcal{A}_{N,r}} \varphi \nu_{r,N} \mathrm{d}\psi \mathrm{d}\varphi = \frac{X_r}{p e_{r,N}}$$

where

$$\begin{aligned} X_r \equiv & \frac{\kappa}{\kappa-1} \frac{1}{\underline{\Psi}_r^{\kappa}} \left[\left(1 - \frac{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} \right) \left(\frac{\underline{\Psi}_r^{\kappa-1}}{\underline{\Phi}_{rr}^{\frac{\kappa-1}{\sigma-1}}} - 1 \right) + \frac{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} \right] \\ & - \frac{\kappa^2}{(\kappa-1)(\sigma-1)\underline{\Psi}_r} \left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}} \right)^{\frac{\kappa-1}{\sigma-1}} B \left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}}, \frac{\kappa}{\sigma-1}, -\frac{1}{\sigma-1} \right]. \quad (38) \end{aligned}$$

5 Symmetric equilibrium

We focus on the case when $\theta_r = \theta_s$, which means that two regions are symmetric. Thus, using (13) yields

$$\underline{\Phi}_{rs} = \underline{\Phi}_{sr} = \phi^{-1} \underline{\Phi}_{rr}. \quad (39)$$

Therefore, the equilibrium values of $\underline{\Phi}_{rr}$ and $\underline{\Phi}_{rs}$ exist under no online technology if

$$\mathcal{F} < \frac{2(\sigma-1)(2\kappa-\sigma+1)}{(\kappa-\sigma+1)^2} f.$$

Furthermore, using (14) and (40) yields

$$\underline{\Psi}_r = \frac{1}{\mathbb{E}\varphi \left[(1+\phi)\,\theta \right]^{1/(\sigma-1)}} \underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}. \quad (40)$$

Substituting (39) and (40) into (25) yields

$$1 < \mathbb{E}\varphi^{\sigma-1}\theta.$$

Thus, the boundary between N-firms and F-firms selling in home market does not cross with the lower bound of Pareto distribution if $\mathbb{E}\varphi$ and/or θ are large enough.

Substituting (39) into (26), we have

$$\underline{\Phi}_{rr} > \theta(1+\phi)\mathbb{E}\varphi^{\sigma-1} > 1+\phi.$$

In other words, some firms exit from the market if $\underline{\Phi}_{rr}$ is sufficiently large and/or if θ, ϕ and $\mathbb{E}\varphi$ are sufficiently small.

The free entry condition in symmetric equilibrium is expressed as

$$\frac{\mathcal{F}}{f} = \mathbb{A} + \mathbb{B} + \mathbb{C}.$$

Substituting (39) and (40) into (35) yields

$$\mathbb{A} \equiv \left[\frac{\kappa^2}{(\kappa - \sigma + 1)^2} - 1 \right] \mathbb{E}\varphi^{\kappa} \left[(1+\phi)\,\theta \right]^{\kappa/(\sigma-1)} \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} + \frac{\kappa^2}{(\kappa - \sigma + 1)^2} \mathbb{Z}(\phi) \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}, \tag{41}$$

where $\mathbb{Z}(\phi) \equiv [\mathbb{E}\varphi^{\sigma-1} (1+\phi)\,\theta\phi^{-1} - 1]\, \phi^{\frac{\kappa}{\sigma-1}} - [\mathbb{E}\varphi^{\sigma-1} (1+\phi)\,\theta - 1]$.

It is readily verified that $\mathbb{Z}(0) = -\infty$, $\mathbb{Z}(1) = 0$, and

$$\frac{\partial \mathbb{Z}(\phi)}{\partial \phi} = \phi^{-2}\theta \mathbb{E}\varphi^{\sigma-1} + \frac{\phi^{\frac{\kappa}{\sigma-1}-2}}{\sigma - 1} \left\{ \kappa\phi(\theta\mathbb{E}\varphi^{\sigma-1} - 1) + \theta[\kappa - (\sigma-1)]\mathbb{E}\varphi^{\sigma-1} \right\} > 0,$$

because of $\kappa > \sigma - 1$ and $\theta\mathbb{E}\varphi^{\sigma-1} > 1$.

Substituting (39) and (40) into (36) yields

$$\begin{aligned} \mathbb{B} \;\; &\equiv \;\; \frac{\kappa(\sigma-1)}{\kappa-\sigma+1} \underline{\Phi}_{rr}^{-\kappa/(\sigma-1)} \log\left(\frac{1}{\mathbb{E}\varphi\left[(1+\phi)\,\theta\right]^{1/(\sigma-1)}} \underline{\Phi}_{rr}^{\frac{1}{\sigma-1}} \right) \\ &\quad + \frac{\kappa(\sigma-1)}{\kappa-\sigma+1} \phi^{\kappa/(\sigma-1)} \underline{\Phi}_{rr}^{-\kappa/(\sigma-1)} \log\left(\frac{\phi^{-1/(\sigma-1)}}{\mathbb{E}\varphi\left[(1+\phi)\,\theta\right]^{1/(\sigma-1)}} \underline{\Phi}_{rr}^{\frac{1}{\sigma-1}} \right) \\ &> \;\; 0 \end{aligned} \tag{42}$$

Finally, substituting (39) and (40) into (37) yields

$$\begin{aligned} \mathbb{C} \;\; &\equiv \;\; \frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1} \left(1 - \mathbb{E}\varphi^{\sigma-1}\theta\right) (1+\phi)^{\frac{\kappa}{\sigma-1}} \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} B\left[\frac{1}{1+\phi^{-1}}, \frac{\kappa-(\sigma-1)}{\sigma-1}, 0 \right] \\ &\quad - \frac{\kappa}{\sigma-1} (1+\phi)^{\kappa/(\sigma-1)} \underline{\Phi}_{rr}^{-\kappa/(\sigma-1)} B\left[\frac{1}{1+\phi^{-1}}, \frac{\kappa}{\sigma-1}, 0 \right]. \end{aligned} \tag{43}$$

We now show $\mathbb{E}\varphi$. Substituting (39) and (40) into (29) and (38) yields

$$\mathbb{E}\varphi = \frac{X}{pe_N} \tag{44}$$

where

$$pe_N \equiv \phi^{\kappa/(\sigma-1)}\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)} - \underline{\Phi}_{rr}^{-\kappa/(\sigma-1)} + \mathbb{E}\varphi^{\kappa}\left[(1+\phi)\,\theta\right]^{\kappa/(\sigma-1)}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}$$
$$-\frac{\kappa}{\sigma-1}(1+\phi)^{\frac{\kappa}{\sigma-1}}\,\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa}{\sigma-1},0\right], \tag{45}$$

and

$$\begin{aligned} X &\equiv \frac{\kappa}{\kappa-1}\mathbb{E}\varphi\left[(1+\phi)\,\theta\right]^{1/(\sigma-1)}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}\left(1-\phi^{\frac{\kappa}{\sigma-1}}\right)+\frac{\kappa}{\kappa-1}\mathbb{E}\varphi^{\kappa}\left[(1+\phi)\,\theta\right]^{\kappa/(\sigma-1)}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \\ &- \frac{\kappa^2\mathbb{E}\varphi\left[(1+\phi)\,\theta\right]^{1/(\sigma-1)}}{(\kappa-1)(\sigma-1)}(1+\phi)^{\frac{\kappa-1}{\sigma-1}}\,\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},-\frac{1}{\sigma-1}\right]. \end{aligned}$$

Note that $\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}$ is cancelled out from pe_N and X. Thus, $\mathbb{E}\varphi$ does not depend on $\underline{\Phi}_{rr}$ in the symmetric case.

We now show when a unique equilibrium value of $\underline{\Phi}_{rr}$ exists. We obtain

$$\lim_{\underline{\Phi}_{rr}\to\infty}\mathbb{A} = \lim_{\underline{\Phi}_{rr}\to\infty}\mathbb{C} = 0.$$

Using l'hopital rule yields $\lim_{\underline{\Phi}_{rr}\to\infty}\frac{\log\underline{\Phi}_{rr}^{1/(\sigma-1)}}{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}} = \lim_{\underline{\Phi}_{rr}\to\infty}\left(\frac{1}{\sigma-1}\frac{\sigma-1}{\kappa}\frac{1}{\underline{\Phi}_{rr}\underline{\Phi}_{rr}^{\kappa/(\sigma-1)-1}}\right) = 0$. Thus, we obtain $\lim_{\underline{\Phi}_{rr}\to\infty}\mathbb{B}=0$. Consequently, we obtain $\lim_{\underline{\Phi}_{rr}\to\infty}(\mathbb{A}+\mathbb{B}+\mathbb{C})=0$.

Meanwhile, using (41), (42) and (43), we obtain

$$\begin{aligned} \frac{\partial\mathbb{A}}{\partial\underline{\Phi}_{rr}} &= -\frac{\kappa}{\sigma-1}\frac{\mathbb{A}}{\underline{\Phi}_{rr}} \\ \frac{\partial\mathbb{B}}{\partial\underline{\Phi}_{rr}} &= -\frac{\kappa}{\sigma-1}\frac{\mathbb{B}}{\underline{\Phi}_{rr}}+\frac{\kappa}{\kappa-(\sigma-1)}\frac{1+\phi^{\frac{\kappa}{\sigma-1}}}{\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}+1}} \\ \frac{\partial\mathbb{C}}{\partial\underline{\Phi}_{rr}} &= -\frac{\kappa}{\sigma-1}\frac{\mathbb{C}}{\underline{\Phi}_{rr}}. \end{aligned}$$

Thus, we obtain

$$\begin{aligned} \frac{\partial\mathbb{A}}{\partial\underline{\Phi}_{rr}}+\frac{\partial\mathbb{B}}{\partial\underline{\Phi}_{rr}}+\frac{\partial\mathbb{C}}{\partial\underline{\Phi}_{rr}} &= -\frac{\kappa}{\sigma-1}\frac{1}{\underline{\Phi}_{rr}}\left[\mathbb{A}+\mathbb{B}+\mathbb{C}-\frac{\sigma-1}{\kappa-(\sigma-1)}\frac{1+\phi^{\frac{\kappa}{\sigma-1}}}{\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}\right] \\ &= -\frac{\kappa}{\sigma-1}\frac{1}{\underline{\Phi}_{rr}}\left[\frac{\mathcal{F}}{f}-\frac{\sigma-1}{\kappa-(\sigma-1)}\frac{1+\phi^{\frac{\kappa}{\sigma-1}}}{\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}\right]. \end{aligned}$$

Since $\underline{\Phi}_{rr}>1$, if

$$\frac{\mathcal{F}}{f} > \frac{\sigma-1}{\kappa-\sigma+1}\left[1+\phi^{\kappa/(\sigma-1)}\right],$$

we have

$$\frac{\partial \mathbb{A}}{\partial \underline{\Phi}_{rr}} + \frac{\partial \mathbb{B}}{\partial \underline{\Phi}_{rr}} + \frac{\partial \mathbb{C}}{\partial \underline{\Phi}_{rr}} < 0.$$

Otherwise, the impact of $\underline{\Phi}_{rr}$ on $\mathbb{A}+\mathbb{B}+\mathbb{C}$ is ambiguous.

We focus on the limit of $\mathbb{A}+\mathbb{B}+\mathbb{C}$ when $\underline{\Phi}_{rr}$ approaches $\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}$ since $\underline{\Phi}_{rr} > \theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}$. Using (41), (42) and (43), we obtain

$$\begin{aligned}
\lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}} \mathbb{A} =& \frac{(\sigma-1)[2\kappa-(\sigma-1)]}{(\kappa-\sigma+1)^2} \\
&+ \frac{\kappa^2}{(\kappa-\sigma+1)^2}\frac{\mathbb{E}\varphi^{\sigma-1}(1+\phi)\,\theta\left(\phi^{\frac{\kappa-\sigma+1}{\sigma-1}}-1\right)-\phi^{\frac{\kappa}{\sigma-1}}+1}{\theta^{\frac{\kappa}{\sigma-1}}(1+\phi)^{\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{\kappa}}, \\
\lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}} \mathbb{B} =& \frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\phi^{\kappa/(\sigma-1)}\theta^{-\frac{\kappa}{\sigma-1}}(1+\phi)^{-\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{-\kappa}\log\left(\phi^{-1/(\sigma-1)}\right), \\
\lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}} \mathbb{C} =& -\frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\mathbb{E}\varphi^{\sigma-1}\theta-1\right)\theta^{-\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{-\kappa}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right] \\
&-\frac{\kappa}{\sigma-1}\theta^{-\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{-\kappa}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa}{\sigma-1},0\right].
\end{aligned}$$

Accordingly, we obtain

$$\begin{aligned}
&\lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}}(\mathbb{A}+\mathbb{B}+\mathbb{C}) \\
=\;& \frac{\kappa^2}{(\kappa-\sigma+1)^2}-1-\frac{\kappa}{\sigma-1}\theta^{-\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{-\kappa}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa}{\sigma-1},0\right] \\
&+\frac{\kappa^2}{(\kappa-\sigma+1)^2}\frac{\mathbb{E}\varphi^{\sigma-1}(1+\phi)\,\theta\left(\phi^{\frac{\kappa-\sigma+1}{\sigma-1}}-1\right)-\phi^{\frac{\kappa}{\sigma-1}}+1}{\theta^{\frac{\kappa}{\sigma-1}}(1+\phi)^{\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{\kappa}}, \\
&+\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\phi^{\kappa/(\sigma-1)}\theta^{-\frac{\kappa}{\sigma-1}}(1+\phi)^{-\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{-\kappa}\log\left(\phi^{-1/(\sigma-1)}\right) \\
&-\frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\mathbb{E}\varphi^{\sigma-1}\theta-1\right)\theta^{-\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{-\kappa}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right].
\end{aligned} \tag{46}$$

Note that, if $\frac{\mathcal{F}}{f} > \frac{\sigma-1}{\kappa-\sigma+1}\left[1+\phi^{\kappa/(\sigma-1)}\right]$ holds, we obtain $\lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}}(\mathbb{A}+\mathbb{B}+\mathbb{C}) > 0$ because $\lim_{\underline{\Phi}_{rr}\to\infty}(\mathbb{A}+\mathbb{B}+\mathbb{C}) = 0$.

Since $\mathbb{A}+\mathbb{B}+\mathbb{C}$ is a continuous function of $\underline{\Phi}_{rr}$ in interval $(\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1},\infty)$, a unique equilibrium exists if

$$\frac{\sigma-1}{\kappa-\sigma+1}\left(1+\phi^{\frac{\kappa}{\sigma-1}}\right) < \frac{\mathcal{F}}{f} < \lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}}(\mathbb{A}+\mathbb{B}+\mathbb{C}). \tag{47}$$

Otherwise, ambiguity remains. For the existence of the interval in (47), it is required that

$$\frac{\sigma-1}{\kappa-\sigma+1}\left(1+\phi^{\frac{\kappa}{\sigma-1}}\right) < \lim_{\underline{\Phi}_{rr}\to\theta(1+\phi)\mathbb{E}\varphi^{\sigma-1}} \left(\mathbb{A}+\mathbb{B}+\mathbb{C}\right),$$

which is equivalent to

$$\begin{aligned}
&\left(\frac{\kappa}{\kappa-\sigma+1}-\phi^{\frac{\kappa}{\sigma-1}}\right)\frac{\sigma-1}{\kappa-\sigma+1}\theta^{\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{\kappa}+\frac{\kappa^2}{(\kappa-\sigma+1)^2}\frac{(1-\phi^{\frac{\kappa}{\sigma-1}})}{(1+\phi)^{\frac{\kappa}{\sigma-1}}}\\
&+\frac{\kappa}{\kappa-\sigma+1}\frac{\phi^{\kappa/(\sigma-1)}}{(1+\phi)^{\frac{\kappa}{\sigma-1}}}\log\left(\phi^{-1}\right)\\
&-\left\{\frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right]+\frac{\kappa^2}{(\kappa-\sigma+1)^2}\frac{(1-\phi^{\frac{\kappa}{\sigma-1}})}{(1+\phi)^{\frac{\kappa}{\sigma-1}}}\left(\frac{1+\phi}{\phi}\right)\right\}\mathbb{E}\varphi^{\sigma-1}\theta\\
&+\frac{\kappa}{\sigma-1}\left\{\frac{\kappa}{\kappa-\sigma+1}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right]-B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa}{\sigma-1},0\right]\right\}>0 \qquad (48)
\end{aligned}$$

by using (46). The terms in the curly bracket of the last term are rewritten as

$$\begin{aligned}
&\frac{\kappa}{\kappa-\sigma+1}\int_0^{\frac{1}{1+\phi^{-1}}} t^{\frac{\kappa}{\sigma-1}-2}(1-t)^{-1}\,\mathrm{d}t-\int_0^{\frac{1}{1+\phi^{-1}}} t^{\frac{\kappa}{\sigma-1}-1}(1-t)^{-1}\,\mathrm{d}t\\
=&\int_0^{\frac{1}{1+\phi^{-1}}} t^{\frac{\kappa}{\sigma-1}-1}(1-t)^{-1}\left[\frac{\kappa}{\kappa-\sigma+1}t^{-1}-1\right]\mathrm{d}t>0.
\end{aligned}$$

Thus, the last term is positive. This means that if $\mathbb{E}\varphi=0$, the function is positive.

Furthermore, taking the derivative of the RHS of (48) with respect to $\mathbb{E}\varphi$ yields

$$\begin{aligned}
&\left(\frac{\kappa}{\kappa-\sigma+1}-\phi^{\frac{\kappa}{\sigma-1}}\right)\theta^{\frac{\kappa}{\sigma-1}-1}\mathbb{E}\varphi^{\kappa-\sigma+1}\\
\gtreqless\ &\left\{\frac{\kappa}{\sigma-1}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right]+\frac{\kappa}{\kappa-\sigma+1}\frac{(1-\phi^{\frac{\kappa}{\sigma-1}})}{(1+\phi)^{\frac{\kappa-\sigma+1}{\sigma-1}}}\left(1-\phi^{\frac{\kappa-\sigma+1}{\sigma-1}}\right)\right\}.
\end{aligned}$$

$\Leftrightarrow$

$$\mathbb{E}\varphi^{\kappa-\sigma+1}\gtreqless\frac{\frac{\kappa}{\sigma-1}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-\sigma+1}{\sigma-1},0\right]+\frac{\kappa}{\kappa-\sigma+1}\frac{(1-\phi^{\frac{\kappa}{\sigma-1}})}{(1+\phi)^{\frac{\kappa-\sigma+1}{\sigma-1}}}\left(1-\phi^{\frac{\kappa-\sigma+1}{\sigma-1}}\right)}{\left(\frac{\kappa}{\kappa-\sigma+1}-\phi^{\frac{\kappa}{\sigma-1}}\right)\theta^{\frac{\kappa-\sigma+1}{\sigma-1}}}\equiv\widetilde{\mathbb{E}\varphi}$$

Thus, the function decreases from a positive value until $\widetilde{\mathbb{E}\varphi}$ and then increases as $\mathbb{E}\varphi$ increases. As θ increases, $\widetilde{\mathbb{E}\varphi}$ becomes smaller. Furthermore, since $\kappa>\sigma-1$, the function becomes positive when $\mathbb{E}\varphi$ approaches to infinity. Thus, the interval (47) exists if $\mathbb{E}\varphi$ is sufficiently large. Otherwise, ambiguity remains.

We now examine $\mathbb{E}\varphi$, using $\mathbb{E}\varphi=X/pe_N$. Rewriting (44), we obtain

$$\left(\mathbb{E}\varphi - \frac{\kappa}{\kappa-1}\right)\mathbb{E}\varphi^{\kappa-1}\left[(1+\phi)\,\theta\right]^{\kappa/(\sigma-1)} = J$$

where

$$\begin{aligned} J \;\equiv\;& \left\{\frac{\kappa}{\kappa-1}\left[(1+\phi)\,\theta\right]^{\frac{1}{\sigma-1}} + 1\right\}\left(1-\phi^{\frac{\kappa}{\sigma-1}}\right) + \frac{\kappa}{\sigma-1}\,(1+\phi)^{\frac{\kappa}{\sigma-1}} \\ & \times\left\{B\left[\frac{\phi}{1+\phi}, \frac{\kappa}{\sigma-1}, 0\right] - \frac{\kappa}{\kappa-1}\theta^{\frac{1}{\sigma-1}}B\left[\frac{\phi}{1+\phi}, \frac{\kappa}{\sigma-1}, -\frac{1}{\sigma-1}\right]\right\}. \end{aligned} \tag{49}$$

It is readily verified that we obtain

$$\frac{\partial(\left(\mathbb{E}\varphi - \frac{\kappa}{\kappa-1}\right)\mathbb{E}\varphi^{\kappa-1})}{\partial\mathbb{E}\varphi} = \kappa\left(\mathbb{E}\varphi - 1\right)\mathbb{E}\varphi^{\kappa-2}.$$

Since $1 < \mathbb{E}\varphi^{\sigma-1}\theta$, we have $\mathbb{E}\varphi > \theta^{-\frac{1}{\sigma-1}} > 1$. As a result, $\left(\mathbb{E}\varphi - \frac{\kappa}{\kappa-1}\right)\mathbb{E}\varphi^{\kappa-1}$ increases with $\mathbb{E}\varphi \in (1,\infty)$. Meanwhile, we have $\lim_{\mathbb{E}\varphi\to\infty}\left(\mathbb{E}\varphi - \frac{\kappa}{\kappa-1}\right)\mathbb{E}\varphi^{\kappa-2} = \infty$. Thus, an equilibrium value of $\mathbb{E}\varphi$ exists if

$$\lim_{\mathbb{E}\varphi\to\theta^{-1/(\sigma-1)}}\left(\mathbb{E}\varphi - \frac{\kappa}{\kappa-1}\right)\mathbb{E}\varphi^{\kappa-2}\left[(1+\phi)\,\theta\right]^{\kappa/(\sigma-1)} < J, \tag{50}$$

which is equivalent to $\left(\theta^{1/(\sigma-1)} - \frac{\kappa}{\kappa-1}\theta^{2/(\sigma-1)}\right)(1+\phi)^{\kappa/(\sigma-1)} < J$. If θ approaches 0, the LHS of (50) becomes zero, and J becomes positive; thus, (50) holds. Using the derivative of the LHS of (50) with respect to θ yields

$$\frac{1}{\sigma-1}\theta^{1/(\sigma-1)-1} - \frac{2}{\sigma-1}\frac{\kappa}{\kappa-1}\theta^{2/(\sigma-1)-1} = \frac{1}{\sigma-1}\theta^{1/(\sigma-1)-1}\left(1 - 2\frac{\kappa}{\kappa-1}\theta^{1/(\sigma-1)}\right),$$

which is negative if and only if

$$\frac{1}{2}\frac{\kappa-1}{\kappa} < \theta^{1/(\sigma-1)}. \tag{51}$$

If (51) holds, the LHS of (50) decreases. Using (49) yields

$$\frac{\partial J}{\partial\theta} > 0 \Leftrightarrow (1+\phi)^{\frac{1-\kappa}{\sigma-1}}\left(1-\phi^{\frac{\kappa}{\sigma-1}}\right) > \frac{\kappa}{\sigma-1}\frac{\kappa}{\kappa-1}B\left[\frac{\phi}{1+\phi}, \frac{\kappa}{\sigma-1}, -\frac{1}{\sigma-1}\right]. \tag{52}$$

We find that (52) holds with $\phi = 0$ but not $\phi = 1$. Furthermore, the LHS of (52) decreases and the RHS increases with ϕ. Thus, (52) is satisfied with low ϕ. That is, if (51) and (52) hold, J increases with θ. Thus, (51) and (52) are sufficient conditions for (50). If

(51) does not hold, (50) is satisfied if θ approaches to 0. Otherwise, ambiguity remains.

Summarizing the results, we have the following proposition:

Proposition 2 *A unique equilibrium value of the expected quality for online firms, $\mathbb{E}\varphi$, exists if transaction costs between consumers and online firms, θ, are sufficiently low as (51) shows, and if transport costs, ϕ, are sufficiently high as (52) shows. The unique equilibrium exists if $\mathcal{F}/f$ takes an intermediate value, as (47) shows, and if $\mathbb{E}\varphi$ is sufficiently large.*

5.1 Impact of online technology

We next compare welfare with and without online technology at symmetric equilibrium. Substituting (40) into the RHS of (28) yields

$$\begin{aligned}&\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)}\log\left(\frac{1}{\mathbb{E}\varphi\left[(1+\phi)\theta\right]^{1/(\sigma-1)}}\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}\right)\\&+\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\phi^{\kappa/(\sigma-1)}\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)}\log\left(\frac{\phi^{-1/(\sigma-1)}}{\mathbb{E}\varphi\left[(1+\phi)\theta\right]^{1/(\sigma-1)}}\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}\right)\equiv\mathbb{B}_{No}.\end{aligned}$$

Using $\sigma = 4$, $\kappa = 4.25$ and $f_x = 0.545$ from Melitz and Redding (2015), $\phi = 0.7$ form Head and Mayer (2004) and $\theta = 0.9$, and normalizing $\mathcal{F}$ to 1, we obtain $\mathbb{E}\varphi = 1.42$. The value of $\underline{\Phi}_{rr}$ with online technology is obtained as $\underline{\Phi}_{rr} = 11.41$ by solving $\mathbb{A}+\mathbb{B}+\mathbb{C} = \mathcal{F}/f$. Meanwhile, the value of $\underline{\Phi}_{rr}$ without online technology is obtained as $\underline{\Phi}_{rr} = 6.88$ by solving $\mathbb{B}_{No} = \mathcal{F}/f$. That is, we compare the cases with and without online technology under the same parameter values. Numerical analysis shows that *the zero cutoff product of productivity and quality level of F-firms selling in home market, $\underline{\Phi}_{rr}$, with online technology is larger than that without online technology.* This is because the expected profit with online technology increases compared with those without online technology. This difference of $\underline{\Phi}_{rr}$ means that the price index with online technology is lower than that without online technology. Furthermore, the difference implies that *the welfare with online technology is higher than the welfare without online technology.* This result is consistent with the empirical findings of Jo, Matsumura, and Weinstein (2022).

5.2 Impact of lower transport and transaction costs

We examine the impacts of two costs with online technology under symmetric equilibrium. We use the same parameter values as in the previous subsection. Accordingly, we use the equilibrium values obtained in the previous subsection, which are $\mathbb{E}\varphi = 1.42$ and $\underline{\Phi}_{rr} = 11.41$.

5.2.1 Decrease in transport costs

We examine the impact of ϕ on $\mathbb{E}\varphi$. Using (49) yields

$$\begin{aligned}\frac{J}{(1+\phi)^{\frac{\kappa}{\sigma-1}}} &= \frac{\kappa}{\kappa-1}\theta^{\frac{1}{\sigma-1}}(1+\phi)^{\frac{1-\kappa}{\sigma-1}}\left(1-\phi^{\frac{\kappa}{\sigma-1}}\right)+\frac{1..-\phi^{\frac{\kappa}{\sigma-1}}}{(1+\phi)^{\frac{\kappa}{\sigma-1}}}+\frac{\kappa}{\sigma-1} \\ &\quad\times\left\{B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},0\right]-\frac{\kappa}{\kappa-1}\theta^{\frac{1}{\sigma-1}}B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},-\frac{1}{\sigma-1}\right]\right\}.\end{aligned}$$

Thus, we obtain

$$\begin{aligned}\frac{\partial\left(\frac{J}{(1+\phi)^{\frac{\kappa}{\sigma-1}}}\right)}{\partial\phi} &= -\frac{\kappa}{\sigma-1}\theta^{\frac{1}{\sigma-1}}(1+\phi)^{\frac{1-\kappa}{\sigma-1}}\left[\frac{1-\phi^{\frac{\kappa}{\sigma-1}}}{1+\phi}+\frac{1-\phi^{\frac{\kappa}{\sigma-1}}}{(1+\phi)^{\frac{\sigma}{\sigma-1}}\theta^{\frac{1}{\sigma-1}}}+\frac{2k}{\kappa-1}\phi^{\frac{\kappa-(\sigma-1)}{\sigma-1}}\right] \\ &< 0.\end{aligned}$$

Since $\left(\mathbb{E}\varphi-\frac{\kappa}{\kappa-1}\right)\mathbb{E}\varphi^{\kappa-1}$ increases with $\mathbb{E}\varphi$, the implicit function theorem provides

$$\frac{\partial\mathbb{E}\varphi}{\partial\phi}<0.$$

In other words, *the expected quality of N-firms decreases when transport costs are lower.* Using numerical analysis, we have $\partial\mathbb{E}\varphi/\partial\phi = -0.59 < 0$.

We then examine the impact of ϕ on $\underline{\Phi}_{rr}$. Using (41), (42) and (43) yields

$$\begin{aligned}&\left(\frac{\partial\mathbb{A}}{\partial\mathbb{E}\varphi}+\frac{\partial\mathbb{B}}{\partial\mathbb{E}\varphi}+\frac{\partial\mathbb{C}}{\partial\mathbb{E}\varphi}\right)\frac{\mathbb{E}\varphi\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}{\kappa} \\ =\ &\left[\frac{\kappa^2}{(\kappa-\sigma+1)^2}-1\right]\mathbb{E}\varphi^{\kappa}\left[(1+\phi)\,\theta\right]^{\kappa/(\sigma-1)}-\frac{\sigma-1}{\kappa-\sigma+1}(1+\phi^{\frac{\kappa}{\sigma-1}}) \\ &-\frac{\kappa\theta(1+\phi)\,\mathbb{E}\varphi^{\sigma-1}}{\kappa-\sigma+1}\left\{\frac{\sigma-1}{\kappa-\sigma+1}\left(1-\phi^{\frac{\kappa}{\sigma-1}-1}\right)+(1+\phi)^{\frac{\kappa-\sigma+1}{\sigma-1}}B\left[\frac{\phi}{1+\phi},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right]\right\}.\end{aligned}$$

and

$$
\begin{aligned}
& \frac{\partial \mathbb{A}}{\partial \phi}+\frac{\partial \mathbb{B}}{\partial \phi}+\frac{\partial \mathbb{C}}{\partial \phi} \\
= & \frac{\kappa}{\sigma-1}\left[\frac{\kappa^2}{(\kappa-\sigma+1)^2}-1\right] \mathbb{E}\varphi^\kappa(1+\phi)^{\frac{\kappa}{\sigma-1}-1} \theta^{\frac{\kappa}{\sigma-1}} \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \\
& +\frac{\kappa^3 \phi^{\frac{\kappa}{\sigma-1}-2}(1+\phi)}{(\sigma-1)(\kappa-\sigma+1)^2}\left(\theta \mathbb{E}\varphi^{\sigma-1}-\frac{\phi}{1+\phi}\right) \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}-\frac{\kappa^2 \theta \mathbb{E}\varphi^{\sigma-1}}{(\kappa-\sigma+1)^2}\left(1+\phi^{\frac{\kappa}{\sigma-1}-2}\right) \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \\
& -\frac{\kappa(1+\phi^{\frac{\kappa}{\sigma-1}})}{(\kappa-\sigma+1)(1+\phi) \underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}-\frac{\kappa \phi^{\frac{\kappa}{\sigma-1}}}{\phi(\kappa-\sigma+1) \underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}\left\{1+\frac{\kappa}{\sigma-1} \log \left[\frac{\theta \phi(1+\phi) \mathbb{E}\varphi^{\sigma-1}}{\underline{\Phi}_{rr}}\right]\right\} \\
& +\frac{\kappa \mathbb{C}}{(\sigma-1)(1+\phi)}-\frac{\kappa[\theta \kappa(1+\phi) \mathbb{E}\varphi^{\sigma-1}-\kappa-\phi(\sigma-1)]}{\phi(\sigma-1)(\kappa-\sigma+1)} \phi^{\frac{\kappa-(\sigma-1)}{\sigma-1}} \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} .
\end{aligned}
$$

Thus, numerical analysis provides $\mathrm{d}\underline{\Phi}_{rr}/\mathrm{d}\phi = -0.51 < 0$. That is, *as transport costs decrease, the zero cutoff product of productivity and quality level of F-firms selling in home market and welfare both decrease*. This differs from the result obtained in Melitz (2003) because online and offline firms coexist in the same industry in our model.

We further examine the impact of ϕ on $\underline{\Psi}_r$. Using (40) yields

$$
\frac{\mathrm{d}\underline{\Psi}_r}{\mathrm{d}\phi} = -\frac{\underline{\Psi}_r}{\mathbb{E}\varphi}\frac{\partial \mathbb{E}\varphi}{\partial \phi} + \frac{1}{\sigma-1}\frac{\underline{\Psi}_r}{\underline{\Phi}_{rr}}\frac{\partial \underline{\Phi}_{rr}}{\partial \phi} - \frac{1}{\sigma-1}\frac{\underline{\Psi}_r}{1+\phi}.
$$

Thus, numerical analysis yields $\mathrm{d}\underline{\Psi}_r/\mathrm{d}\phi = 0.28 > 0$. The zero cutoff productivity level of N-firms increases as transport costs decrease, indicating that the horizontal line in Figure 1 increases. This means that it becomes *more difficult for some N-firms to survive with lower transport costs*. Note that the impact of transport costs on $\underline{\Psi}_r$ is the same as that on the cutoff productivity of domestic firms in Melitz (2003).

Since $\underline{\Phi}_{rr}$ decreases and $\underline{\Psi}_r$ increases with a decrease of transport costs, the boundary between N-firms and F-firms selling in home market shifts leftward in Figure 1. This suggests that *the competition between N- and F-firms becomes tougher for N-firms as transport costs decrease.*

Finally, we examine the impact of ϕ on pe_N. Using (45) yields

$$
\begin{aligned}
\frac{\mathrm{d}pe_N}{\mathrm{d}\phi} = & \kappa \mathbb{E}\varphi^{\kappa-1}\left[(1+\phi)\theta\right]^{\kappa/(\sigma-1)} \underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \frac{\partial \mathbb{E}\varphi}{\partial \phi} - \frac{\kappa}{\sigma-1}\frac{pe_N}{\underline{\Phi}_{rr}}\frac{\partial \underline{\Phi}_{rr}}{\partial \phi} \\
& +\frac{\kappa}{\sigma-1}\mathbb{E}\varphi^\kappa (1+\phi)^{\kappa/(\sigma-1)-1}\theta^{\kappa/(\sigma-1)}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \\
& -\left(\frac{\kappa}{\sigma-1}\right)^2 (1+\phi)^{\frac{\kappa}{\sigma-1}-1} \underline{\Phi}_{rr}^{-\kappa/(\sigma-1)} B\left[\frac{1}{1+\phi^{-1}}, \frac{\kappa}{\sigma-1}, 0\right].
\end{aligned}
$$

Thus, numerical analysis yields $\mathrm{d}pe_N/\mathrm{d}\phi = -0.24 < 0$. In other words, the ex-ante probability of choosing to be an N-firm decreases as transport costs decreases. This is because lower transport costs leads to tougher competition for N-firms.

5.2.2 Decrease in transaction costs

We next examine the impact of θ on $\mathbb{E}\varphi$. Using (49) yields

$$\begin{aligned}\frac{J}{\theta^{\frac{\kappa}{\sigma-1}}} &= \frac{\kappa}{\kappa-1}\theta^{\frac{1-\kappa}{\sigma-1}}(1+\phi)^{\frac{1}{\sigma-1}}\left(1-\phi^{\frac{\kappa}{\sigma-1}}\right)+\frac{1-\phi^{\frac{\kappa}{\sigma-1}}}{\theta^{\frac{\kappa}{\sigma-1}}}+\frac{\kappa(1+\phi)^{\frac{\kappa}{\sigma-1}}}{(\sigma-1)\theta^{\frac{\kappa}{\sigma-1}}} \\ &\quad\times\left\{B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},0\right]-\frac{\kappa\theta^{\frac{1}{\sigma-1}}}{\kappa-1}B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},-\frac{1}{\sigma-1}\right]\right\}.\end{aligned}$$

Thus, we obtain

$$\begin{aligned}&\frac{\partial\left(\frac{J}{\theta^{\kappa/(\sigma-1)}}\right)}{\partial\theta}=-\frac{\kappa(1-\phi^{\frac{\kappa}{\sigma-1}})}{(\sigma-1)\theta^{\frac{\kappa+\sigma-1}{\sigma-1}}}\left\{1+[\theta(1+\phi)]^{\frac{1}{\sigma-1}}\right\} \\ &-\frac{\kappa^2(1+\phi)^{\frac{\kappa}{\sigma-1}}}{(\sigma-1)^2\theta^{\frac{\kappa+\sigma-1}{\sigma-1}}}\left\{B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},0\right]-\theta^{\frac{1}{\sigma-1}}B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},-\frac{1}{\sigma-1}\right]\right\}. \qquad (53)\end{aligned}$$

Examining the last term in the RHS of (53), we obtain

$$B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},0\right]-\theta^{\frac{1}{\sigma-1}}B\left[\frac{\phi}{1+\phi},\frac{\kappa}{\sigma-1},-\frac{1}{\sigma-1}\right]=\int_0^{\frac{\phi}{1+\phi}}\left\{\frac{t^{\frac{\kappa}{\sigma-1}-1}}{1-t}\left[1-\theta^{\frac{1}{\sigma-1}}(1-t)^{-\frac{1}{\sigma-1}}\right]\right\}dt.$$

Since $1>\theta^{\frac{1}{\sigma-1}}(1-t)^{-\frac{1}{\sigma-1}}\Leftrightarrow 1-\theta>t$ and $t\in[0,\phi/(1+\phi)]$, we have

$$1-\theta>t>\frac{\phi}{1+\phi}\Rightarrow\frac{1}{1+\phi}>\theta.$$

Thus, if $1/(1+\phi)>\theta$ holds, we have $\partial\left(J/\theta^{\kappa/(\sigma-1)}\right)/\partial\theta<0$, which implies that

$$\partial\mathbb{E}\varphi/\partial\theta<0.$$

In other words, *the expected quality of N-firms decrease with a decrease in transaction costs if transaction or transport costs are sufficiently high.* Otherwise, the sign of $\partial\mathbb{E}\varphi/\partial\theta$ is ambiguous. The numerical analysis shows $\partial\mathbb{E}\varphi/\partial\theta=-0.38<0$.

We then examine the impact of θ on $\underline{\Phi}_{rr}$. Using (41), (42) and (43), we obtain

$$\begin{aligned}
\frac{\partial \mathbb{A}}{\partial \theta}+\frac{\partial \mathbb{B}}{\partial \theta}+\frac{\partial \mathbb{C}}{\partial \theta} &= \frac{\kappa}{(\sigma-1)}\left[\frac{\kappa^2}{(\kappa-\sigma+1)^2}-1\right]\mathbb{E}\varphi^{\kappa}(1+\phi)^{\frac{\kappa}{\sigma-1}}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}\theta^{\frac{\kappa}{\sigma-1}-1} \\
&\quad -\frac{\kappa^2}{(\kappa-\sigma+1)^2}\mathbb{E}\varphi^{\sigma-1}(1+\phi)\left(1-\phi^{\frac{\kappa}{\sigma-1}-1}\right)\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \\
&\quad -\frac{\kappa}{\kappa-\sigma+1}\frac{1+\phi^{\frac{\kappa}{\sigma-1}}}{\theta}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}} \\
&\quad -\frac{\kappa^2\mathbb{E}\varphi^{\sigma-1}(1+\phi)^{\frac{\kappa}{\sigma-1}}}{(\sigma-1)(\kappa-\sigma+1)}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}B\left[\frac{1}{1+\phi^{-1}},\frac{\kappa-(\sigma-1)}{\sigma-1},0\right].
\end{aligned}$$

Thus, numerical analysis provides $\mathrm{d}\underline{\Phi}_{rr}/\mathrm{d}\theta = 3.91 > 0$. That is, as transaction costs decrease, the zero cutoff productivity and quality level of F-firm selling in home market increases, implying that *the welfare increases with lower transaction costs.*

We further examine the impact of θ on $\underline{\Psi}_r$. Using (40) yields

$$\frac{\mathrm{d}\underline{\Psi}_r}{\mathrm{d}\theta} = -\frac{\underline{\Psi}_r}{\mathbb{E}\varphi}\frac{\partial \mathbb{E}\varphi}{\partial \theta}+\frac{1}{\sigma-1}\frac{\underline{\Psi}_r}{\underline{\Phi}_{rr}}\frac{\partial \underline{\Phi}_{rr}}{\partial \theta}-\frac{1}{\sigma-1}\frac{\underline{\Psi}_r}{\theta}.$$

Thus, numerical analysis yields $\mathrm{d}\underline{\Psi}_r/\mathrm{d}\theta = 0.01 > 0$. In other words, the zero cutoff productivity level of N-firm becomes larger as transaction costs decreases, implying that the horizontal line in Figure 1 increases. This implies that *survival becomes more difficult for some N-firms as transaction costs decrease.*

Since

$$\begin{aligned}
\frac{\partial \varphi_{r1}}{\partial \theta} &= \frac{1}{\sigma-1}\frac{\varphi_{r1}}{\underline{\Phi}_{rr}}\frac{\mathrm{d}\underline{\Phi}_{rr}}{\mathrm{d}\theta}-\frac{\varphi_{r1}}{\underline{\Psi}_r}\frac{\mathrm{d}\underline{\Psi}_r}{\mathrm{d}\theta} \\
&= \varphi_{r1}\left(\frac{1}{3}\frac{3.91}{11.41}-\frac{0.01}{1.37}\right) \\
&= 0.107\times\varphi_{r1},
\end{aligned}$$

the boundary between N-firms and F-firms selling in home market shifts rightward, which means that *the competition between N-firms and F-firms becomes milder for N-firms as transaction costs decrease.*

Finally, we examine the impact of θ on pe_N. Using (45) yields

$$\begin{aligned}\frac{\mathrm{d}pe_N}{\mathrm{d}\theta} &= \kappa\mathbb{E}\varphi^{\kappa-1}\left[(1+\phi)\,\theta\right]^{\kappa/(\sigma-1)}\underline{\Phi}_{rr}^{-\frac{\kappa}{\sigma-1}}\frac{\partial\mathbb{E}\varphi}{\partial\theta} \\ &\quad -\frac{\kappa}{\sigma-1}\underline{\Phi}_{rr}^{-\kappa/(\sigma-1)}pe_N\frac{\partial\underline{\Phi}_{rr}}{\partial\theta}+\frac{\kappa\theta^{\frac{\kappa-(\sigma-1)}{\sigma-1}}(1+\phi)^{\frac{\kappa}{\sigma-1}}\mathbb{E}\varphi^{\kappa}}{(\sigma-1)\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}.\end{aligned}$$

Thus, numerical analysis yields $\mathrm{d}pe_N/\mathrm{d}\theta = 0.01 > 0$. In other words, the ex-ante probability of choosing to be N-firms increases as transaction costs decrease.

6 Conclusion

We show that online technology improves welfare with numerical analysis. More precisely, the case with online technology provides higher welfare than that without online technology. A decrease in transaction costs between online firms and consumers improves welfare and increases the ex-ante probability of the successful entry of online firms. Transport costs have the opposite impacts on them. Specifically, our result differ from that of Melitz (2003) on transport costs. This is because our model includes competition between online and offline firms. Consequently, the zero-cutoff product of productivity and quality for offline firms selling in home market are not the main boundary for firms' exit. However, welfare is still evaluated by the size of the zero-cutoff product of productivity and quality for offline firms selling in home market because all online firms sell in two markets. As a policy implication, investments that lower transaction costs in the online market are more prefered than ones that lower firms' transport costs.

Finally, further studies are needed to clarify whether the impact of transport costs on welfare changes qualitatively by the emergence of online market. As a future research direction, an analysis of the hierarchy of online market platforms, which is not included in this paper, still needs to be conducted.

Appendix A

Assuming (12), we now determine the conditions under which firm (ψ, φ) in region r choose to exit, to be an online firm, to be a single-market offline firm, or to be an exporting offline firm, respectively.

A.1 No online technology

(i) Inactive firms

Firm (ψ, φ) in region r exits immediately when its profit from being an offline firm is negative. Accordingly, solving $\pi_{rr}^F(\psi, \varphi) < 0$ with (16) and $\pi_{rs}^F(\psi, \varphi) < 0$ with (17), we obtain (18).

(ii) Single-market offline firms

Firm (ψ, φ) in region r chooses to be an offline firm selling in home market if and only if $\pi_{rr}^F(\psi, \varphi) > 0$ and $\pi_{rs}^F(\psi, \varphi) < 0$ hold. Solving $\pi_{rr}^F(\psi, \varphi) > 0$ with (16) and $\pi_{rs}^F(\psi, \varphi) < 0$ with (17), we obtain (19).

(iii) Exporting offline firms

Firm (ψ, φ) in region r chooses to be an offline firm selling in both home and foreign markets if and only if $\pi_{rr}^F(\psi, \varphi) > 0$ and $\pi_{rs}^F(\psi, \varphi) > 0$ hold. Solving $\pi_{rr}^F(\psi, \varphi) > 0$ with (16) and $\pi_{rs}^F(\psi, \varphi) > 0$ with (17), we obtain (20).

A.2 Online technology utilization

(i) Inactive firms

Firm (ψ, φ) in region r exits immediately if and only if $\max\{\pi_r^N(\psi, \varphi), \pi_{rr}^F(\psi, \varphi), \pi_{rs}^F(\psi, \varphi)\} < 0$ holds. Solving $\pi_r^N(\psi, \varphi) < 0$ with (15), $\pi_{rr}^F(\psi, \varphi) < 0$ with (16), and $\pi_{rs}^F(\psi, \varphi) < 0$ with (17), we obtain (21).

(ii) Online firms

Firm (ψ, φ) in region r chooses to be an online firm if and only if $\pi_r^N(\psi, \varphi) > \max\{0, \pi_{rr}^F(\psi, \varphi), \pi_r^F(\psi, \varphi)\}$ hold. Solving $\pi_r^N(\psi, \varphi) > 0$ with (15), $\pi_r^N(\psi, \varphi) > \pi_{rr}^F(\psi, \varphi)$ with (15) and (16), and $\pi_r^N(\psi, \varphi) > \pi_r^F(\psi, \varphi)$ with (15), (16) and (17), we obtain (22).

(iii) Single-market offline firms

Firm (ψ, φ) in region r chooses to be an offline firm selling only in home market if and only if $\pi_{rr}^F(\psi, \varphi) > \max\{0, \pi_r^N(\psi, \varphi), \pi_r^F(\psi, \varphi)\}$ holds. Solving $\pi_{rr}^F(\psi, \varphi) > 0$ with (16), $\pi_{rr}^F(\psi, \varphi) > \pi_r^N(\psi, \varphi)$ with (15) and (16), and $\pi_{rs}^F(\psi, \varphi) < 0$ with (17), we obtain (23).

(iv) Exporting offline firms

Firm (ψ, φ) in region r chooses to be an offline firm selling in both home and foreign markets if and only if $\pi_r^F(\psi, \varphi) > \max\{0, \pi_r^N(\psi, \varphi), \pi_{rr}^F(\psi, \varphi)\}$ holds. Solving $\pi_r^F(\psi, \varphi) > 0$ with (16) and (17), $\pi_r^F(\psi, \varphi) > \pi_r^N(\psi, \varphi)$ with (15), (16) and (17), and $\pi_{rs}^F(\psi, \varphi) > 0$ with (17), we obtain (24).

Appendix B

Under $\psi > 1$ and $\varphi > 1$, we can further determine the specific domain of firm (ψ, φ)'s choice in the following.

B.1 No online technology

(i) offline firms selling home market

We can rewrite (19) as follows: $1 < \varphi < \underline{\Phi}_{rr}^{1/(\sigma-1)}$ and $\underline{\Phi}_{rr}^{1/(\sigma-1)}/\varphi \equiv \psi_{r1}(\varphi) < \psi < \infty$, and $\underline{\Phi}_{rr}^{1/(\sigma-1)} < \varphi < \infty$ and $1 < \psi < \infty$.

(ii) Exporting offline firms

We can rewrite (20) as follows: $1 < \varphi < \underline{\Phi}_{rs}^{1/(\sigma-1)}$ and $\underline{\Phi}_{rs}^{1/(\sigma-1)}/\varphi < \psi < \infty$, and $\underline{\Phi}_{rs}^{1/(\sigma-1)} < \varphi < \infty$ and $1 < \psi < \infty$.

B.2 Online technology utilization

(i) Online firms

If $\underline{\Psi}_r^{\sigma-1} < (1/\underline{\Phi}_{rr} + 1/\underline{\Phi}_{rs})^{-1} = \frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$ hold, we can rewrite (22) as follows: $\underline{\Psi}_r < \psi < \psi_{r2}$ and $1 < \varphi < \varphi_{r1}$, and $\psi_{r2} < \psi < \infty$ and $1 < \varphi < \varphi_{r2}(\psi)$.

Using $\varphi_{r2}(\psi) = 1 \Leftrightarrow \psi = \left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}} - \frac{1}{\underline{\Psi}_r^{\sigma-1}}\right)^{-1/(\sigma-1)} \equiv \psi_{r3}$, if $\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs} < \underline{\Psi}_r^{\sigma-1} < \underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$, we can rewrite (22) as follows: $\underline{\Psi}_r < \psi < \psi_{r2}$ and $1 < \varphi < \varphi_{r1}$, and $\psi_{r2} < \psi < \psi_{r3}$ and $1 < \varphi < \varphi_{r2}(\psi)$.

(ii) Offline firms

If $\underline{\Psi}_r^{\sigma-1} < \frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$, we can rewrite (23) as follows: $\varphi_{r1} < \varphi < \underline{\Phi}_{rr}^{1/(\sigma-1)}$ and $\psi_{r1}(\varphi) < \psi < \infty$, $\underline{\Phi}_{rr}^{1/(\sigma-1)} < \varphi < \infty$ and $1 < \psi < \infty$, and $\psi_{r2} < \psi < \infty$ and $\varphi_{r2}(\psi) < \varphi < \varphi_{r1}$.

If $\frac{\Phi_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs} < \underline{\Psi}_r^{\sigma-1} < \underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$, we can rewrite (23) as follows: $\varphi_{r1} < \varphi < \underline{\Phi}_{rr}^{1/(\sigma-1)}$ and $\psi_{r1}(\varphi) < \psi < \infty$, $\underline{\Phi}_{rr}^{1/(\sigma-1)} < \varphi < \infty$ and $1 < \psi < \infty$, $\psi_{r2} < \psi < \psi_{r3}$ and $\varphi_{r2}(\psi) < \varphi < \varphi_{r1}$, and $\psi_{r3} < \psi < \infty$ and $1 < \varphi < \varphi_{r1}$.

If $\underline{\Psi}_r^{\sigma-1} > \underline{\Phi}_{rs}$, we can rewrite (23) as follows: $1 < \varphi < \underline{\Phi}_{rr}^{1/(\sigma-1)}$ and $\psi_{r1}(\varphi) < \psi < \infty$, and $\underline{\Phi}_{rr}^{1/(\sigma-1)} < \varphi < \infty$ and $1 < \psi < \infty$.

(iii) Exporting offline firms

If $\underline{\Psi}_r^{\sigma-1} < \frac{\Phi_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$, we can rewrite (24) as follows: $1 < \psi < \psi_{r2}$ and $\underline{\Phi}_{rs}^{1/(\sigma-1)}/\psi \equiv \varphi_{r3}(\psi) < \varphi < \underline{\Phi}_{rs}^{1/(\sigma-1)}$, $1 < \psi < \psi_{r2}$ and $\underline{\Phi}_{rs}^{1/(\sigma-1)} < \varphi < \infty$, and $\psi_{r2} < \psi < \infty$ and $\varphi_{r2}(\psi) < \varphi < \infty$.

If $\frac{\Phi_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs} < \underline{\Psi}_r^{\sigma-1} < \underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$, we can rewrite (24) as follows: $1 < \psi < \psi_{r2}$ and $\varphi_{r3}(\psi) < \varphi < \underline{\Phi}_{rs}^{1/(\sigma-1)}$, $1 < \psi < \psi_{r2}$ and $\underline{\Phi}_{rs}^{1/(\sigma-1)} < \varphi < \infty$, $\psi_{r2} < \psi < \psi_{r3}$ and $\varphi_{r2}(\psi) < \varphi < \infty$, and $\psi_{r3} < \psi < \infty$ and $1 < \varphi < \infty$.

Using $\varphi_{r3}(\psi) = 1 \Leftrightarrow \psi = \underline{\Phi}_{rs}^{1/(\sigma-1)}$, if $\underline{\Psi}_r^{\sigma-1} > \underline{\Phi}_{rs}$, we can rewrite (24) as follows: $1 < \psi < \underline{\Phi}_{rs}^{1/(\sigma-1)}$ and $\varphi_{r3}(\psi) < \varphi < \infty$, and $\underline{\Phi}_{rs}^{1/(\sigma-1)} < \psi < \infty$ and $1 < \varphi < \infty$.

Appendix C

Under $\underline{\Psi}_r^{\sigma-1} < \frac{\Phi_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}\underline{\Phi}_{rs}$ and $\underline{\Psi}_r^{\sigma-1} > 1$, we obtain the ex-ante probabilities, the products of ex-ante probability and aggregate product of productivity and quality when online technology exists, and part of $\mathbb{E}\varphi$.

C.1 Online firms

(i) $pe_{r,N}$

Using Appendix B, the ex-ante probability of successful entry for N-firms producing in region r is expressed as follows:

$$pe_{r,N} = \int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi + \int_{\psi_{r2}}^{\infty} \int_1^{\varphi_{r2}(\psi)} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi.$$

The first term is obtained as

$$\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi = \frac{1}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} - \frac{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)} \underline{\Psi}_r^{\kappa}} - \frac{1}{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}} + \frac{1}{\underline{\Psi}_r^{\kappa}}.$$

We calculate the send term. By assuming $\gamma = \log \psi^{\sigma-1} \Leftrightarrow e^{\gamma/(\sigma-1)} = \psi$ and $\varkappa = \log \varphi^{\sigma-1} \Leftrightarrow e^{\varkappa/(\sigma-1)} = \varphi$, we obtain

$$\begin{aligned}
&\int_{\psi_{r2}}^{\infty} \int_{1}^{\varphi_{r2}(\psi)} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi \\
= \;& \int_{\log \underline{\Phi}_{rs} \underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \int_{0}^{\log \frac{\frac{1}{e^{\gamma}} + \frac{1}{\underline{\Psi}_r^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}}} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} (\sigma-1)^{-2} \psi \varphi \mathrm{d}\varkappa \mathrm{d}\gamma \\
= \;& -\frac{\kappa(\sigma-1)^{-1}}{\left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}\right)^{-\frac{\kappa}{\sigma-1}}} \int_{\log \underline{\Phi}_{rs} \underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \left(1 + \frac{1}{\underline{\Psi}_r^{\sigma-1}} e^{\gamma}\right)^{-\frac{\kappa}{\sigma-1}} \mathrm{d}\gamma + \frac{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)} \underline{\Psi}_r^{\kappa}}.
\end{aligned}$$

Setting $1 + \frac{1}{\underline{\Psi}_r^{\sigma-1}} e^{\gamma} = v$, we obtain $\gamma = \log \left[\underline{\Psi}_r^{\sigma-1}(v-1)\right]$ and $\mathrm{d}v = \frac{1}{\underline{\Psi}_r^{\sigma-1}} e^{\gamma} \mathrm{d}\gamma$. Thus, we have

$$\int_{\log \underline{\Phi}_{rs} \underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \left(1 + \frac{1}{\underline{\Psi}_r^{\sigma-1}} e^{\gamma}\right)^{-\frac{\kappa}{\sigma-1}} \mathrm{d}\gamma = \int_{1+\underline{\Phi}_{rs}/\underline{\Phi}_{rr}}^{\infty} \frac{1}{v^{\frac{\kappa}{\sigma-1}}(v-1)} \mathrm{d}v.$$

Setting $v = 1/t$, we obtain

$$\int_{1+\underline{\Phi}_{rs}/\underline{\Phi}_{rr}}^{\infty} \frac{1}{v^{\frac{\kappa}{\sigma-1}}(v-1)} \mathrm{d}t = B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}}, \frac{\kappa}{\sigma-1}, 0\right].$$

As a result, we have (29).

(ii) $\widetilde{\Psi}_{r,N} \cdot pe_{r,N}$

Using Appendix B, the product of the ex-ante probability of successful entry for N firms producing in region r and aggregate productivity level of N-firms in region r is expressed as

$$\widetilde{\Psi}_{r,N} \cdot pe_{r,N} = \int_{\underline{\Psi}_r}^{\psi_{r2}} \int_{1}^{\varphi_{r1}} \varphi^{\sigma-1} \psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi + \int_{\psi_{r2}}^{\infty} \int_{1}^{\varphi_{r2}(\psi)} \varphi^{\sigma-1} \psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi.$$

The first term is calculated as follows:

$$\begin{aligned}
&\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_{1}^{\varphi_{r1}} \varphi^{\sigma-1} \psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi \\
= \;& \frac{\kappa^2}{(\sigma-\kappa-1)^2} \left[\underline{\Phi}_{rs}^{(\sigma-\kappa-1)/(\sigma-1)} - \frac{\underline{\Phi}_{rs}^{(\sigma-\kappa-1)/(\sigma-1)} \underline{\Psi}_r^{\sigma-\kappa-1}}{\underline{\Phi}_{rr}^{(\sigma-\kappa-1)/(\sigma-1)}} - \underline{\Phi}_{rr}^{\frac{\sigma-\kappa-1}{\sigma-1}} + \underline{\Psi}_r^{\sigma-\kappa-1}\right].
\end{aligned}$$

Setting $\gamma = \log \psi^{\sigma-1} \Leftrightarrow e^{\gamma/(\sigma-1)} = \psi$ and $\varkappa = \log \varphi^{\sigma-1} \Leftrightarrow e^{\varkappa/(\sigma-1)} = \varphi$, the second term is calculated as follows:

$$
\begin{aligned}
&\int_{\psi_{r2}}^{\infty}\int_{1}^{\varphi_{r2}(\psi)}\varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi\\
=\;&\kappa^{2}(\sigma-1)^{-2}\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\int_{0}^{\log\frac{\frac{1}{e^{\gamma}}+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}}}\varphi^{\sigma-\kappa-1}\psi^{\sigma-\kappa-1}\mathrm{d}\varkappa\mathrm{d}\gamma\\
=\;&-\kappa^{2}\frac{(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{-(\sigma-\kappa-1)/(\sigma-1)}\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}e^{\gamma}\right)^{(\sigma-\kappa-1)/(\sigma-1)}\mathrm{d}\gamma\\
&+\frac{\kappa^{2}}{(\sigma-\kappa-1)^{2}}\left(\frac{\underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}}{\underline{\Phi}_{rr}}\right)^{(\sigma-\kappa-1)/(\sigma-1)}.
\end{aligned}
$$

We calculate

$$
\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}e^{\gamma}\right)^{(\sigma-\kappa-1)/(\sigma-1)}\mathrm{d}\gamma.
$$

Setting $1+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}e^{\gamma}=v$, we obtain $\gamma=\log\left[\underline{\Psi}_{r}^{\sigma-1}(v-1)\right]$ and $\mathrm{d}v=\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}e^{\gamma}\mathrm{d}\gamma$. Thus, we have:

$$
\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}e^{\gamma}\right)^{(\sigma-\kappa-1)/(\sigma-1)}\mathrm{d}\gamma=\int_{1+\underline{\Phi}_{rs}/\underline{\Phi}_{rr}}^{\infty}v^{(\sigma-\kappa-1)/(\sigma-1)}(v-1)^{-1}\mathrm{d}v
$$

Setting $v=1/t$ yields

$$
\int_{1+\underline{\Phi}_{rs}/\underline{\Phi}_{rr}}^{\infty}v^{(\sigma-\kappa-1)/(\sigma-1)}(v-1)^{-1}\mathrm{d}v=B\left[\underline{\Phi}_{rr}/(\underline{\Phi}_{rr}+\underline{\Phi}_{rs}),\frac{\kappa-(\sigma-1)}{\sigma-1},0\right].
$$

As a result, we obtain (30).

C.2 Offline firms

(i) $pe_{rr,F}$

Using Appendix B, the ex-ante probability of successful entry for F-firms producing and selling in region r is expressed as follows:

$$
\begin{aligned}
pe_{rr,F}=&\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_{r}}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}\int_{\underline{\Phi}_{rr}^{1/(\sigma-1)}/\varphi}^{\infty}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi+\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}^{\infty}\int_{1}^{\infty}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi\\
&+\int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_{r}/\underline{\Phi}_{rr}^{1/(\sigma-1)}}^{\infty}\int_{\varphi_{r2}(\psi)}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_{r}}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi.
\end{aligned}
$$

The first and second terms are calculated as follows:

$$\begin{aligned}
&\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}\int_{\underline{\Phi}_{rr}^{1/(\sigma-1)}/\varphi}^{\infty}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi+\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}^{\infty}\int_{1}^{\infty}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi\\
=\;&\frac{\kappa}{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}\log\underline{\Psi}_r+\frac{1}{\underline{\Phi}_{rr}^{\frac{\kappa}{\sigma-1}}}
\end{aligned}$$

Setting $\gamma=\log\psi^{\sigma-1}\Leftrightarrow e^{\gamma/(\sigma-1)}=\psi$ and $\varkappa=\log\varphi^{\sigma-1}\Leftrightarrow e^{\varkappa/(\sigma-1)}=\varphi$, we calculate the last term of $pe_{rr,F}$ as follows:

$$\begin{aligned}
&\int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r/\underline{\Phi}_{rr}^{1/(\sigma-1)}}^{\infty}\int_{\varphi_{r2}(\psi)}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi\\
=\;&-\left(\frac{\underline{\Phi}_{rr}}{\underline{\Psi}_r^{\sigma-1}}\right)^{-\kappa/(\sigma-1)}\left(\frac{\underline{\Phi}_{rs}\underline{\Psi}_r^{\sigma-1}}{\underline{\Phi}_{rr}}\right)^{-\kappa/(\sigma-1)}\\
&+(\sigma-1)^{-1}\kappa\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\kappa/(\sigma-1)}\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_r^{\sigma-1}}e^{\gamma}\right)^{-\kappa/(\sigma-1)}\mathrm{d}\gamma.
\end{aligned}$$

As shown in Appendix C.1(i), we obtain

$$\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_r^{\sigma-1}}e^{\gamma}\right)^{-\kappa/(\sigma-1)}\mathrm{d}\gamma=B[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}},\frac{\kappa}{\sigma-1},0].$$

Thus, we obtain (31).

(ii) $\widetilde{\Phi}_{rr}\cdot pe_{rr,F}$

Using Appendix B, the product of the ex-ante probability of successful entry for F-firms producing and selling in region r and aggregate productivity and quality level of F-firms producing and selling in region r is expressed as follows:

$$\begin{aligned}
\widetilde{\Phi}_{rr}\cdot pe_{rr,F}=&\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}\int_{\underline{\Phi}_{rr}^{1/(\sigma-1)}/\varphi}^{\infty}\varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi+\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}^{\infty}\int_{1}^{\infty}\varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi\\
&+\int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r/\underline{\Phi}_{rr}^{1/(\sigma-1)}}^{\infty}\int_{\varphi_{r2}(\psi)}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r}\varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi.
\end{aligned}$$

The first and second terms of $\widetilde{\Phi}_{rr} \cdot pe_{rr,F}$ are calculated as follows:

$$\begin{aligned}
&\int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}} \int_{\underline{\Phi}_{rr}^{1/(\sigma-1)}/\varphi}^{\infty} \varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi + \int_{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}}^{\infty}\int_{1}^{\infty} \varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\psi\mathrm{d}\varphi \\
= \;& \frac{\kappa^2}{\kappa-\sigma+1}\underline{\Phi}_{rr}^{-(\kappa-\sigma+1)/(\sigma-1)}\log\underline{\Psi}_r + \frac{\kappa^2}{(\kappa-\sigma+1)^2}\underline{\Phi}_{rr}^{\frac{\sigma-\kappa-1}{\sigma-1}}
\end{aligned}$$

Setting $\gamma = \log\psi^{\sigma-1} \Leftrightarrow e^{\gamma/(\sigma-1)} = \psi$ and $\varkappa = \log\varphi^{\sigma-1} \Leftrightarrow e^{\varkappa/(\sigma-1)} = \varphi$, we calculate the last term of $\widetilde{\Phi}_{rr} \cdot pe_{rr,F}$ as follows:

$$\begin{aligned}
&\int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}\underline{\Psi}_r/\underline{\Phi}_{rr}^{1/(\sigma-1)}}^{\infty}\int_{\varphi_{r2}(\psi)}^{\underline{\Phi}_{rr}^{\frac{1}{\sigma-1}}/\underline{\Psi}_r} \varphi^{\sigma-1}\psi^{\sigma-1}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi \\
= \;& \kappa^2(\sigma-1)^{-2}\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\int_{\log\frac{\frac{1}{e^\gamma}+\frac{1}{\underline{\Psi}_r^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}}}^{\log\left(\underline{\Phi}_{rr}/\underline{\Psi}_r^{\sigma-1}\right)} \varphi^{\sigma-\kappa-2}\psi^{\sigma-\kappa-2}\psi\varphi\mathrm{d}\varkappa\mathrm{d}\gamma \\
= \;& -\frac{\kappa^2}{(\sigma-\kappa-1)^2}\underline{\Phi}_{rs}^{\frac{\sigma-\kappa-1}{\sigma-1}} \\
&+\frac{\kappa^2(\sigma-1)^{-1}}{\kappa-\sigma+1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa-\sigma+1}{\sigma-1}}\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_r^{\sigma-1}}e^{\gamma}\right)^{\frac{\sigma-\kappa-1}{\sigma-1}}\mathrm{d}\gamma.
\end{aligned}$$

As shown in Appendix C.1(ii), we obtain

$$\int_{\log\underline{\Phi}_{rs}\underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{1}{\underline{\Psi}_r^{\sigma-1}}e^{\gamma}\right)^{(\sigma-\kappa-1)/(\sigma-1)}\mathrm{d}\gamma = B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}}, \frac{\kappa-(\sigma-1)}{\sigma-1}, 0\right].$$

Consequently, we obtain (32).

C.3 Exporting offline firms

(i) $pe_{rs,F}$

Using Appendix B, the ex-ante probability of successful entry for F-firms producing in region r and selling in region s is expressed as follows:

$$pe_{rs,F} = \int_{1}^{\psi_{r2}}\int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}/\psi}^{\infty}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi + \int_{\psi_{r2}}^{\infty}\int_{\varphi_{r2}(\psi)}^{\infty}\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi.$$

The first term of $pe_{rs,F}$ is calculated as follows:

$$\int_{1}^{\psi_{r2}} \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}/\psi}^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi = \frac{\kappa}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} \log \frac{\underline{\Phi}_{rs}^{1/(\sigma-1)} \underline{\Psi}_{r}}{\underline{\Phi}_{rr}^{1/(\sigma-1)}}.$$

Setting $\gamma = \log \psi^{\sigma-1} \Leftrightarrow e^{\gamma/(\sigma-1)} = \psi$ and $\varkappa = \log \varphi^{\sigma-1} \Leftrightarrow e^{\varkappa/(\sigma-1)} = \varphi$, we calculate the last term of $pe_{rs,F}$ as follows:

$$\begin{aligned}
& \int_{\psi_{r2}}^{\infty} \int_{\varphi_{r2}(\psi)}^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi \\
= \;& (\sigma-1)^{-2} \int_{\log \underline{\Phi}_{rs} \underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \int_{\log \frac{\frac{1}{e^{\gamma}} + \frac{1}{\underline{\Psi}_{r}^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}}}^{\infty} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \psi \varphi \mathrm{d}\varkappa \mathrm{d}\gamma \\
= \;& (\sigma-1)^{-1} \kappa \left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}} \right)^{\kappa/(\sigma-1)} \int_{\log \underline{\Phi}_{rs} \underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \left(1 + \frac{1}{\underline{\Psi}_{r}^{\sigma-1}} e^{\gamma} \right)^{-\kappa/(\sigma-1)} \mathrm{d}\gamma.
\end{aligned}$$

As shown in Appendix C.1(i), we obtain

$$\int_{\log \underline{\Phi}_{rs} \underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \left(1 + \frac{1}{\underline{\Psi}_{r}^{\sigma-1}} e^{\gamma} \right)^{-\kappa/(\sigma-1)} \mathrm{d}\gamma = B[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}}, \frac{\kappa}{\sigma-1}, 0].$$

Thus, we obtain (33).

(ii) $\widetilde{\Phi}_{rs} \cdot pe_{rs,F}$

Using Appendix B, the product of the ex-ante probability of successful entry for F-firms producing in region r and selling in region s and aggregate productivity and quality level of F-firms producing in region r and selling in region s is expressed as follows:

$$\widetilde{\Phi}_{rs} \cdot pe_{rs,F} = \int_{1}^{\psi_{r2}} \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}/\psi}^{\infty} \varphi^{\sigma-1} \psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} d\varphi d\psi + \int_{\psi_{r2}}^{\infty} \int_{\varphi_{r2}(\psi)}^{\infty} \varphi^{\sigma-1} \psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} d\varphi d\psi.$$

The first term of $\widetilde{\Phi}_{rs} \cdot pe_{rs,F}$ is calculated as follows:

$$\int_{1}^{\psi_{r2}} \int_{\underline{\Phi}_{rs}^{1/(\sigma-1)}/\psi}^{\infty} \varphi^{\sigma-1} \psi^{\sigma-1} \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} d\varphi d\psi = \frac{\kappa^2}{\kappa-\sigma+1} \underline{\Phi}_{rs}^{(\sigma-\kappa-1)/(\sigma-1)} \log \frac{\underline{\Phi}_{rs}^{1/(\sigma-1)} \underline{\Psi}_{r}^{1}}{\underline{\Phi}_{rr}^{1/(\sigma-1)}}.$$

Setting $\gamma = \log \psi^{\sigma-1} \Leftrightarrow e^{\gamma/(\sigma-1)} = \psi$ and $\varkappa = \log \varphi^{\sigma-1} \Leftrightarrow e^{\varkappa/(\sigma-1)} = \varphi$, we calculate the last term of $\widetilde{\Phi}_{rs} \cdot pe_{rs,F}$ as follows:

$$\begin{aligned}
& \kappa^2 \int_{\psi_{r2}}^{\infty} \int_{\varphi_{r2}(\psi)}^{\infty} \varphi^{\sigma-\kappa-2} \psi^{\sigma-\kappa-2} d\varphi d\psi \\
= \quad & (\sigma-1)^{-2} \kappa^2 \int_{\log \underline{\Phi}_{rs} \underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \int_{\log \frac{\frac{1}{e^{\gamma}} + \frac{1}{\underline{\Psi}_r^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}}}}^{\infty} \varphi^{\sigma-\kappa-2} \psi^{\sigma-\kappa-2} \psi \varphi d\varkappa d\gamma \\
= \quad & \frac{(\sigma-1)^{-1}}{\kappa-\sigma+1} \kappa^2 \left(\frac{1}{\underline{\Phi}_{rr}} + \frac{1}{\underline{\Phi}_{rs}} \right)^{(\kappa-\sigma+1)/(\sigma-1)} \int_{\log \underline{\Phi}_{rs} \underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \left(1 + \frac{1}{\underline{\Psi}_r^{\sigma-1}} e^{\gamma} \right)^{(\sigma-\kappa-1)/(\sigma-1)} d\gamma.
\end{aligned}$$

As shown in Appendix C.1(ii), we obtain

$$\int_{\log \underline{\Phi}_{rs} \underline{\Psi}_r^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty} \left(1 + \frac{1}{\underline{\Psi}_r^{\sigma-1}} e^{\gamma} \right)^{(\sigma-\kappa-1)/(\sigma-1)} \mathrm{d}\gamma = B \left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr} + \underline{\Phi}_{rs}}, \frac{\kappa-(\sigma-1)}{\sigma-1}, 0 \right].$$

Thus, we obtain (34).

C.1 Part of $\mathbb{E}\varphi$

Using Appendix B, we obtain the part of $\mathbb{E}\varphi$ as follows:

$$\begin{aligned}
\int\int_{\mathcal{A}_{N,r}} \varphi \nu_{r,N} \mathrm{d}\psi \mathrm{d}\varphi \quad &= \quad \frac{\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} \varphi g(\psi) g(\varphi) d\varphi d\psi + \int_{\psi_{r2}}^{\infty} \int_1^{\varphi_{r2}(\psi)} \varphi g(\psi) g(\varphi) d\varphi d\psi}{\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} g(\psi) g(\varphi) d\varphi d\psi + \int_{\psi_{r2}}^{\infty} \int_1^{\varphi_{r2}(\psi)} g(\psi) g(\varphi) d\varphi d\psi} \\
&= \quad \frac{\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} \varphi \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi + \int_{\psi_{r2}}^{\infty} \int_1^{\varphi_{r2}(\psi)} \varphi \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi}{pe_{r,N}}.
\end{aligned}$$

We calculate

$$\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} \varphi \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi + \int_{\psi_{r2}}^{\infty} \int_1^{\varphi_{r2}(\psi)} \varphi \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi.$$

The first term is calculated as

$$\int_{\underline{\Psi}_r}^{\psi_{r2}} \int_1^{\varphi_{r1}} \varphi \frac{\kappa}{\psi^{\kappa+1}} \frac{\kappa}{\varphi^{\kappa+1}} \mathrm{d}\varphi \mathrm{d}\psi = \frac{\kappa}{\kappa-1} \frac{1}{\underline{\Psi}_r^{\kappa}} \left(1 - \frac{\underline{\Phi}_{rr}^{\kappa/(\sigma-1)}}{\underline{\Phi}_{rs}^{\kappa/(\sigma-1)}} \right) \left(\frac{\underline{\Psi}_r^{\kappa-1}}{\underline{\Phi}_{rr}^{\frac{\kappa-1}{\sigma-1}}} - 1 \right).$$

Setting $\gamma = \log \psi^{\sigma-1} \Leftrightarrow e^{\gamma/(\sigma-1)} = \psi$ and $\varkappa = \log \varphi^{\sigma-1} \Leftrightarrow e^{\varkappa/(\sigma-1)} = \varphi$, we calculate the last term as follows:

$$
\begin{aligned}
&\int_{\psi_{r2}}^{\infty}\int_{1}^{\varphi_{r2}(\psi)} \varphi\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}\mathrm{d}\varphi\mathrm{d}\psi \\
=\ &\int_{\log \underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\int_{0}^{\log\frac{\frac{1}{e^{\gamma}}+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}}{\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}}} \varphi\frac{\kappa}{\psi^{\kappa+1}}\frac{\kappa}{\varphi^{\kappa+1}}(\sigma-1)^{-2}\psi\varphi\mathrm{d}\varkappa\mathrm{d}\gamma \\
=\ &\kappa^{2}\frac{(\sigma-1)^{-1}}{-\kappa+1}\left(\frac{1}{\underline{\Phi}_{rr}}+\frac{1}{\underline{\Phi}_{rs}}\right)^{\frac{\kappa-1}{\sigma-1}}\int_{\log \underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(\frac{1}{e^{\gamma}}+\frac{1}{\underline{\Psi}_{r}^{\sigma-1}}\right)^{\frac{-\kappa+1}{\sigma-1}} e^{\gamma\frac{-\kappa}{\sigma-1}}\mathrm{d}\gamma \\
&-\frac{\kappa}{-\kappa+1}\left(\frac{\underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}}{\underline{\Phi}_{rr}}\right)^{-\kappa/(\sigma-1)}
\end{aligned}
$$

Setting $1+\frac{e^{\gamma}}{\underline{\Psi}_{r}^{\sigma-1}} = v$ yields $1+\frac{e^{\gamma}}{\underline{\Psi}_{r}^{\sigma-1}} = v \Leftrightarrow \gamma = \log\left[(v-1)\,\underline{\Psi}_{r}^{\sigma-1}\right]$ and $dv = \underline{\Psi}_{r}^{-(\sigma-1)}e^{\gamma}d\gamma$. Thus, we have

$$
\int_{\log \underline{\Phi}_{rs}\underline{\Psi}_{r}^{\sigma-1}/\underline{\Phi}_{rr}}^{\infty}\left(1+\frac{e^{\gamma}}{\underline{\Psi}_{r}^{\sigma-1}}\right)^{\frac{-\kappa+1}{\sigma-1}} e^{-\frac{\gamma}{\sigma-1}}\mathrm{d}\gamma = \underline{\Psi}_{r}^{-1}\int_{1+\underline{\Phi}_{rs}/\underline{\Phi}_{rr}}^{\infty} v^{\frac{-\kappa+1}{\sigma-1}}(v-1)^{-\frac{\sigma}{\sigma-1}}\mathrm{d}v.
$$

Setting $v = 1/t$, we have

$$
\int_{1+\underline{\Phi}_{rs}/\underline{\Phi}_{rr}}^{\infty} v^{\frac{-\kappa+1}{\sigma-1}}(v-1)^{-\frac{\sigma}{\sigma-1}}\mathrm{d}v = B\left[\frac{\underline{\Phi}_{rr}}{\underline{\Phi}_{rr}+\underline{\Phi}_{rs}},\frac{\kappa}{\sigma-1},-\frac{1}{\sigma-1}\right].
$$

Thus, we have (38).

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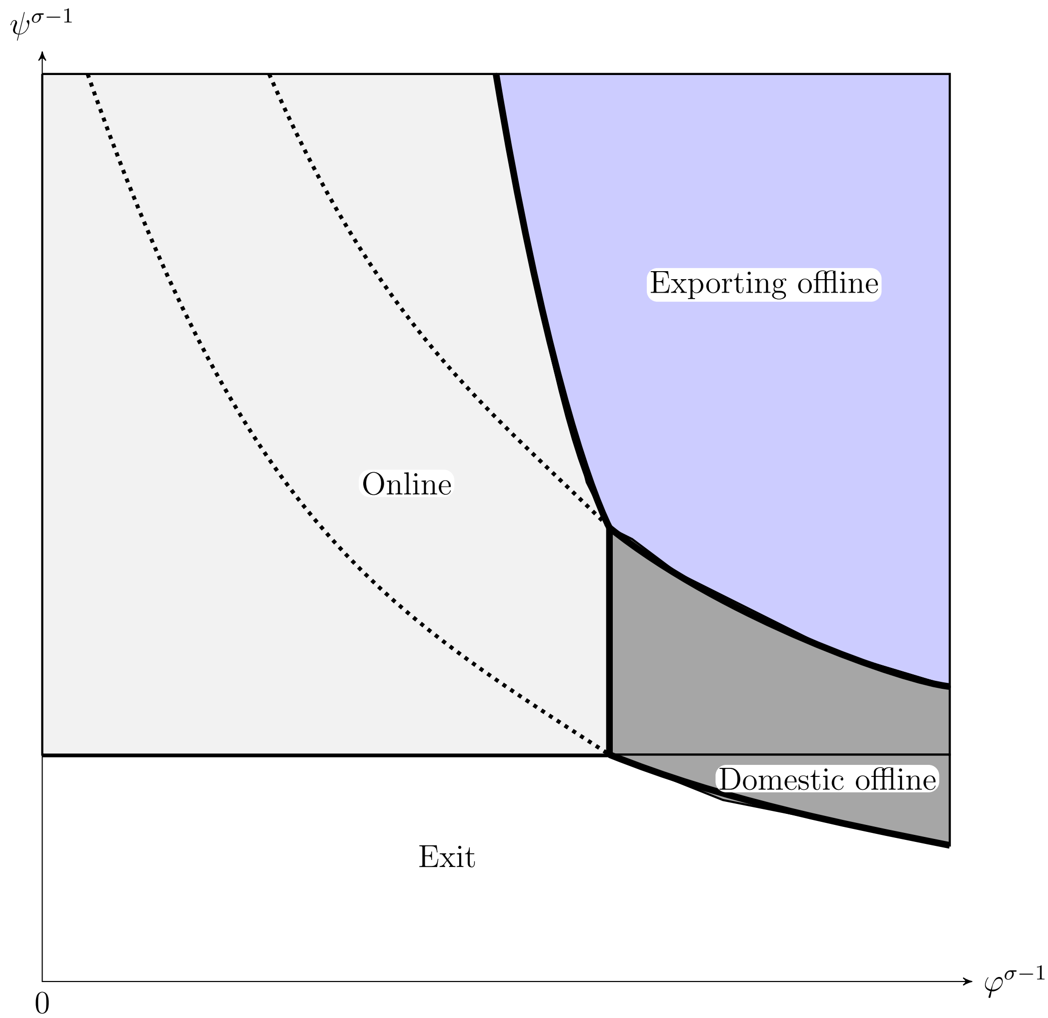

Figure 1: The pattern of equilibrium

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