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Model of Banking Behavior: Specification and Estimation

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March 2019

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This study aims to construct a model of the economic behavior of banks as intermediate sectors in macroeconomics that play an important role interplaying between financial and real markets based on a theoretical and empirical perspective. There are two distinguishing characteristics of our approach. First, we specify an economic model of banking behavior, following partially assets and liabilities management (ALM). As the primary object of our paper is to provide an understanding of a bank's role in macroeconomics rather than the details of ALM, we abstract from that concept and embed its crucial factors into the model specification. Second, we assume that banks have two optimization problems: profit maximization and asset allocation. Banks are assumed to maximize the profit function which takes into consideration items of the balance sheet and income statement. Then, banks are assumed to attempt to determine their optimal portfolio. Although our model is remarkably simple, we consider that our framework is valid to illustrate the mechanism of the loan market.

Keywords: Banking Behavior, Loan Market, Asset and Liability Management, Asset Allocation

JEL classification: G00, G11, G21

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Model of Banking Behavior: Specification and Estimation

Tsubasa SHIBATA[†] Hiroyuki KOSAKA[‡]

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Abstract

This study aims to construct a model of the economic behavior of banks as intermediate sectors in macroeconomics that play an important role interplaying between financial and real markets based on a theoretical and empirical perspective.

There are two distinguishing characteristics of our approach. First, we specify an economic model of banking behavior, following partially assets and liabilities management (ALM). ALM is a practical system that commercial banks introduce for assessing and managing a bank's risk exposure and a huge and complex system. As the primary object of our paper is to provide an understanding of a bank's role in macroeconomics rather than the details of ALM, we abstract from that concept and embed its crucial factors into the model specification. Second, we assume that banks have two optimization problems: profit maximization and asset allocation. Banks are assumed to maximize the profit function which takes into consideration items of the balance sheet and income statement. Simultaneously, banks are assumed to attempt to determine their optimal portfolio among choices of federal and state federal and state government bonds, municipal bonds, corporate bonds, stocks, and foreign securities.

Although our model is remarkably simple, we consider that our framework is valid to illustrate the mechanism of the loan market. Especially, we believe that our approach could provide us a deeper understanding of the mechanisms of the lending market by linking it to a macroeconomic model that includes various markets.

Keywords: Banking Behavior, Loan Market, Asset and Liability Management, Asset Allocation
JEL Codes: G00, G11, G21

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1. Introduction

The history of unconventional monetary policy by the Bank of Japan (BOJ) dates back to its zero-interest-rate policy that was implemented in order to overcome deflation in February 1999. Since then, Japan undertook a quantitative easing (QE) policy from 2001 to 2006. After the global financial crisis, the BOJ further expanded its QE program in 2010 by purchasing additional risky assets such as ETF, J-REIT, and corporate bonds. The BOJ once again expanded the QE program on a larger scale in 2013 and late 2015. Furthermore, in early 2016, the BOJ introduced a negative interest rate policy. Thus, the low-interest-rate environment has continued more than two decades now, when the first unconventional monetary policy was introduced. However, Japan has not achieved its price stability target of two percent yet. Indeed, these monetary policies have reached the limits of their effectiveness.

In response to the global financial crisis, a number of studies have attempted to understand the mechanism of financial crisis and to evaluate optimal monetary policies. In particular, a recent strand within the literature focuses on banks of financial intermediaries linking them between financial markets and the real economy, a motif which has been neglected as a research topic, examining how the financial intermediation sector affects money flowing into the real economy. On the basis of a macroeconomic model approach, Eggertsson, et al. (2017) evaluated the impact of low and negative interest rates on the macroeconomy by applying a New-Keynesian model with a certain banking behavior. As for the studies on the partial and specific perspectives, Bario, et al. (2017a; 2017b), Jobst and Lin (2016), and Lopez (2018) shed light on the relation between low and negative interest rates and bank profitability by using bank-level data. While these studies have provided important insights into understanding transmission channel of monetary policy, they are insufficient to capture the more practical aspects of banking behavior.

Conventionally, the principal operation for banks is considered to rely on its loans and deposits. Their primary profits are based on the net interest margin between the interest they pay on customer deposits and the interest they receive on loans. However, this banking business model has changed due to the prolonged low-interest-rate environment. Each panel in Figure 1 shows outstanding loans, outstanding discounts applied, and securities holdings of domestic banks, city banks, and local banks, respectively. It also represents the policy interest rate (uncollateralized overnight call rates) and the government bond interest rate. Overall, the amount of lending by banks has been increasing. In particular, loans by local banks (Panel C of Figure 1) have risen steadily. On the one hand, since rate policies have keep them close to zero and long-term interest rates have continued to decline, suggesting that the profits from increasing loans would be offset by available interest rates. On the other hand, one may see a tendency to increase securities holdings by each bank, at least compared to the early 2000s. In order to see implications of this

fact shown in Figure 1, we consider further details about banks' securities holdings.

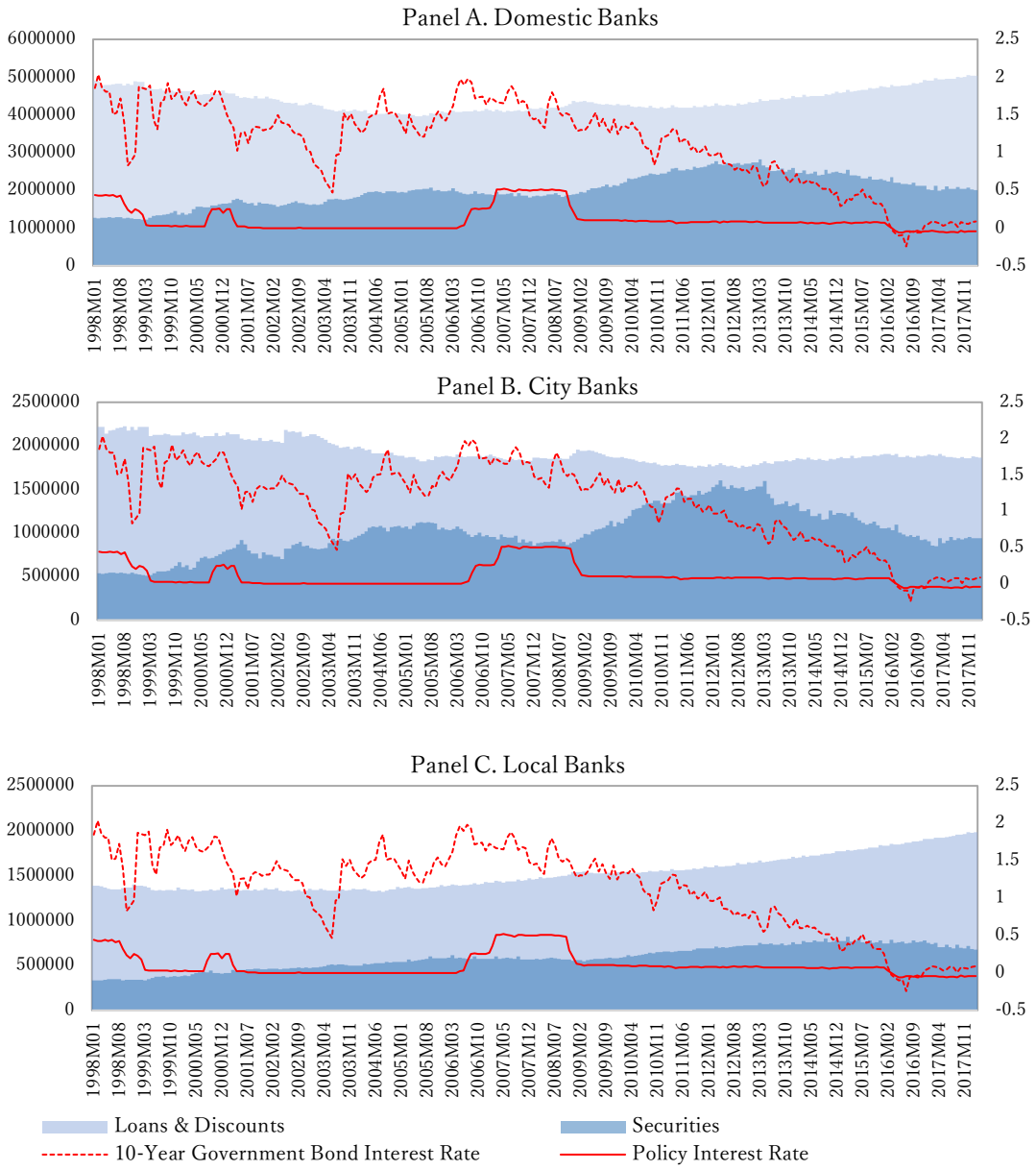

Figure 1. Japanese Banks' Historical Performances of Loans and Securities Holding

Figure 2 illustrates composition ratio of securities holdings: federal and state government bonds, federal and state government bonds, municipal bonds, corporate bonds, stocks, and foreign securities. While holdings of low-risk assets (i.e., federal and state government bonds) have decreased steadily, holdings of higher-risk assets (i.e., corporate bonds and stocks) have expanded. More concretely, city banks (Panel B of Figure 2) have shifted their asset allocations from favoring federal and state government bonds to stocks and foreign securities. Local banks (Panel B of Figure 2) have increased corporate bonds rather than federal and state government bonds.

That is to say, Figure 2 explains that asset allocation has changed from low-return assets to high-return ones. It implies that the banks have attempted to generate their profits through financial assets rather than by expanding their loan base.

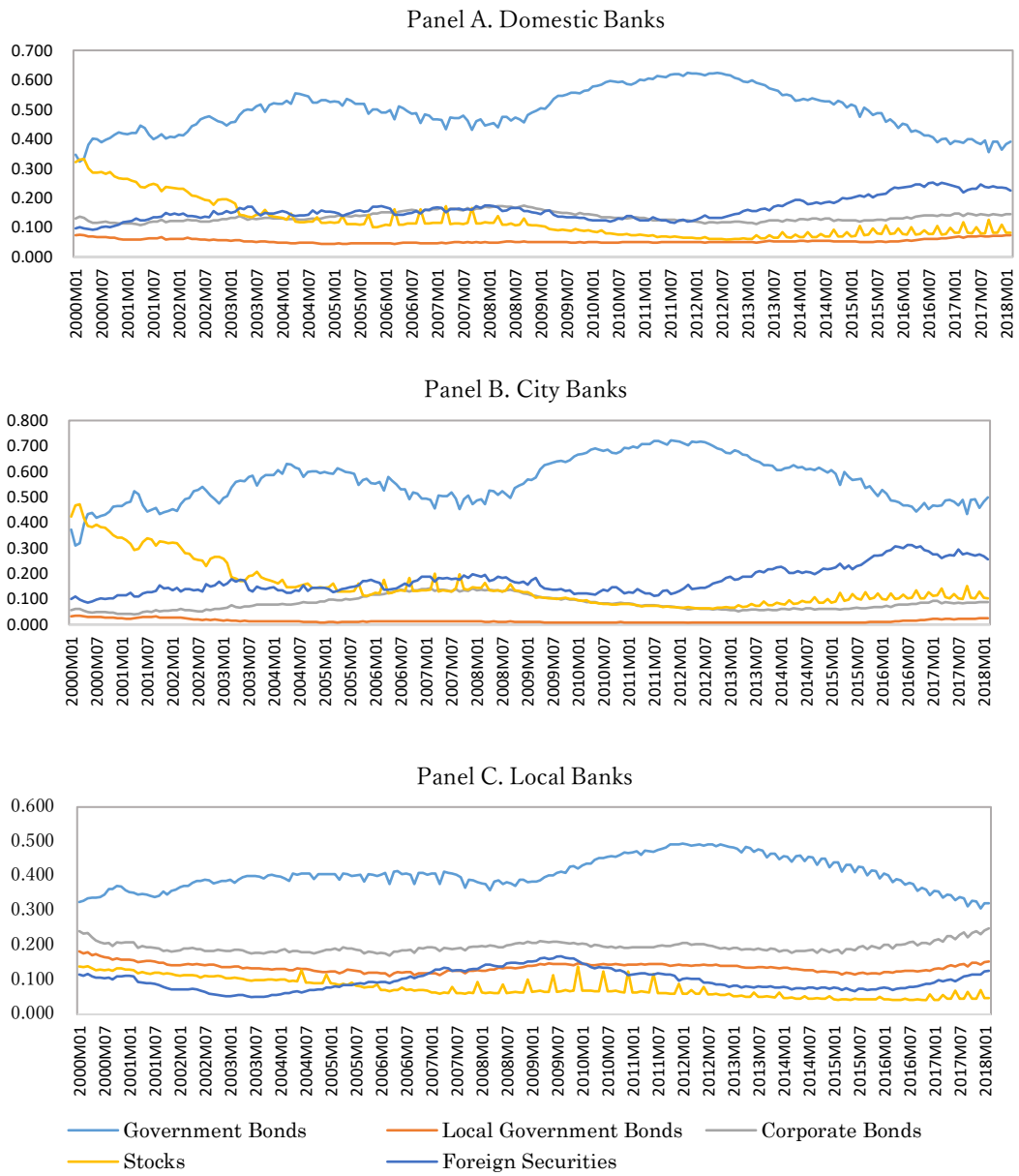

Figure 2. Composition Ratio of Securities Holdings in of Japanese Banks

As noted above, two facts have been discovered. First, a prolonged low-interest-rate environment has led to narrowing of net interest margins for banks, resulting in an impact on bank profitability. Second, banks responded sensitively to this serious situation by attempting to ensure their profits through reinforcing greater investment in financial assets rather than expanding their loan base, in order to make up the losses in net interest income (owing to low interest rate). It is

noteworthy that banks have been expanded the holdings of higher-risk assets like stocks.

In this study, in order to reflect such real aspects of banks, we aim at specifying an economic model of banking behavior, partially following the concept of assets and liabilities management (ALM). ALM is a huge and complex system that commercial banks introduce to assess and manage their risk exposure practically. As the primary objective of our paper is to provide an understanding of the role of banks macroeconomically rather than highlight the details of ALM, we abstract from ALM and embed its crucial factors into the model specification. Specifically, we assume that banks have two optimization problems: profit maximization and asset allocation. First, banks are assumed to maximize their profit function, which leads them to consider balance sheet and income statement items. Second, banks are assumed to attempt to determine their optimal portfolios among federal and state government bonds, municipal bonds, corporate bonds, stocks, and foreign securities. Although our model is quite simple, we consider the framework to be valid for illustrating the mechanism of the loan market. In particular, we believe that our approach provides a deeper understanding about the loan market mechanism by linking it with a macroeconomic model that includes various markets.

The rest of this paper is organized as follows. In Section 2, we give a brief literature review and explain our approach. Section 3 outlines the analytical framework of the loan market by specifying banking behavior. Section 4 provides a description of the data that are utilized in this study. In Section 5, we present the empirical results. Our concluding remarks are given in Section 6.

2. Literature Review and Study Approach

The primary focus of our paper is to specify banks' behaviors. By linking this aim to the whole model developed in our previous studies (Shibata and Kosaka 2018, Shibata 2016), we believe that we can obtain a better guide to explaining how a central bank's monetary policy impacts the real economy via various markets. This section illustrates the "big picture" of our model by reviewing previous contributions related to our approach.

2.1 A Literature Review

Over the past eighty years, numerous studies have contributed to developing financial models both theoretically and empirically. In this section, we summarize representative works below.

A) One of the cornerstones of monetary models is liquidity preference theory in the book entitled *The General Theory of Employment. Interest and Money* by Keynes (1936) who argues that an inverse relationship exists between the demand for cash balances and the rate of interest. One year later, in 1937, in order to capture Keynes' essential ideas (1936), Hicks (1937) developed the IS-LM model that is composed of simultaneous equalities between goods supplied and goods demanded (IS), and between money demanded and money supplied (LM), to obtain equilibrium figures for GNP and interest rates. R. Klein and W.E. Krelle (1983) then asserted that endogenous money supply should be taken into consideration. Hence, liquidity preference theory has become more sophisticated since the 1960s. Yet, these conventional models were based on the simple framework wherein the long-term interest rate is determined based on equilibrium in the market for money.

B) Tobin (1969) challenged Keynes' theory of demand for money from the viewpoint of a general framework for monetary analysis. The main drawback of Keynes' perspective demand for money was to assume that individuals would hold all their liquid assets either as money or bonds. It implied that holding money was the alternative to holding bonds. This assumption did not reflect reality. Thus, Tobin developed a general equilibrium to monetary theory by extending Keynes' model of the transaction demand for money and introducing portfolio theory. Following Tobin (1969) and Brunner-Meltzer (1972), Bernanke and Blinder (1988) and Ueda (1993) constructed the modified monetary model where loans and bonds are imperfect substitutes. Ogawa and Kitasaka (1998) further incorporated real estate market into the framework of Ueda (1993a;1993b), resulting in providing crucial insights about the important role of the collateral channel within the call market, short-term monetary market, government bond market, real estate market and the real marketplace in general.

C) Modigliani, Rasche and Cooper (1970) provided a new framework, the reserve market equilibrium where short-term interest rates are determined, and the long-term interest rate is set. In addition, Modigliani and Shiller (1973) introduced the term structure of interest rates, along with the relationship between short- and long-term rates. Following the term structure by Modigliani and Shiller (1973), Sadahiro (1992) established a Japanese financial model.

Table 1. Chart of Historical Monetary Models

	Keynes (1936)	Klein (1983)	Shibata & Kosaka	Modigliani (1970)	Brainard & Tobin (1969)	Ogawa & Kitasaka (1998)
Reserve Market		○	○	○		× Walras'Law
Money Market	○	○	○			
Call Market			△			○
Short Term Monetary Market			△	× Walras'Law		
Government Bond Market	× Walras'Law	× Walras'Law		○		
Lending Market			△			○
Corporate Bond Market			△			
Japan Stock Market			△			
U.S. Bond Market			△			
U.S. Government Bond Market			△			
Commodity Market (including future market)			○			
Land Market			○			○
Real Market	○	○	○	○		○

Note: ○ shows that models are based on an equilibrium. △ represents that models are illustrated on the equation.

2.2 Our Approach

Our approach is summarized as follows:

1) Diversification of Assets in the Market

Generalizing the basic model of Tobin (1958) that consists of two assets, i.e., money and bonds, we re-specify the money-demand function (liquidity preference) by expanding the portfolio from domestic bonds to short-term assets, stocks, corporate bonds, and even U.S. Treasury bonds (see Shibata and Kosaka, 2018; Kosaka, 2017). Then, as the short-term interest rate is closely linked to policy instruments (namely, the call rate), the money-demand function is linked with a policy instrument via the short-term market; at the same time, it is connected to U.S. government bonds.

2) Endogenizing M1

As Table 1 shows, there are few studies which focus on banks as intermediaries in macro economy. The Japanese bank-lending market constructed by Ogawa and Kitasaka (1998) has the mechanism that an interest rate of loans is determined when the loan market is in equilibrium. However, in reality, the loan interest rate can be affected by factors other than the demand or supply for loans.

We endogenize money stock (M1) in Shibata and Kosaka (2018),[1] which explains M1 in terms of reserve money and a bank's loans. The money supply is assumed to relate with loans via

[1] Shibata and Kosaka (2018) assume that money supply M^S is determined by reserve money RM and lending by banks like $M^S = f_{m^s}(RM, L_B)$. This study specifies L_B.

money creation—the so-called "money multiplier." By developing banks' lending behavior in this study and introducing it into the model of Shibata and Kosaka (2018) in the future, the transmission channel between the loan market and money supply is enhanced much further.

3) Assets and Liabilities Management (ALM) of Banks

We specify banking behavior that is partially motivated by the ALM concept being injected into our model for the purpose of reflecting the reality of the lending mechanism. ALM is a system for assessing and managing a banking company's risk exposure comprehensively, based on its balance sheet and income statement.

3. Model

In this section, we formalize the economic behavior of the financial intermediary sector. The model is composed of loan and portfolio optimization. Simplifying the concept of ALM, loan optimization stems from an understanding of the profit or loss statement, and portfolio optimization represents ideal balance-sheet management. Optimal lending is determined by profit maximization, and optimal portfolio by asset allocation theory (Markowitz, 1952). This model reflects current banks' behaviors.

3.1 Determining Optimal Banking Loans

We begin with considering the banks' profit maximization problem. Basically, banks attempt to choose optimal loans to households and firms in non-financial industries that maximize their profits generated from the spread between the interest rates charged on lending and that paid to depositors. Banks have some expenses like fees and commissions' expense other than interest expense for deposits. In addition, holdings of bad loans discourage banks from lending. The profit maximization problem of banks can be formalized as follows:

$$\pi_{LB} = \underbrace{-\frac{1}{2}w_{B1}\big(L_B - \phi_B(Deposit + CD)\big)^2}_{\text{the difference between loans and deposits}}$$

$$\underbrace{-\frac{1}{2}w_{B2}(L_{B1} - \alpha_{B1}I_F)^2}_{\text{the difference between capital investment loans and borrowing demand}}$$

$$\underbrace{-\frac{1}{2}w_{B3}(L_{B2} - \alpha_{B2}I_H)^2}_{\text{the difference between housing loans and borrowing demand}} \tag{1}$$

$$+\frac{1}{r_{LB}^*}\Bigg(\underbrace{r_{LB}L_B}_{\text{the interest income from loans}} \quad \underbrace{-r_{DT}(DP_0 + \beta_{B1}L_B)}_{\text{the interest expense for deposits}}$$

$$\underbrace{-\alpha_B\psi_{LB-1}L_B}_{\text{holding of bad debts}} \quad \underbrace{-\beta_B\omega_{LB-1}(Deposit + CD)}_{\text{fees and comissions' expenses}}\Bigg)$$

L_B	: outstanding loans	r_{LB}	: lending rate
L_{B1}	: capital investment loans	r_{DT}	: interest rate on deposits
L_{B2}	: housing loans	φ_{LB}	: loan to deposit ratio
$Deposit$	: outstanding deposits	$\psi_{LB,t-1}$	: holding ratio of bad debts during the previous
CD	: certificate of deposits	ω_{LB-1}	month
I_F	: fixed investments (SNA)	θ_{LB}	: operating costs
I_H	: housing investments (SNA)		: reserve requirement ratio

where $w_0, w_1, w_2, w_3, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ are parameters. Profit function (1) is an optimization problem with three constraints that are represented by a quadratic-loss term: i) the gap between loans and deposits, ii) the gap between loans and borrowing demand for capital investments, iii) the gap between loans and borrowing demand for housing investments. The quadratic-loss terms impose more penalties as the level of banks' lending become distant from ideal points. In addition, equation (1) illustrates the reality that the amount of bad debt leads to banks' reluctance to lend. With respect to L_{B1}, the first order condition for the banks' problems are defined as:

$$\frac{\partial \pi_{LB}}{\partial L_{B1}} = -w_{B1}\big(L_B - \phi_B(Deposit + CD)\big) - w_{B2}(L_{B1} - \alpha_{B1}I_F)\left(1 - \alpha_{B1}\frac{\partial I_F}{\partial L_{B1}}\right)$$
$$+\frac{r_{LB}}{r_{LB}^*} - \frac{r_{DT}\beta_{B1}}{r_{LB}^*} - \frac{\alpha_B\psi_{LB-1}}{r_{LB}^*} = 0 \tag{2}$$

where investment I_F is presumably affected by macroeconomic factors.[2] Here, the term of partial derivatives $\partial I_F / \partial L_{B1}$, we set δ_{LB1} as conjecture variation by banks.[3] Additionally, substituting r_{LB}^* to r_{LB} and rearranging equation (2), the revised equation is given as:

$$\begin{aligned}\big(w_{B1} + w_{B2}(1 - \alpha_{B1}\delta_{LB})\big)L_{B1} &= w_{B1}\phi_B(Deposit + CD) - w_{B1}L_{B2} \\ &+(1 - \alpha_{B1}\delta_{LB})w_{B2}\alpha_{B1}I_F - \frac{r_{DT}\beta_{B1}}{r_{LB}} - \frac{\alpha_B\psi_{LB-1}}{r_{LB}} + 1\end{aligned} \tag{3}$$

For simplicity, we insert $w_{B1} + w_{B2}(1 - \alpha_{B1}\delta_{LB}) = H_{LB1}$ for equation (4). By doing so, the optimal lending for capital investment given by,

$$\begin{aligned}L_{B1} = \frac{w_{B1}\phi_B}{H_{LB1}}(Deposit + CD) - \frac{w_{B1}}{H_{LB1}}L_{B2} + \frac{(1 - \alpha_{B1}\delta_{LB1})w_{B2}\alpha_{B1}}{H_{LB1}}I_H \\ - \frac{\beta_{B1}}{H_{LB1}}\frac{r_{DT}}{r_{LB}} - \frac{\alpha_B}{H_{LB1}}\frac{\psi_{LB-1}}{r_{LB}} + \frac{1}{H_{LB1}}\end{aligned} \tag{4}$$

In addition, following the foregoing process, the optimal lending for housing investment L_{B2} is defined as:

$$\begin{aligned}L_{B2} = \frac{w_{B1}\phi_B}{H_{LB2}}(Deposit + CD) - \frac{w_{B1}}{H_{LB2}}L_{B1} + \frac{(1 - \alpha_{B2}\delta_{LB2})w_{B3}\alpha_{B2}}{H_{LB}}I_F \\ - \frac{\beta_{B1}}{H_{LB2}}\frac{r_{DT}}{r_{LB}} - \frac{\alpha_B}{H_{LB2}}\frac{\psi_{LB-1}}{r_{LB}} + \frac{1}{H_{LB2}}\end{aligned} \tag{5}$$

Equations (4) and (5) state that the amount of the loan base depends on the level of deposits and investment demand from the macroeconomy. Also, if the interest rates earned on deposits are relatively higher (lower) than those paid on loans, banks tend to be encouraged to increase (decrease) loans. Simultaneously, the level of lending becomes negative in relation to the degree of bad loan holdings.

[2] This study assumes macroeconomic condition as given. However, this is one of important monetary transmission channels connecting between financial market and real economy. Linking to macro model would be realized in future studies.

[3] We assume that banks conjecture how the investment will be adjusted with respect to potential adjustments in macro economy.

3.2 Determining Optimal Asset Allocation of Banks

Next, we consider banks' asset allocation. We assume that banks invest the loan-deposit gap in assets as follows:

$$Deposit + CD - \bar{L}_B = SEC_{ALL} \tag{6}$$

where $\bar{L}_B$ means the optimal loans, which are determined by equations (4) and (5). SEC_{ALL} represents banks' holding securities. SEC_{ALL} contains these five assets:

$$SEC_{ALL} = SEC_{Gbond} + SEC_{Lbond} + SEC_{CBond} + SEC_{SEC} + SEC_{FOR} \tag{7}$$

SEC_{Gbond} means holding of federal and state government bonds, SEC_{Lbond} means municipal bonds, SEC_{CBond} means corporate bonds, SEC_{SEC} means stocks, SEC_{FOR} means foreign stocks. Also, the ratio of securities held is shown by:

$$w1_{Gbond} + w1_{Lbond} + w1_{Cbond} + w1_{SEC} + w1_{FOR} = 1 \tag{8}$$

where $w_{Gbond} = SEC_{Gbond}/SEC_{ALL}$, $w_{Lbond} = SEC_{Lbond}/SEC_{ALL}$, $w_{Cbond} = SEC_{Cbond}/SEC_{ALL}$, $w_{SEC} = SEC_{SEC}/SEC_{ALL}$ and $w_{FOR} = SEC_{FOR}/SEC_{ALL}$. They show the share ratio of government bond holdings, municipal bond holdings, corporate bond holdings, stock holdings, and foreign securities holdings, respectively.

Then, banks attempt to construct a portfolio of five assets which maximizes expected return. We assume optimization of portfolio by applying Markowitz (1952)'s mean-variance portfolio theory. Here, we begin with its general formulation. Portfolio return is defined as:

$$E(R) = w'f \tag{9}$$

$\sum$ is defined as the covariance matrix of returns.

$$\sigma^2 = w'\sum w \tag{10}$$

Investors are assumed to choose portfolios w in order to maximize expected return subject to the target level of risks. The mean-variance portfolio optimization is formulated as:

$$\max_{w} \qquad w'f = -\frac{1}{2\rho} w'\sum w \tag{11}$$

$$subject\ \ to \quad \sigma^2 = w'\Sigma w$$

$$w'u = 1$$

where f shows ρ represents risk capacity of the investor. To solve the constrained maximization problem (11), the Lagrangian is defined as:

$$F = w'f - \frac{1}{2\rho} w'\sum w + \lambda(w'u - 1) \tag{12}$$

where λ implies the Lagrangian multiplier. And the first order conditions for maximization are:

$$\frac{\partial F}{\partial w} = f - \frac{1}{\rho}\sum w + \lambda u = 0 \tag{13}$$

$$\frac{\partial F}{\partial \lambda} = (w'u - 1) = 0 \tag{14}$$

Rearranging equations (13) and (14), the following equations are obtained,

$$\sum w - \lambda\rho u = \rho f \tag{15}$$

$$w'u = 1 \tag{16}$$

The matrix representation is as follows;

$$\begin{pmatrix} \Sigma & -\rho u \\ u' & 0 \end{pmatrix} \begin{pmatrix} w \\ \lambda \end{pmatrix} = \begin{pmatrix} \rho f \\ 1 \end{pmatrix} \tag{17}$$

In this study, we assume that banks invest in five assets, namely federal and state government bonds, municipal bonds, corporate bonds, domestic stocks, and foreign securities. Namely, the covariance matrix of five risk assets is represented as follows:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} \end{pmatrix} \tag{18}$$

Replacing Σ of equation (17) for (18), we yield the following equation.

$$\begin{pmatrix} w1_{Gbond} \\ w1_{Lbond} \\ w1_{Cbond} \\ w1_{SEC} \\ w1_{FOR} \\ \gamma \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} & -\rho \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} & -\rho \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} & -\rho \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} & -\rho \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} & -\rho \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} \rho f_{GB} \\ \rho f_{LG} \\ \rho f_{C} \\ \rho f_{TOPIX} \\ \rho f_{S\&P} \\ 1 \end{pmatrix} \tag{19}$$

f_{GB} : expected government bond yield

f_{LG} : expected local government bond yield

f_{C} : expected corporate bond yield

f_{TOPIX} : expected stock return

$f_{S\&P}$: expected foreign securities return

We simplify the inverse covariance matrix of equation (19) as follows:

$$\begin{pmatrix} w1_{Gbond} \\ w1_{Lbond} \\ w1_{Cbond} \\ w1_{SEC} \\ w1_{FOR} \\ \gamma \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{pmatrix} \begin{pmatrix} \rho f_{GB} \\ \rho f_{LG} \\ \rho f_{C} \\ \rho f_{TOPIX} \\ \rho f_{S\&P} \\ 1 \end{pmatrix} \tag{20}$$

The optimal shares of asset holdings are generated as:

$$\begin{pmatrix} w1_{Gbond} \\ w1_{Lbond} \\ w1_{Cbond} \\ w1_{SEC} \\ w1_{FOR} \\ \gamma \end{pmatrix} = \begin{pmatrix} a_{11}\rho f_{GB} + a_{12}\rho f_{LG} + a_{13}\rho f_{C} + a_{14}\rho f_{TOPIX} + a_{15}\rho f_{S\&P} + a_{16} \\ a_{21}\rho f_{GB} + a_{22}\rho f_{LG} + a_{23}\rho f_{C} + a_{24}\rho f_{TOPIX} + a_{25}\rho f_{S\&P} + a_{26} \\ a_{31}\rho f_{GB} + a_{32}\rho f_{LG} + a_{33}\rho f_{C} + a_{34}\rho f_{TOPIX} + a_{35}\rho f_{S\&P} + a_{36} \\ a_{41}\rho f_{GB} + a_{42}\rho f_{LG} + a_{43}\rho f_{C} + a_{44}\rho f_{TOPIX} + a_{45}\rho f_{S\&P} + a_{46} \\ a_{51}\rho f_{GB} + a_{52}\rho f_{LG} + a_{53}\rho f_{C} + a_{54}\rho f_{TOPIX} + a_{55}\rho f_{S\&P} + a_{56} \\ a_{61}\rho f_{GB} + a_{62}\rho f_{LG} + a_{63}\rho f_{C} + a_{64}\rho f_{TOPIX} + a_{65}\rho f_{S\&P} + a_{66} \end{pmatrix} \tag{21}$$

Here, we modify equation (21) for estimation. The estimation of regression coefficients in equation (21) are likely to be unstable because the values of the dependent variables are ratios. Hence, we decompose them into two components, e.g., holdings of each portfolio and all securities holdings, and then utilize a variable for all securities holdings as the independent variable. The modified equation is shown as follows:

$$\begin{pmatrix} SEC1_{Gbond} \\ SEC1_{Lbond} \\ SEC1_{Cbond} \\ SEC1_{SEC} \\ SEC1_{FOR} \end{pmatrix} = \begin{pmatrix} a_{11}\rho f_{GB} + a_{12}\rho f_{LG} + a_{13}\rho f_{C} + a_{14}\rho f_{TOPIX} + a_{15}\rho f_{S\&P} + a_{16}SEC_{ALL} + a_{17} \\ a_{21}\rho f_{GB} + a_{22}\rho f_{LG} + a_{23}\rho f_{C} + a_{24}\rho f_{TOPIX} + a_{25}\rho f_{S\&P} + a_{26}SEC_{ALL} + a_{27} \\ a_{31}\rho f_{GB} + a_{32}\rho f_{LG} + a_{33}\rho f_{C} + a_{34}\rho f_{TOPIX} + a_{35}\rho f_{S\&P} + a_{36}SEC_{ALL} + a_{37} \\ a_{41}\rho f_{GB} + a_{42}\rho f_{LG} + a_{43}\rho f_{C} + a_{44}\rho f_{TOPIX} + a_{45}\rho f_{S\&P} + a_{46}SEC_{ALL} + a_{47} \\ a_{51}\rho f_{GB} + a_{52}\rho f_{LG} + a_{53}\rho f_{C} + a_{54}\rho f_{TOPIX} + a_{55}\rho f_{S\&P} + a_{56}SEC_{ALL} + a_{57} \end{pmatrix} \tag{22}$$

We apply equation (22) for empirical analysis.

4. Data

We consider tracing the bank's balance sheet and accounts in order to conduct empirical analysis for the conceptual discussion in the previous section. We use data from several sources.

(1) Financial Statements

Our main data sources for the financial statements of domestic banks are *Financial and Economic Statistics Monthly*, which were published by the Bank of Japan from February 1998 to May 2018. This statistic provides key data about the financial accounts and balance sheets from each banking category, e.g., like city and local banks, as well as total figures for domestic banks, including assets (loans/securities) and liabilities (deposits/certificates of deposit). The data of banks' holdings of securities are categorized into five kinds of assets like federal and state government bonds, municipal bonds, corporate bonds, stock or shares, and foreign securities.

(2) Interest Rates

Additionally, we use monthly data for interest rates published by Bank of Japan: uncollateralized overnight call rate, lending rate, certificate of deposit rate, ten-year government bond interest rate. Federal Funds rate, discount policy rate of the United States, garnered from the Federal Reserve Bank.

(3) Other Data

We also utilize financial statements and related data pertaining to all banks (i.e., balance sheets and income statements) complied by Japanese Bankers Association. The Japanese Bankers Association discloses these items, both on consolidated and non-consolidated bases, of individual banks, as well as the totals for each category of bank every six months (year-end/interim). In particular, our study employs data of risk-monitored loans (i.e., four levels of risk: loans to bankrupt borrowers, delinquent loans, loans past due three months or more, and restructured loans) to estimate an optimal lending model. However, the frequency of this data is different from those found in the monthly financial statements. We interpolate missing values for the time series by inserting the same value. The supply-side data of investments (i.e., real estate investments and capital investments) that banks provide to households and firms in non-financial industries are from bank accounts of domestically licensed banks (by the Bank of Japan). In contrast, the demand-side data pertaining to housing and capital investments for gross fixed-capital formation comes from the national accounts published by the Cabinet Office, government of Japan. As for the exchange rate, we use data from the *International Financial Statistics* database.

5. Empirical Results

5.1 Estimation Results

We estimate the stochastic equations of banking behavior by applying ordinary least squares. This section provides the estimation results.

5.1.1 Estimation Results of Optimal Loans

Table 2 gives the estimated results for optimal bank loans for capital investments as discussed in equation (4) of subsection 3.1.1. Statistics show that the optimal loan level for capital investment is well-estimated. The response of bank lending to the interest rate and bad loan holding is statistically explained well, too. The empirical results suggest that bad debt risk is associated with the banks' lending activities, resulting in them being reluctant to lend. Lending by banks is less responsive to private demand for investments. Further, Table 3 describes the results of optimal bank loan levels for housing investment seen in equation (5). Similarly, just as our results pertaining to capital investment showed, we confirm the response of bank lending to interest rate changes and the level of bad debts. Also, housing investments cannot confirm the effect of private demand for housing investment. We conclude that the optimal lending model of banks is acceptable.

Table 2. Optimal Loan for Capital Investment：Sample 1998M2-2018M05

Explanatory Variables	Coefficient	S.E.
Deposit (*t*-3) plus Certificate of Deposit (*t*-3)	0.302***	0.007
Outstanding of Lending for Housing	-0.423***	0.039
Demand for Investment (*t*-12)	0.385	0.278
Interest Rate on Deposit (*t*-12)/ Interest Rate on Lending (*t*-12)	-12413.430*	6779.996
Holding of Bankruptcy Debts (*t*-12) / Interest Rate on Lending (*t*-9)	-0.367***	0.054
Constant	461012.6***	39869.04
Observation		232
Adj. R-squared		0.988

Table 3. Optimal Loans for Housing: Sample 1998M10-2018M05

Explanatory Variables	Coefficient	S.E.
Deposit (*t*-3) plus Certificate of Deposit (*t*-3)	0.464***	0.020
Outstanding of Lending for Fixed Investment	-1.280***	0.082
Interest Rate on Deposit (*t*-12)/ Interest Rate on Lending (*t*-12)	-120766.8**	46114.470
Holding of Bankruptcy Debts (*t*-12) / Interest Rate on Lending (*t*-9)	-1.001***	0.077
Dummy from 2008M09 to 2009M12	33813.790***	11529.810
Constant	685876.700***	42788.060
Observation		236
Adj. R-squared		0.987

Note: Adj. *R*-Squared is adjusted *R*-squared. *** $p < 0.001$, ** $p < 0.05$, and * $p < 0.1$, respectively.

5.1.2 Estimation Results of the Optimal Portfolio

In this subsection, we show estimated results for our portfolio selection model of equation (22). The result of each asset is reported below.

(1) Optimal Government Bond Holdings

Banks are assumed to determine the optimal investment amount in each asset class based on expected returns of each asset and its variance. In Table 4, the optimal portfolio for federal and state government bonds is reported. Table 4 is divided into three panels: Panel A for domestic banks, Panel B for city banks, and Panel C for local banks. All panels have the same signs on their coefficients across the panels. Expected returns from government bonds are negative in relation to expected returns on other assets. Overall, the results are robust.

Table 4. Portfolio for Government Bonds: Sample 1998M1-2018M2

	Dependent Variables		
	Panel A:	Panel B:	Panel C:
Independent Variable	All Banks	City Banks	Local Banks
Interest Rate of Government Bond 10 Year	202488.000***	81104.200**	78546.180***
	[50788.25]	[25546.29]	[16716.76]
Interest Rate of Local Government Bond	-133845.900**	-51143.66**	-50800.760**
	[52855.34]	[24008.71]	[16860.27]
Tokyo Stock Price Index (TOPIX)	-93.527***	-90.949***	-53.880***
	[26.655]	[15.432]	[5.333]
Standard & Poor's 500 Stock Index	-122.531***	7.202	3.290
	[32.374]	[14.606]	[6.428]
All Holding Securities & Bonds	0.902***	0.919***	0.594***
	[0.019]	[0.014]	[0.016]
Constant	-604569.400***	-288549.9***	-76066.730***
	[65489.32]	[24004.82]	[9555.003]
Observation	242	242	242
Adj. R-squared	0.987	0.992	0.957

Note: Adj. *R*-Squared is adjusted *R*-squared. *** $p < 0.001$, ** $p < 0.05$, and * $p < 0.1$, respectively.

(2) Optimal Corporate and Local Bond Holdings

As Table 5 and Table 6 show a portfolio of municipal and corporate bonds. The blank spaces signify that the estimation is conducted by omitting items from the model due to statistical problems. Table 5 tells that local government bond holdings tend to be motivated by expected returns from foreign securities. Although the result of corporate bond holdings by city banks (Panel B of Table 6) is weak, the others are well-estimated.

Table 5. Portfolio of Local Government Bonds: Sample 1998M1-2018M2

	Dependent Variables		
	Panel A:	Panel B:	Panel C:
Independent Variable	All Banks	City Banks	Local Banks
Interest Rate of Government Bond 10 Year	-41060.44***		
	[8505.419]		
Interest Rate of Local Government Bonds	25258.34**	4319.123***	-5363.709***
	[9211.27]	[689.788]	[628.781]
Tokyo Stock Price Index (TOPIX)	4.956**	-4.849***	-4.95**
	[2.263]	[1.634]	[1.292]
Standard & Poor's 500 Stock Index		5.96***	
		[1.436]	
All Holding Securities & Bonds	0.024***	-0.016***	0.078***
	[0.003]	[0.001]	[0.004]
Constant	72010.43***	23594.84***	47238.33***
	[8562.443]	[2334.28]	[3271.221]
Observation	242	293	293
Adj. R-squared	0.706	0.702	0.885

Note: Adj. *R*-Squared is adjusted *R*-squared. *** $p < 0.001$, ** $p < 0.05$, and * $p < 0.1$, respectively.

Table 6. Portfolio of Corporate Bonds: Sample 1998M1-2018M2

	Dependent Variables		
	Panel A:	Panel B:	Panel C:
Independent Variable	All Banks	City Banks	Local Banks
Interest Rate of Government Bond 10 Year			-8084.94***
			(6242.767]
Interest Rate of Local Government Bond	20648.94***	3202.5730.1561	25956.64**
	[2156.306]	[2251.874]	[6790.84]
Tokyo Stock Price Index (TOPIX)	34.225***	52.597***	-27.622***
	[5.477]	[7.13]	[3.128]
Standard & Poor's 500 Stock Index		-27.929***	39.183***
		[5.469]	[3.304]
All Holding Securities & Bonds	0.13***	0.1***	0.114***
	[0.005]	[0.005]	[0.008]
Constant	-55214.92***	-47874.87***	17046.49***
	[15216.63]	[9811.584]	(6184.475]
Observation	293	293	242
Adj. R-squared	0.75	0.475	0.910

Note: Adj. *R*-Squared is adjusted *R*-squared. *** $p < 0.001$, ** $p < 0.05$, and * $p < 0.1$, respectively.

(3) <u>Optimal Stock Holdings</u>

In Table 7, the blank spaces signify that the estimation was conducted by omitting certain parameters from model due to statistical problems encountered. Overall, the results are weak. The results show an inverse relationship between bonds yields and stocks, which has changed over time. Additionally, according to result of domestic banks, the relationship between the domestic stock market and foreign ones is negative.

Table 7. Portfolio of Japanese Stocks: Sample 1998M1-2018M2

	Dependent Variables		
	Panel A:	Panel B:	Panel C:
Independent Variable	All Banks	City Banks	Local Banks
Interest Rate of Government Bond 10 Year	52584.1***	26931.14***	1248.0240.2982
	[14563.38]	[4494.233]	[1197.153]
Tokyo Stock Price Index (TOPIX)	-80.956***	-35.465***	-7.318**
	[26.931]	[8.568]	[1.516]
Standard & Poor's 500 Stock Index	77.238***		
	[26.028]		
All Holding Securities & Bonds	-0.214***	-0.177***	-0.032***
	[0.013]	[0.01]	[0.005]
Constant	617031.8***	348543***	70565.72***
	[37955.43]	[20177.89]	[4441.92]
Observation	242	242	242
Adj. R-squared	0.693	0.674	0.361

Note: Adj. *R*-Squared is adjusted *R*-squared. *** $p < 0.001$, ** $p < 0.05$, and * $p < 0.1$, respectively.

(4) Optimal Foreign Securities Holdings

Table 8[4] summarizes the results of foreign securities holdings by all national banks and city banks. It represents that the foreign securities holdings are strongly motivated by the returns generated by the U.S. stock market index. As city banks have a positive sign with respect to TOPIX, we can see the interrelation of movement between the Japanese stock index and the U.S.'s.

Table 8. Portfolio of Foreign Securities: Sample 1998M1-2018M2

	Dependent Variables	
	Panel A:	Panel B:
Independent Variable	All Banks	City Banks
Interest Rate of Government Bond 10 Year	-71796.710***	-64692.49***
	[15169.96]	[6353.378]
Tokyo Stock Price Index (TOPIX)		51.415***
		[10.235]
Standard & Poor's 500 Stock Index	91.388***	18.1160.07
	[16.181]	[9.952]
All Holding Securities & Bonds	0.118***	0.144***
	[0.016]	[0.007]
Constant	33513.420	10943.680.4628
	[44452.59]	[14881.04]
Observation	242	242
Adj. R-squared	0.877	0.877

Note: Adj. *R*-Squared is adjusted *R*-squared. *** $p < 0.001$, ** $p < 0.05$, and * $p < 0.1$, respectively.

[4] As local banks' foreign securities holdings is quite slight, we do not estimate local banks' model.

6. Conclusion

This study specifies a model of banking behavior based on an optimization process: profit maximization and efficient asset allocation. In particular, our model definitely differs from other previous studies in that the crucial concept of ALM is partially embedded into our model in order to reflect the practical aspects of bank management. Some variables might be unsatisfactory. Nevertheless, the overall performance of this system is acceptable. The estimated results suggest that our theoretical approach has grasped the reality of the loan market.

However, some improvements must be made in order to make the model into a more applicable framework for analyzing the reality of the loan market sufficiently. First, we consider that a profit function should include other elements such as an equity ratio by following the concept of ALM. Second, as our model based on ALM does not introduce the foreign exchange market, we cannot trace the reality of the currency risk that banks face. Third, we should take in to consideration the land market, which affects the real economy through the collateral channel. As a bad loan is assumed to be an exogenous variable in the profit function in our model, it is insufficient to calculate economic impacts, e.g., the bubble burst in Japan and the global financial crisis.

This study is also rather simplistic and tentative, but it is just getting started. In the future, we are planning to introduce this banking model into the macromonomeric model that we have developed so far. By doing so, our model may not only investigate the relevancy between negative- and low-interest rates and a bank's lending behavior, but may also evaluate the adverse effects of such rates on banking behavior and macroeconomic issues. We believe that a full model would render better guidance in explaining how a central bank's monetary policy generates impacts on the real economy via various financial instrument markets.

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