INSTITUTE OF DEVELOPING ECONOMIES

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IDE DISCUSSION PAPER No. 757

Online and Offline Sales in a Spatial Economy

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March 2019

Abstract

This study analyzes the interplay between cost-sharing and imperfect information among online firms competing with offline firms within an industry. By incorporating firm level quality, it shows how the intensity of cost-sharing, local market size, the number of regions being serviced, and the transport costs affect the expected quality of products, the "richness" of the varieties sold in the online market and social welfare. One of our results shows that a high intensity of cost-sharing, such as costs relating to the warehouse provided by the online platform, forces the lowest-quality firms to exit the online market but has no impact on the entry of higher-quality firms into the offline market. As a result, the average quality of products sold in the online market improves and the product variety decreases. Consequently, the welfare remains unchanged.

Keywords: Heterogeneous firms; Online; Offline

JEL classification: D04, R12

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1 Introduction

The purpose of this paper is to contribute to a better understanding of electronic commerce by showing how the interplay between imperfect information and cost-sharing among online firms affects whether firms choose online or offline sales. Building a setting with the heterogeneous quality of products allows us to investigate how the combination of imperfect information and cost-sharing among online firms affects welfare.

Electronic commerce has many aspects, as explained in Borenstein and Saloner (2001). Online sales are characterized by easy access to many types of information, asynchronized communication, and tailored information. As a result, better matching between consumers and sellers can be achieved, and the costs related to product handling, theft, rents and selling costs are saved. Furthermore, geographically dispersed offline stores incur inventory costs, whereas online firms may enjoy economies of centralized inventories. The uncertainty or imperfect information of offline shopping can be considered the primary shortcoming of purchasing an item from offline firms as all information on the item is not transmitted perfectly via the internet. However, the consumers often prefer not to wait for the arrival of an item bought from an online marketplace.

Online sellers outsource many tasks to the selling platform (such as Amazon.com) in order to avoid the activities for which it is difficult for a single seller to achieve economies of scale. Thus, online sellers can enjoy more outputs and higher labor productivity as demonstrated in a study on outsourcing IT (Han, Kauffman, and Nault, 2011). The development and operation costs may decrease if a large number of sellers gather in the platform as Nocke, Peitz, and Sthal (2007) discussed that development costs are shared equally among platform owners. Hounde, Newberry, and Seim (2017) clarify that the economies of density works in Amazon.com's delivery network.

We consider firms with heterogeneous product quality to express imperfect information as well as endogenous fixed costs of online firms. The greater the number of users of an online selling platform, the lower are the fixed costs of the firms because of cost-sharing. To understand these two mechanisms, we do not focus on the search and matching and the waiting costs associated with online purchases among the characteristics of online sales, although Goldmanis, Hortaçsu, Syverson, and Emre (2009) and Williams (2018) analyze the former, and Loginova (2009) focuses on the latter. In a multiple-region setting, online firms can share the fixed costs as the number of regions increases while offline firms need to incur the same fixed costs when entering each region.

Our paper's focus is similar to that of Chen, Hu, and Li (2017). Firms of heterogeneous quality choose an online or offline market, and then the quality of products is disclosed in the offline market while remaining hidden in the online market. Furthermore, the higher fixed costs of the offline market corresponds to the cost for disclosing information. The analytical framework of Chen, Hu, and Li (2017) concerns vertical product differentiation under oligopolies in the literature of industrial organization. In contrast, our analytical framework is based on the Dixit-Stiglitz model of monopolistic competition, which is popular in trade, economic growth and the spatial economy. Furthermore, Chen, Hu, and Li (2017) do not consider the cost sharing among online sellers, but their consumers do have heterogeneous preference. Consequently, they show that products with very low quality might be sold in the offline market, whereas our result shows that the lowest quality products are sold online.

The importance of sensory examination differs among products. Using the results of consumer survey on clothes, books and digital cameras in online and offline markets, Gruber (2009) shows that offline (resp. online) channel for clothes (resp. digital cameras) generally reveals more price dispersion, while books take up a moderate position. Higher price dispersion could be regarded as an indicator of differentiating with quality or services. The case extremely relying on sensory examination will be art auction. Kazumori and McMillan (2005) shows that higher value items are more likely to be sold live than online auction empirically. Furthermore, they show that the lower valuation uncertainty leads for sellers to choose online auction theoretically and empirically. The low value uncertainty can be interpreted as low-quality products. Thus, our model illustrates the market on the product in which there is a huge gap of information between online and offline markets like clothes.

Our main findings are as follows. The impact of cost sharing differs between low and high-quality firms in the online market. As a result, a larger market benefits consumers and embraces fewer firms in the economy and fewer varieties with higher quality in the online market. Comparing firm with and without an online sales presence, we find that the number of offline firms in the case without an online presence is larger than of the case with an online presence. However, consumers receive the same levels of welfare in both cases due to the same price indices induced by imperfect information. This result differs from Brynjolfsson, Smith, and Hu (2003) which estimates that consumer welfare is enhanced by the increased product variety in the case of online bookstores. Furthermore, without online firms, the market outcome is the same as the second-best optimum obtained by choosing the product quality threshold. That is, both the case with two markets and the case without an online market achieve the same level of social welfare as the second-best optimum. In a multiple-region setting, firms choose to be online firms, offline domestic firms and offline exporters according to their quality. We find that the more integrated economy provides a higher level of social welfare because of the higher expected quality of the online market and the smaller number of online firms.

Krautheim (2012) introduces cost sharing in the Dixit-Stiglitz monopolistic competition model for heterogeneous firms, accounting for the fixed costs of exporting which, in turn, decreases with the number of exporters. To determine the number of exporters, the study assumes that the total number of firms in an industry is fixed, and under these conditions, the entry and exit of firms into an industry is not affected, but we endogenize total number of firms.

The paper is organized as follows. Section 2 explains a one-region model, showing the equilibrium, comparative analysis, and the social welfare. Section 3 analyzes a multiple-region model. Section 4 provides concluding remarks.

2 The model

2.1 Basic setup

A country comprises a continuum of firms producing horizontally differentiated products under the Dixit-Stiglitz (1977) model of monopolistic competition. We denote the population of the country as L. Each individual inelastically supplies one unit of labor, which is the only production factor. Without loss of generality, we take labor as the numéraire. That is, the wage rate $w = 1$ holds. Thus, the individual income and regional income are, respectively, given by $y^* = 1$ and $Y^* = y^* L = L$.

2.1.1 Demand

All consumers share the same homothetic preference and the utility function is given by:

$$U \equiv \left\{ \int_{\omega\in\Omega} \varphi(\omega) \left[q(\omega)\right]^{\frac{\sigma-1}{\sigma}} d\omega \right\}^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where Ω is the set of available varieties, $\varphi(\omega)$ is the *product attractiveness* or *product quality* of variety ω, and $\sigma > 1$ is the elasticity of substitution between any two varieties.

Utility maximization of the representative consumer yields the demand for variety ω given by:

$$q(\omega) = \frac{L}{\mathcal{P}} \left[\frac{p(\omega)/\varphi(\omega)}{\mathcal{P}} \right]^{-\sigma}, \tag{2}$$

where the price index is given by

$$\mathcal{P} \equiv \left[\int_{\omega\in\Omega} \varphi(\omega)^{\sigma} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

2.1.2 Production

Following Melitz and Ottaviano (2008), we assume that firms only exist for a certain period. There are $\mathcal{N}$ potential firms, which operate under increasing returns to scale with no economies of scope. Thus, each firm produces a single variety and each variety is produced by a single firm. To produce, a firm needs a marginal requirement c units of labor. Choosing the unit of each variety, we set $c = (\sigma - 1)/\sigma$. Prior to entry, firms are identical and face uncertainty about their quality level φ. Entry requires a sunk cost of F_e units of labor. Once this cost is paid, firms observe their quality index $\varphi \in (0, +\infty)$ from the common distribution $g(\varphi)$ which has positive supports over $(0, +\infty)$ and has the cumulative distribution $G(\varphi)$.

We assume that there exist two channels for selling: online and offline. Prior to selling its product, each firm incurs a fixed labor requirement. Consumers have perfect information about firms' quality index φ through offline shopping, while they have imperfect information about the online firms. Specifically, consumers have the only information about the highest quality, $\bar{\varphi}$, and the lowest quality, $\underline{\varphi}$, in the the online market. For simplicity, we assume that consumers have an expected value of quality index $\mathbb{E}\varphi$, which is a sufficient statistics such that

$$\mathbb{E}\varphi \equiv \int_{\underline{\varphi}}^{\bar{\varphi}} \varphi \mu_N(\varphi) d\varphi, \tag{3}$$

where $\underline{\varphi}$ and $\bar{\varphi}$ are, respectively, the lowest and highest quality indices of firms using online sale; $\mu_N(\varphi)$ is the conditional density function of $g(\varphi)$ on $[\underline{\varphi}, \bar{\varphi}]$. Therefore, the demand of an online firm $\varphi \in [\underline{\varphi}, \bar{\varphi}]$ is given by:

$$q^N(\varphi) = \frac{L}{\mathcal{P}} \left[\frac{p(\varphi)/\mathbb{E}\varphi}{\mathcal{P}} \right]^{-\sigma}. \tag{4}$$

As there are positive externalities of cost-sharing among online firms, each online firm incurs a fixed labor requirement as follows:

$$f_x = f n_x^{-\alpha}, \quad \alpha \geq 1,$$

where f is the cost ceiling for online firms and $n_x > 1$ is the mass of available varieties sold in the online market. Correspondingly, the profit of an online firm $\varphi \in [\underline{\varphi}, \bar{\varphi}]$ is given by:

$$\pi^N(\varphi) = [p^N(\varphi) - c] q^N(\varphi) - f_x, \tag{5}$$

where $q^N(\varphi)$ is given by (4).

The profit maximization yields an online firm φ's optimal price:

$$p^N(\varphi) = \frac{\sigma}{\sigma - 1} c = 1. \tag{6}$$

Substituting (2) and (6) into (5) yields the profit of the online firm $\varphi \in [\underline{\varphi}, \bar{\varphi}]$ and can be rewritten as:

$$\pi^N(\varphi) = \frac{L}{\sigma} \frac{(\mathbb{E}\varphi)^\sigma}{\mathcal{P}^{1-\sigma}} - f_x. \tag{7}$$

Thus, (7) indicates that the profits of all online firms are at the same level regardless of their types.

Before entering the offline market, each firm must pay a fixed requirement of $F > f$ units of labor. Meanwhile, we assume the iceberg form of transportation costs in the offline market: $T \in (1, t)$ units of a variety should be shipped in order to ensure the delivery of one unit to a consumer. Correspondingly, the profit of an offline firm φ is given by:

$$\pi^F(\varphi) = [p^F(\varphi) - c]q^F(\varphi) - F, \tag{8}$$

where $q^F(\varphi)$ is given by (2). The profit maximization yields the offline firm φ's optimal price:

$$p^F(\varphi) = \frac{\sigma}{\sigma - 1}c = 1. \tag{9}$$

Substituting (2) and (9) into (8) provides the demand and profit of the offline firm φ:

$$\begin{aligned} q^F(\varphi) &= L\frac{\varphi^\sigma}{\mathcal{P}^{1-\sigma}}, \\ \pi^F(\varphi) &= \frac{L}{\sigma}\frac{\varphi^\sigma}{\mathcal{P}^{1-\sigma}} - F. \end{aligned} \tag{10}$$

2.1.3 Zero cutoff profit condition

A firm disguise himself in the online market such that the zero cutoff profit condition for an online firm $\pi^N(\varphi) = 0, \forall \varphi \in (\underline{\varphi}, \bar{\varphi})$. Then, *the equilibrium expected quality index of online firms* $\mathbb{E}\varphi$ is obtained as:

$$\mathbb{E}\varphi = \left[\frac{\sigma f_x \mathcal{P}^{1-\sigma}}{L}\right]^{\frac{1}{\sigma}}. \tag{11}$$

The zero cutoff profit condition in the offline market $\pi^F(\varphi) = 0$ yields *the threshold offline firm* φ_F, which is the lowest quality index of active offline firms:

$$\varphi_F \equiv \left[\frac{\sigma F \mathcal{P}^{1-\sigma}}{L}\right]^{\frac{1}{\sigma}}. \tag{12}$$

Assuming that $f_x < F$ holds, we obtain:

$$\mathbb{E}\varphi < \varphi_F, \tag{13}$$

which is the sufficient condition for the coexistence of online firms and offline firms. The indifference between entering the online and offline markets determines *the marginal firm* φ_I, which is the root to $\pi^F(\varphi) = \pi^N(\varphi)$. Since $\pi^N(\varphi) = 0, \forall \varphi \in (\underline{\varphi}, \bar{\varphi})$, setting $\pi^F(\varphi) = \pi^N(\varphi) = 0$ leads to:

$$\frac{L}{\sigma}\frac{\varphi_I^\sigma}{\mathcal{P}^{1-\sigma}} - F = \frac{L}{\sigma}\frac{(\mathbb{E}\varphi)^\sigma}{\mathcal{P}^{1-\sigma}} - f_x = 0 \Longrightarrow \quad \varphi_I = \varphi_F = \bar{\varphi}.$$

Therefore, firm $\varphi \in [\underline{\varphi}, \varphi_F]$ chooses to be an online firm and earns zero profit, and firm $\varphi \in (\varphi_F, +\infty)$ chooses to be an offline firm and earns a positive profit. As a result, all potential firms are active but operate differently due to their market choices.

2.1.4 Aggregation

Equilibrium is characterized by the mass of survival firms N and the distribution of productivity levels. Since firm $\varphi \in [\underline{\varphi}, \varphi_F]$ enters the online market, let $\mu_N(\varphi)$ be the conditional distribution of $g(\varphi)$ on online firms:

$$\mu_N(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_F) - G(\underline{\varphi})} & \text{if} \quad \underline{\varphi} \leq \varphi \leq \varphi_F, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The mass of active firms in the online market n_x is given by

$$n_x = \mathcal{N} \int_{\underline{\varphi}}^{\varphi_F} g(\varphi) d\varphi. \tag{15}$$

Let $\mu_F(\varphi)$ be the conditional distribution of $g(\varphi)$ on offline firms:

$$\mu_F(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi_F)} & \text{if} \quad \varphi_F \leq \varphi < +\infty, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

The mass of active firms in the offline market n_o is determined by

$$n_o = \mathcal{N} \int_{\varphi_F}^{+\infty} g(\varphi) d\varphi. \tag{17}$$

Using (14) and (16), the consumer price index is given by

$$\mathcal{P}^{1-\sigma} \equiv \mathcal{N} \left[(\mathbb{E}\varphi)^\sigma \int_{\underline{\varphi}}^{\varphi_F} g(\varphi) d\varphi + \int_{\varphi_F}^{+\infty} \varphi^\sigma g(\varphi) d\varphi \right] \tag{18}$$

2.1.5 Free Entry

Free entry of firms is expressed as

$$F_e = \int_{\underline{\varphi}}^{\varphi_F} \pi^N(\varphi) g(\varphi) d\varphi + \int_{\varphi_F}^{+\infty} \pi^F(\varphi) g(\varphi) d\varphi. \tag{19}$$

Since $\pi^N(\varphi) = 0$, $\forall \varphi \in (\underline{\varphi}, \bar{\varphi})$, the first term in the LHS of (19) becomes 0. Thus, (19) can be written as:

$$F_e = \int_{\varphi_F}^{+\infty} \left[\frac{L\varphi^\sigma}{\sigma \mathcal{P}^{1-\sigma}} - F \right] g(\varphi) d\varphi.$$

Although all online firms are indifferent between operations and exit, we assume that online firms are active in offering jobs for workers. Thus, the market-clearing condition for labor yields:

$$\frac{L}{\mathcal{N}} = F_e + \int_{\underline{\varphi}}^{\varphi_F} [f_x + cq^N(\varphi)] g(\varphi) d\varphi + \int_{\varphi_F}^{\infty} [F + cq^F(\varphi)] g(\varphi) d\varphi. \tag{20}$$

Note that the resource constraint affects both online and offline firms.

2.2 Equilibrium

Substituting (6) and (11) into (4) yields the expected demand of online firms:

$$q^N(\varphi) = \sigma f_x. \tag{21}$$

Combining (12) and (19) yields:

$$\mathcal{H}(\varphi_F) \equiv \int_{\varphi_F}^{+\infty} \left(\frac{\varphi^\sigma}{\varphi_F^\sigma} - 1 \right) g(\varphi) d\varphi - \frac{F_e}{F} = 0. \tag{22}$$

It is readily verified that $\mathcal{H}(0) = +\infty$ and $\mathcal{H}(+\infty) = -F_e/F < 0$ hold. Meanwhile, we have

$$\frac{\partial \mathcal{H}(\varphi_F)}{\partial \varphi_F} = -\frac{\sigma}{\varphi_F^{\sigma+1}} \int_{\varphi_F}^{+\infty} \varphi^\sigma g(\varphi) d\varphi < 0.$$

Therefore, there exists a unique solution $\varphi_F^* > 0$ in (22). Different from Melitz (2003), φ_F^* is determined by the free entry condition and the zero cutoff condition of only offline firms because of imperfect information such that $\pi^N(\varphi) = 0, \forall \varphi \in (\underline{\varphi}, \bar{\varphi})$.

Substituting (18) into (12) yields the mass of potential firms given by:

$$\mathcal{N} = \frac{L(\varphi_F^*)^\sigma}{\sigma F \left\{ \left[\int_{\underline{\varphi}}^{\varphi_F^*} \varphi g(\varphi) d\varphi \right]^\sigma \left[G(\varphi_F^*) - G(\underline{\varphi}) \right]^{1-\sigma} + \int_{\varphi_F^*}^{+\infty} \varphi^\sigma g(\varphi) d\varphi \right\}}. \tag{23}$$

Combining (3), (11), (12), and (23) yields

$$\begin{aligned}\mathcal{M}(\underline{\varphi}) \equiv& 1+\int_{\varphi_F^*}^{+\infty}\varphi^\sigma g(\varphi)d\varphi\frac{\left[\int_{\underline{\varphi}}^{\varphi_F^*} g(\varphi)d\varphi\right]^{\sigma-1}}{\left[\int_{\underline{\varphi}}^{\varphi_F^*}\varphi g(\varphi)d\varphi\right]^{\sigma}}\\ &-\frac{L}{\sigma F}\left(\frac{F}{f}\right)^{\frac{1}{\alpha}}\left[\frac{\varphi_F^*\int_{\underline{\varphi}}^{\varphi_F^*} g(\varphi)d\varphi}{\int_{\underline{\varphi}}^{\varphi_F^*}\varphi g(\varphi)d\varphi}\right]^{\frac{(\alpha-1)\sigma}{\alpha}}=0. \end{aligned}\tag{24}$$

We assume $g(\varphi)$ has a heavy tail such that the following inequality holds:

$$\sigma\underline{\varphi}\int_{\underline{\varphi}}^{\varphi_F^*} g(\varphi)d\varphi-(\sigma-1)\int_{\underline{\varphi}}^{\varphi_F^*}\varphi g(\varphi)d\varphi>0. \tag{25}$$

Under (25), we obtain $\partial\mathcal{M}(\underline{\varphi})/\partial\underline{\varphi}>0$. Furthermore, we have $\lim_{\underline{\varphi}\to\varphi_F^*}\mathcal{M}(\underline{\varphi})=+\infty$. By defining $\mathbb{M}(L)\equiv\lim_{\underline{\varphi}\to 0}\mathcal{M}(\underline{\varphi})$, it is readily verified that $\mathbb{M}'(L)<0$ holds. We further assume that the market size L surpasses a critical level such that:

$$L>\underline{L}\equiv \mathrm{I}/\mathrm{II}, \tag{26}$$

where

$$\begin{aligned}\mathrm{I}\equiv& 1+\int_{\varphi_F^*}^{+\infty}\varphi^\sigma g(\varphi)d\varphi\frac{\left[\int_0^{\varphi_F^*} g(\varphi)d\varphi\right]^{\sigma-1}}{\left[\int_0^{\varphi_F^*}\varphi g(\varphi)d\varphi\right]^{\sigma}},\\ \mathrm{II}\equiv&\frac{1}{\sigma F}\left(\frac{F}{f}\right)^{\frac{1}{\alpha}}\left[\frac{\varphi_F^*\int_0^{\varphi_F^*} g(\varphi)d\varphi}{\int_0^{\varphi_F^*}\varphi g(\varphi)d\varphi}\right]^{\frac{(\alpha-1)\sigma}{\alpha}}.\end{aligned}$$

Under (26), we have $\mathbb{M}(L)<0$ holds. As a result, (24) has a unique solution $\underline{\varphi}^*\in(0,\varphi_F^*)$ under (25) and (26). Note that $\underline{\varphi}^*$ is determined by a combination of zero cutoff conditions of online and offline firms that share the same price index and market size. The profit of the threshold online firms $\underline{\varphi}$ becomes positive if $\mathcal{M}(\underline{\varphi})<0$, otherwise the profit becomes negative.

Substituting $\underline{\varphi}^*$ and φ_F^* into (3) and (14), we obtain the equilibrium expected quality index of online firms given by:

$$(\mathbb{E}\varphi)^*=\frac{1}{G(\varphi_F^*)-G(\underline{\varphi}^*)}\int_{\underline{\varphi}^*}^{\varphi_F^*}\varphi g(\varphi)d\varphi. \tag{27}$$

Substituting φ_F^* into (23) yields the equilibrium mass of potential firms $\mathcal{N}^*$. Thus, the equilibrium mass of active firms N^* is given by:

$$N^* = \mathcal{N}^* \left[1 - G(\underline{\varphi}^*)\right].$$

Therefore, (12) yields the price index:

$$\mathcal{P}^* = \left[\frac{L(\varphi_F^*)^\sigma}{\sigma F}\right]^{\frac{1}{1-\sigma}}. \tag{28}$$

Combining (11) and (28) yields the equilibrium mass of online firms (n_x^*):

$$n_x^* = \left(\frac{f}{F}\right)^{\frac{1}{\alpha}} \left[\frac{\varphi_F^*}{(\mathbb{E}\varphi)^*}\right]^{\frac{\sigma}{\alpha}}. \tag{29}$$

and combining (15) and (17) yields the equilibrium mass of offline firms (n_o^*):

$$n_o^* = \frac{\int_{\varphi_F^*}^{+\infty} g(\varphi)d\varphi}{\int_{\underline{\varphi}^*}^{\varphi_F^*} g(\varphi)d\varphi} n_x^*. \tag{30}$$

2.3 Comparative analysis

2.3.1 General distribution

We now focus on the impact of market size on the equilibrium. From (22) and (24), we have:

$$\frac{\partial \varphi_F^*}{\partial L} = 0, \qquad \frac{\partial \underline{\varphi}^*}{\partial L} = -\frac{\partial \mathcal{M}(\underline{\varphi}^*)/\partial L}{\partial \mathcal{M}(\underline{\varphi}^*)/\partial \underline{\varphi}^*} > 0. \tag{31}$$

As Melitz (2003) clarified, the market size has no impact on the threshold offline firm φ_F^* because the market expansion effect equals to the competition effect for the marginal firm as the market size increases. However, a larger market size increases the competition effect more than the market expansion effect for online firms, resulting in an increase in the threshold quality of online firms $\underline{\varphi}^*$, which differs from the result on the threshold productivity of firms in Melitz (2003).

The case without cost sharing is expressed by $f_x = f$. Setting $\alpha = 0$, which means $f_x = f$, and applying it to (24), we obtain an expression which is equivalent to $\mathcal{M}(\underline{\varphi}) = 0$. Then, we find that $\underline{\varphi}^*$ remains unchanged with an increase in market size if cost sharing is not allowed. That is, each firm does not receive the same benefit in the online market.

Furthermore, (28) implies that $\partial\mathcal{P}^*/\partial L < 0$. In other words, consumers benefit from a larger economy due to a decrease of the price index.

From (27) and (31), we have

$$\frac{d(\mathbb{E}\varphi)^*}{dL} = \underbrace{\frac{\partial(\mathbb{E}\varphi)^*}{\partial\underline{\varphi}^*}}_{(+)}\underbrace{\frac{\partial\underline{\varphi}^*}{\partial L}}_{(+)} > 0.$$

A larger market size intensifies competition among online firms and increases the threshold quality of online firms. Thus, the expected quality of online firms increases with the market size.

Differentiating (29) with respect to L, we have:

$$\frac{dn_x^*}{dL} = \underbrace{\frac{\partial n_x^*}{\partial(\mathbb{E}\varphi)^*}}_{(-)}\underbrace{\frac{d(\mathbb{E}\varphi)^*}{dL}}_{(+)} < 0.$$

This decrease of n_x^* by an increase in market size leads to an increase of f_x, which implies that the size of online firms are larger in larger region. In Melitz (2003), an increase in fixed costs means a higher zero cutoff profit and cutoff level. That is, our model is qualitatively different from Melitz (2003). Since a larger market size increases the expected quality of online firms, consumers tend to buy more from each online firm. As a result, online firms rely less on the cost-sharing mechanism to break even, resulting in a decrease in the mass of online firms. In other words, online firms mitigate competition by increasing the fixed costs.

Taking the derivative of n_o^* with respect to L yields:

$$\frac{dn_o^*}{dL} = \frac{n_o^*\left[\int_{\underline{\varphi}^*}^{\varphi_F^*}\varphi g(\varphi)d\varphi\right]\left[\int_{\underline{\varphi}^*}^{\varphi_F^*} g(\varphi)d\varphi\right]}{\alpha g(\underline{\varphi}^*)}\left[\sigma\underline{\varphi}^*\int_{\underline{\varphi}^*}^{\varphi_F^*} g(\varphi)d\varphi - (\sigma-\alpha)\int_{\underline{\varphi}^*}^{\varphi_F^*}\varphi g(\varphi)d\varphi\right] > 0.$$

It is readily verified that $\partial n_o^*/\partial L > 0$ holds because of (25) and $\alpha \geq 1$. Combining (27), (29) and (30), we obtain

$$\frac{dN^*}{dL} = \frac{d\left(n_x^* + n_o^*\right)}{dL} < 0, \qquad \frac{d\mathcal{N}^*}{dL} = \frac{d}{dL}\left(\frac{n_x^* + n_o^*}{1 - G(\underline{\varphi}^*)}\right) > 0.$$

In other words, a larger market size attracts more potential entrants, but causes fewer firms to survive because of tougher competition.

Proposition 1 *An increase in market size increases competition among firms; thus, consumers benefit from the decrease in the price index. Tougher competition among firms due to cost sharing increases the threshold quality of online firms while keeping the marginal firm, which is indifferent between online and offline markets, unchanged. This results in a decrease in the mass of online firms and an increase in the mass of offline firms. Therefore, an increase in market size attracts more potential entrants and causes fewer firms to survive.*

Owing to the flexibility of the model, we can explore the impact of the intensity of cost-sharing α on the equilibrium. From (22) and (24), we have:

$$\frac{\partial \underline{\varphi}^*}{\partial \alpha} = -\underbrace{\frac{\partial \mathcal{M}(\underline{\varphi}^*)}{\partial \alpha}}_{(+)} \underbrace{\left[\frac{\partial \mathcal{M}(\underline{\varphi}^*)}{\partial \underline{\varphi}^*}\right]^{-1}}_{(-)} > 0.$$

Meanwhile, combining (22) and (27), we have:

$$\frac{d(\mathbb{E}\varphi)^*}{d\alpha} = \underbrace{\frac{\partial (\mathbb{E}\varphi)^*}{\partial \underline{\varphi}^*}}_{(+)} \underbrace{\frac{\partial \underline{\varphi}^*}{\partial \alpha}}_{(+)} > 0. \tag{32}$$

Thus, the stronger the intensity of cost sharing in the online market, the higher the threshold quality of online firms and the expected quality index of online firms. This is due to the tougher competition in the online market. As (22) shows, a change of α has no impact on φ_F^*. Then, using (32) and (29), we obtain $\partial n_x^*/\partial \alpha < 0$, which implies that the fewer number of online firms operate in the presences of a higher intensity of cost-sharing, owing to the reduction in competition in the online market because we obtain $\partial f_x/\partial \alpha > 0$, which implies that more intensity of cost-sharing leads to the larger size of online firms. In other words, the intensity of cost-sharing improves the average quality of products sold in the online market, but causes a decrease in the mass of varieties offered.

Proposition 2 *A higher intensity of the cost sharing forces firms of the least quality to exit from the online market, but has no impact on the entrance of high-quality firms to the offline market. As a result, the average quality of products sold in the online market improves, and the number of varieties offered decreases. However, the welfare remains unchanged.*

2.3.2 Pareto distribution

We now consider a special heavy-tailed distribution to identify the impact of the similarity of firms. We assume that each firm's quality index $\varphi > 1$ follows the Pareto distribution given by:

$$g(\varphi) = \kappa\varphi^{-(\kappa+1)}, \tag{33}$$

where $\kappa > \sigma$ is the shape parameter. The Pareto distribution offers an advantage in that the shape parameter κ is a measure for the similarity of firms (von Ehrlich and Seidel, 2013) or an inverse measure of "evenness" defined as the similarity between the probabilities of those different draws to happen (Ottaviano, 2012). In specific, a high value of κ implies that it becomes less likely to draw a high quality index φ. Thus, smaller κ leads to an increase in heterogeneity along the evenness dimension.

Under the Pareto distribution, the sufficient condition (25) to $\partial\mathcal{M}(\underline{\varphi})/\partial\underline{\varphi} > 0$ can be written by:

$$(\kappa - \sigma)[(\varphi_F^*)^\kappa - \underline{\varphi}^\kappa] + \kappa(\sigma - 1)\underline{\varphi}^{\kappa-1}(\varphi_F^* - \underline{\varphi}) > 0.$$

which always holds because of $\varphi_F^* > \underline{\varphi} > 1$. Meanwhile, (26) can be written by

$$L > \underline{L} \equiv \frac{1 + \frac{(\kappa-1)^\sigma}{\kappa^{\sigma-1}(\kappa-\sigma)} \frac{[(\varphi_F^*)^\kappa - 1]^{\sigma-1}}{[(\varphi_F^*)^{\kappa-1} - 1]^\sigma}}{\frac{1}{\sigma F}\left(\frac{F}{f}\right)^{\frac{1}{\alpha}} \left\{\frac{(\kappa-1)[(\varphi_F^*)^\kappa - 1]}{\kappa[(\varphi_F^*)^{\kappa-1} - 1]}\right\}^{\frac{\sigma(\alpha-1)}{\alpha}}}.$$

Plugging (33) into (22) yields:

$$\varphi_F^* = \left(\frac{\sigma}{\kappa - \sigma}\frac{F}{F_e}\right)^{1/\kappa} > 1. \tag{34}$$

and substituting (34) into the price index (28) yields:

$$\mathcal{P}^* = \left[\frac{L}{\sigma F}\left(\frac{\sigma}{\kappa - \sigma}\frac{F}{F_e}\right)^{\sigma/\kappa}\right]^{\frac{1}{1-\sigma}}. \tag{35}$$

We now turn to the impacts of the similarity of firms κ on the equilibrium variables. From (34), we have

$$\frac{\partial\varphi_F^*}{\partial\kappa}\frac{\kappa}{\varphi_F^*} = -\left(\frac{1}{\kappa - \sigma} + \ln\varphi_F^*\right) < 0.$$

Although the market size has no impact on φ_F^*, an increase in κ, which implies that the distribution becomes more concentrated at the lowest level of quality, weakens competition and results in a low threshold quality of offline firms.

Next, from (35), we have:

$$\frac{\partial \mathcal{P}^*}{\partial \kappa}\frac{\kappa}{\mathcal{P}^*} = \frac{\sigma}{\sigma - 1}\left(\frac{1}{\kappa - \sigma} + \ln \varphi_F^*\right) > 0.$$

An increase in κ weakens competition, results in an increase of the price index. Therefore, consumers suffer from an increase in the similarity of firms.

2.4 Rethinking the online market

In order to understand the function of the online market, we consider a special case in which there are only offline firms. Specifically, we compare the case of the offline-only market with the case of online and offline markets.

2.4.1 Equilibrium

In the case of the offline-only market, *the offline firm threshold* φ_F^{NOM} is determined by (12) with the new price index given by:

$$(\mathcal{P}^{\mathrm{NOM}})^{1-\sigma} \equiv \mathcal{N}^{\mathrm{NOM}} \int_{\varphi_F^{\mathrm{NOM}}}^{+\infty} \varphi^\sigma g(\varphi) d\varphi. \tag{36}$$

where $\mathcal{P}^{\mathrm{NOM}}$ and $\mathcal{N}^{\mathrm{NOM}}$ show the price index and the mass of potential firms in the case of the offline-only market, respectively.

The free entry condition is the same as (19). Thus, combining (12) and (19) yields (22), which implies that the threshold quality of offline firm in the case of offline-only market satisfies $\varphi_F^{\mathrm{NOM}} = \varphi_F^*$.

Substituting (36) into (12) yields the equilibrium mass of potential firms without an online market $\mathcal{N}^{\mathrm{NOM}}$ given by:

$$\mathcal{N}^{\mathrm{NOM}} = \frac{L}{\sigma F}\frac{(\varphi_F^{\mathrm{NOM}})^\sigma}{\int_{\varphi_F^{\mathrm{NOM}}}^{+\infty} \varphi^\sigma g(\varphi) d\varphi}. \tag{37}$$

Substituting (37) into (36) yields:

$$(\mathcal{P}^{\mathrm{NOM}})^{1-\sigma} = \frac{L}{\sigma F}(\varphi_F^{\mathrm{NOM}})^\sigma. \tag{38}$$

Since $\varphi_F^{\mathrm{NOM}} = \varphi_F^*$ holds, (28) and (38) imply that the two price indices, with and without the online market, are equal, i.e., $\mathcal{P}^{\mathrm{NOM}} = \mathcal{P}^*$ holds. On one hand, price index is a "sufficient statistics "for the magnitude of competition among firms. Thus, all firms face the same intensity of competition in both cases. On the other hand, price index is also an inverse index of indirect utility. From the view of social welfare, the above two cases have the same market performance.

Combining (23), (24), and (37) yields the relationship between $\mathcal{N}^*$ and $\mathcal{N}^{\text{NOM}}$ as follows:

$$\mathcal{N}^* = \left[1 - \frac{\sigma f}{L(n_x^*)^{\alpha-1}}\right] \mathcal{N}^{\text{NOM}} > 0.$$

Thus, we obtain $\mathcal{N}^* < \mathcal{N}^{\text{NOM}}$. The intuition is as follows: since the threshold quality of offline firm and the price indices are equal in both cases (i.e., $\varphi_F^{\text{NOM}} = \varphi_F^*$ and $\varphi_F^{\text{NOM}} = \varphi_F^*$ hold), the demands for labor of the offline firm $\varphi > \varphi_F^* = \varphi^{\text{NOM}}$ are also equal in both cases. The market-clearing condition for labor (20) implies that more potential entrants in the case without online firms. In other words, the reallocation of labor from online firms to offline firms increases the number of entrants in the case without online firms.

Last but not the least, there exists the following relationship between n_o^* and n_o^{NOM} as follows:

$$n_o^* \equiv \mathcal{N}^*[1 - G(\varphi_F^*)] < n_o^{\text{NOM}} \equiv \mathcal{N}^{\text{NOM}}[1 - G(\varphi_F^{\text{NOM}})] = N^{\text{NOM}}.$$

In other words, the mass of offline firms in the case without online firms is larger than that of the case with online firms.

Proposition 3 *The case with two kinds of markets has both a greater richness of firm quality and a smaller mass of potential entrants than those in the case without an online market, i.e.,* $\underline{\varphi}^* < \varphi_F^{NOM} = \varphi_F^*$ *and* $\mathcal{N}^{NOM} > \mathcal{N}^*$. *Thus, the mass of offline firms is larger in the case without online firms than that of the case with online firms. However, consumers have the same levels of welfare in both cases due to the same price indices by imperfect information.*

2.4.2 Second-best optimum

We now consider the second-best optimum when there are only offline firms. In this case, the social planner chooses the optimal threshold quality φ_F to maximize the social welfare.

The timing is as follows. In Step 1, the social planner chooses the optimal threshold quality. In Step 2, each firm draws its product attractiveness after paying entry costs. The firms with product quality lower than the optimal threshold exit the market. Otherwise, firms can choose to produce or quit. In Step 3, each consumer chooses his or her demands for all available varieties in order to maximize his utility.

We use backward induction to solve the second-best optimum problem. At the first stage, in observing prices of all available varieties, the representative consumer maximizes his or her utility given by (1). Thus, we obtain the demand for variety ω with product quality $\varphi(\omega)$ given by (2). At the second stage, facing consumers' demand (2) and the optimal threshold quality $\varphi_F^{\text{SB}} \geq \varphi_F^*$[1] chosen by the social planner, firm $\varphi \geq \varphi_F^{\text{SB}}$ chooses its price strategy to maximize profit where φ_F^{SB} is the threshold quality of offline firm chosen as the second-best optimum. Otherwise, firm $\varphi < \varphi_F^{\text{SB}}$ exit from the market. Profit maximization yields firm φ's optimal price given by (9). Accordingly, its profit is given by (10). Note that each active firm $\varphi \geq \varphi_F^{\text{SB}}$ generates a nonnegative profit.

At the third stage, the welfare maximization problem of the social planner is described by as follows:

$$\varphi_F^{\text{SB}} = \arg\max_{\varphi_F \geq \varphi_F^*} \mathcal{P}^{1-\sigma} \equiv \mathcal{N} \int_{\varphi_F}^{+\infty} \varphi^\sigma g(\varphi) d\varphi, \tag{39}$$

The social planner has a resource constraint, which is equivalent to the market-clearing condition for labor given by:

$$\frac{L}{\mathcal{N}} = F_e + \int_{\varphi_F}^{+\infty} [F + cq^F(\varphi)] g(\varphi) d\varphi. \tag{40}$$

Substituting (40) into (39), the welfare maximization problem can be written by

$$\varphi_F^{\text{SB}} = \arg\max_{\varphi_F \geq \varphi_F^*} \mathcal{P}^{1-\sigma} = \frac{1}{\sigma} \frac{L \int_{\varphi_F}^{+\infty} \varphi^\sigma g(\varphi) d\varphi}{F_e + F \int_{\varphi_F}^{\infty} g(\varphi) d\varphi}. \tag{41}$$

Thus, the FOC of welfare maximization (41) is obtained as:

$$\frac{\partial \mathcal{P}^{1-\sigma}}{\partial \varphi_F} = -\frac{1}{\sigma} \frac{L g(\varphi_F)}{F_e + F \int_{\varphi_F}^{\infty} g(\varphi) d\varphi} \left[\varphi_F^\sigma - \frac{F \int_{\varphi_F}^{+\infty} \varphi^\sigma g(\varphi) d\varphi}{F_e + F \int_{\varphi_F}^{\infty} g(\varphi) d\varphi} \right] = 0. \tag{42}$$

[1]The optimal threshold quality φ_F^{SB} should be no less than the equilibrium threshold quality φ_F^*, i.e., $\varphi_F^{\text{SB}} \geq \varphi_F^*$. Otherwise, φ_F^{SB} will never be binding.

Note that (42) is equivalent to (22), which implies $\varphi_F^{\text{SB}} = \varphi_F^*$. Meanwhile, the SOC of welfare maximization (41) is given by:

$$\left.\frac{\partial^2 \mathcal{P}^{1-\sigma}}{\partial \varphi_F^2}\right|_{\varphi_F \to \varphi_F^{\text{SB}} = \varphi_F^*} = -\frac{(\varphi_F^*)^{\sigma-1} g(\varphi_F^*) L}{F_e + F \int_{\varphi_F^*}^{\infty} g(\varphi) d\varphi} < 0.$$

Since the resource constraint (40) is the same as the market-clearing condition when there is only an offline market, the mass of potential entrants in the second-best optimum $\mathcal{N}^{SB}$ is the same as $\mathcal{N}^*$. Accordingly, the free entry condition (19) holds in the second-best optimum. Therefore, the market outcome is the same as the second-best optimum when there is only an offline market.

Proposition 4 *If there are only offline firms, the market outcome is the same as the second-best optimum. Furthermore, both the case with two markets and the case without an online market achieve the same level of social welfare as the second-best optimum.*

3 Multiple Regions

3.1 Setup

We now consider that the economy consists of a number of symmetric regions indexed by $r = 1, 2, ..., R$. Consumers in each region share the same homothetic preferences given by (1). We assume that each region is endowed with L population that supplies L units of labor inelastically. Without loss of generality, we take labor in a region as numéraire. The symmetricity implies that the equilibrium wage rates in any two different regions are equal, i.e., $w_r = w_s = 1$, $\forall r$, $\forall s \neq r$ holds.

Firm heterogeneity, $G_r(\varphi)$, takes the same form among all regions such that $G_r(\varphi) = G(\varphi)$, $\forall r$. After paying the same sunk entry costs ($F_{e,r} = F_e$, $\forall r$), a firm in region r draws its quality index φ from the cumulative distribution $G(\varphi)$. To provide a certain variety to region s, an offline firm in region r needs to incur F units of the fixed requirement of labor and $c = (\sigma - 1)/\sigma$ units of the marginal requirement of labor. In other words, F units of labor are required as an entry cost for an offline firm to sell its product to a region. In contrast, all online firms in the economy benefit from the *unified* online market and can access to all regions after paying the one-time fixed requirement f_x. The fixed requirement of an online firm is given as follows:

$$f_x = f\left(\sum_{r=1}^{R} n_{x,r}\right)^{-\alpha} = f(Rn_x)^{-\alpha}. \tag{43}$$

where $n_{x,r}$ is the mass of online firms in region r. As in Samuelson (1954) and in common with studies in New Economic Geography, the same iceberg transport costs exist among any two regions. In specific, $\tau > 1$ units of goods must be shipped from region r in order to ensure delivery of one unit to region $s \neq r$.

Profit maximization, respectively, yields the consumer prices in region r and $s \neq r$ of an online or offline firm φ located in region r:

$$p_{rr}(\varphi) = \frac{\sigma}{\sigma-1}c = 1, \qquad p_{rs}(\varphi) = \frac{\sigma}{\sigma-1}c\tau = \tau.$$

Owing to the unified online market, there are no further fixed costs required for online firms to meet the demand of interregional consumers. Thus, each active online firm will serve consumers in all regions. The profit of online firm $\varphi \in [\underline{\varphi}, \varphi_F)$ is given by

$$\pi^N(\varphi) = \frac{L}{\sigma}\frac{[1+(R-1)\phi](\mathbb{E}\varphi)^{\sigma}}{\mathcal{P}^{1-\sigma}} - f_x,$$

where $\phi \equiv \tau^{1-\sigma} \in (0,1)$ represent the degree of trade freeness. Thus, the zero cutoff profit condition of online firms yields:

$$\mathbb{E}\varphi = \left\{\frac{\sigma f_x \mathcal{P}^{1-\sigma}}{L[1+(R-1)\phi]}\right\}^{\frac{1}{\sigma}}. \tag{44}$$

While, each offline firm in region r needs to pay an extra fixed entry cost of F units of labor to serve the consumers in region $s \neq r$. Thus, there exists a gap between the operating profit of the home market, $\pi_d^F(\varphi)$, and that of an external market, $\pi_t^F(\varphi)$, for offline firm $\varphi > \varphi_F$ as follows:

$$\pi_d^F(\varphi) \equiv \frac{r_d^F(\varphi)}{\sigma} - F > \pi_t^F(\varphi) \equiv \phi\frac{r_d^F(\varphi)}{\sigma} - F,$$

where $r_d^F(\varphi)$ is the revenue of offline firm $\varphi > \varphi_F$ from the home market and $\pi_d^F(\varphi_F) = 0$ holds. Thus, a threshold exporting offline firm $\varphi_X(> \varphi_F)$ earns zero operating profit from an external market , i.e., $\pi_t^F(\varphi_X) = 0$ holds. Thus, the operating profit of offline firm $\varphi \in (\varphi_F, \varphi_X)$, who only serves its home market, is given by:

$$\pi^F(\varphi) = \pi_d^F(\varphi) \equiv \frac{L\varphi^\sigma}{\sigma \mathcal{P}^{1-\sigma}} - F.$$

The zero cutoff profit condition of offline firm φ_F yields (12). It is readily verified that $\pi_t^F(\varphi_X) = 0$ yields the threshold exporting offline firm φ_X given by

$$\varphi_X = \left(\frac{\sigma F \mathcal{P}^{1-\sigma}}{L\phi}\right)^{\frac{1}{\sigma}}. \tag{45}$$

Note that, using (12) and (45), we obtain

$$\varphi_X = \widetilde{\tau}\varphi_F, \tag{46}$$

where $\widetilde{\tau} \equiv \tau^{1-1/\sigma} = \phi^{-1/\sigma}$. Therefore, firm $\varphi < \underline{\varphi}$ is inactive; firm $\varphi \in [\underline{\varphi}, \varphi_F)$ chooses to be an online firm and earns zero profit; firm $\varphi \in [\varphi_F, \varphi_X)$ chooses to be an offline firm serving only its home market and earns positive profit; and firm $\varphi \geq \varphi_X$ chooses to be an offline firm serving all regions and earns a large positive profit. Thus, the operating profit of offline firm $\varphi \geq \varphi_X$, who serves both home and external markets, is given by

$$\pi^F(\varphi) = \pi_d^F(\varphi) + (R-1)\pi_t^F(\varphi) = \frac{L}{\sigma}\frac{[1+(R-1)\phi]\varphi^\sigma}{\mathcal{P}^{1-\sigma}} - RF.$$

As a result, the conditional distribution of $g(\varphi)$ on online firms, $\mu_N(\varphi)$, is given by (14), and the conditional distribution of $g(\varphi)$ on offline firms serving only home market, $\mu_d(\varphi)$, is given by

$$\mu_d(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_X)-G(\varphi_F)} & \text{if} \quad \varphi_F \leq \varphi < \varphi_X, \\ 0 & \text{otherwise}, \end{cases}$$

and the conditional distribution of $g(\varphi)$ on offline firms serving all regions, $\mu_T(\varphi)$, is given by

$$\mu_t(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_X)} & \text{if} \quad \varphi_X \leq \varphi < +\infty, \\ 0 & \text{otherwise}. \end{cases}$$

The mass of active firms in the online market n_x is the same as (15). The mass of active offline firms who serves only their home markets, n_d, and those who serves both home and external markets, n_t, are, respectively, given by:

$$n_d = \mathcal{N}\int_{\varphi_F}^{\varphi_X} g(\varphi)d\varphi, \qquad n_t = \mathcal{N}\int_{\varphi_X}^{+\infty} g(\varphi)d\varphi.$$

The price index is now rewritten as:

$$\mathcal{P} \equiv \left\{ [1+\phi(R-1)]\, \mathcal{N} \left[(\mathbb{E}\varphi)^{\sigma} \int_{\underline{\varphi}}^{\varphi_F} g(\varphi)d\varphi + \int_{\varphi_X}^{+\infty} \varphi^{\sigma} g(\varphi)d\varphi \right] + \mathcal{N} \int_{\varphi_F}^{\varphi_X} \varphi^{\sigma} g(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}}. \tag{47}$$

Free entry condition yields:

$$F_e = \int_{\varphi_F}^{+\infty} \pi_d^F(\varphi) g(\varphi)d\varphi + (R-1) \int_{\varphi_X}^{+\infty} \pi_t^F(\varphi) g(\varphi)d\varphi. \tag{48}$$

and the market-clearing condition for labor can be rewritten as:

$$\begin{aligned} \frac{L}{\mathcal{N}} =& F_e + \int_{\underline{\varphi}}^{\varphi_F} \left\{ f_x + c[q_{rr}^N(\varphi) + (R-1) q_{rs}^N(\varphi)\tau] \right\} g(\varphi)d\varphi \\ &+ \int_{\varphi_F}^{+\infty} [F + c q_{rr}^F(\varphi)] g(\varphi)d\varphi + (R-1) \int_{\varphi_X}^{+\infty} [F + c q_{rs}^F(\varphi)\tau] g(\varphi)d\varphi. \end{aligned} \tag{49}$$

3.2 Equilibrium

Substituting (46) into (48) yields:

$$\mathcal{H}_M(\varphi_F) \equiv \int_{\varphi_F}^{+\infty} \left(\frac{\varphi^{\sigma}}{\varphi_F^{\sigma}} - 1 \right) g(\varphi)d\varphi + (R-1) \int_{\widetilde{\tau}\varphi_F}^{+\infty} \left(\frac{\phi\varphi^{\sigma}}{\varphi_F^{\sigma}} - 1 \right) g(\varphi)d\varphi - \frac{F_e}{F} = 0. \tag{50}$$

It is readily verified that $\mathcal{H}_M(0) = +\infty$, $\mathcal{H}_M(+\infty) = -F_e/F < 0$, and

$$\frac{\partial \mathcal{H}_M(\varphi_F)}{\partial \varphi_F} = -\frac{\sigma}{\varphi_F^{\sigma+1}} \int_{\varphi_F}^{+\infty} \varphi^{\sigma} g(\varphi)d\varphi - \frac{\sigma\phi(R-1)}{\varphi_F^{\sigma+1}} \int_{\widetilde{\tau}\varphi_F}^{+\infty} \varphi^{\sigma} g(\varphi)d\varphi < 0.$$

hold. Therefore, there exists a unique solution $\varphi_F^* > 0$ to (50). Thus, we obtain $\varphi_X^* = \widetilde{\tau}\varphi_F^* > \varphi_F^*$ because of (46).

Substituting (44) and (46) into (47) yields the mass of potential firms given by

$$\mathcal{N} = \frac{L}{\sigma F} \frac{(\varphi_F)^{\sigma}}{[1+\phi(R-1)] \left[(\mathbb{E}\varphi)^{\sigma} \int_{\underline{\varphi}}^{\varphi_F} g(\varphi)d\varphi + \int_{\varphi_X}^{+\infty} \varphi^{\sigma} g(\varphi)d\varphi \right] + \int_{\varphi_F}^{\varphi_X} \varphi^{\sigma} g(\varphi)d\varphi}. \tag{51}$$

Combining (3), (12), (43), (44), and (51) yields

$$\begin{aligned} \mathcal{M}_M(\underline{\varphi}) \equiv& 1 + \phi(R-1) - \frac{LR}{\sigma F} \left\{ \frac{F}{f} [1+\phi(R-1)] \right\}^{1/\alpha} \left[\frac{\varphi_F^* \int_{\underline{\varphi}}^{\varphi_F^*} g(\varphi)d\varphi}{\int_{\underline{\varphi}}^{\varphi_F^*} \varphi g(\varphi)d\varphi} \right]^{\frac{(\alpha-1)\sigma}{\alpha}} \\ &+ \left[\phi(R-1) \int_{\widetilde{\tau}\varphi_F^*}^{+\infty} \varphi^{\sigma} g(\varphi)d\varphi + \int_{\varphi_F^*}^{+\infty} \varphi^{\sigma} g(\varphi)d\varphi \right] \frac{\left[\int_{\underline{\varphi}}^{\varphi_F^*} g(\varphi)d\varphi \right]^{\sigma-1}}{\left[\int_{\underline{\varphi}}^{\varphi_F^*} \varphi g(\varphi)d\varphi \right]^{\sigma}} = 0. \end{aligned} \tag{52}$$

We assume $g(\varphi)$ has a heavy tail such that (25) holds. Under (25), we obtain $\partial \mathcal{M}_M(\underline{\varphi})/\partial \underline{\varphi} > 0$. Furthermore, we have $\lim_{\underline{\varphi}\to\varphi_F^*} \mathcal{M}_M(\underline{\varphi}) = +\infty$. By defining $\mathbb{M}_M(L) \equiv \lim_{\underline{\varphi}\to 0} \mathcal{M}_M(\underline{\varphi})$, it is readily verified that $\mathbb{M}'_M(L) < 0$ holds. We further assume that the market size L surpasses a critical level such that

$$L > \underline{L}_R \equiv \mathrm{I}_M/\mathrm{II}_M, \tag{53}$$

where

$$\mathrm{I}_M \equiv 1 + \phi(R-1) + \left[\phi(R-1)\int_{\tilde{\tau}\varphi_F^*}^{+\infty} \varphi^\sigma g(\varphi)d\varphi + \int_{\varphi_F^*}^{+\infty} \varphi^\sigma g(\varphi)d\varphi\right] \frac{\left[\int_0^{\varphi_F^*} g(\varphi)d\varphi\right]^{\sigma-1}}{\left[\int_0^{\varphi_F^*} \varphi g(\varphi)d\varphi\right]^{\sigma}},$$

$$\mathrm{II}_M \equiv \frac{R}{\sigma F}\left\{\frac{F}{f}\left[1+\phi(R-1)\right]\right\}^{1/\alpha} \left[\frac{\varphi_F^* \int_0^{\varphi_F^*} g(\varphi)d\varphi}{\int_0^{\varphi_F^*} \varphi g(\varphi)d\varphi}\right]^{\frac{(\alpha-1)\sigma}{\alpha}}.$$

Under (53), we confirm that $\mathbb{M}_M(L) < 0$ holds. As a result, equation $\mathcal{M}_M(\underline{\varphi}) = 0$ has a unique root $\underline{\varphi}^* \in (0, \varphi_F^*)$ under (25) and (53).

Meanwhile, because (12) is the same as the single region case, the equilibrium expected quality index of online firm $(\mathbb{E}\varphi)^*$ and the price index $\mathcal{P}^*$ are, respectively, given by (27) and (28). Thus, combining (44) and (28) yields the equilibrium mass of online firms (n_x^*) given by:

$$n_x^* = \frac{1}{R}\left\{\frac{f}{\left[1+(R-1)\,\phi\right]F}\right\}^{\frac{1}{\alpha}} \left[\frac{\varphi_F^*}{(\mathbb{E}\varphi)^*}\right]^{\frac{\sigma}{\alpha}}. \tag{54}$$

Finally, the equilibrium mass of offline firms serving only the home market, n_d^*, and the equilibrium mass of offline firms serving all regions, n_t^*, are,respectively, obtained as:

$$n_d^* = \frac{G(\varphi_X^*) - G(\varphi_F^*)}{G(\varphi_F^*) - G(\underline{\varphi}^*)} n_x^*, \qquad n_t^* = \frac{1 - G(\varphi_X^*)}{G(\varphi_F^*) - G(\underline{\varphi}^*)} n_x^*.$$

3.3 Comparative analysis

From (50), we have:

$$\frac{\partial \mathcal{H}_M(\varphi_F^*)}{\partial \phi} = (R-1)\int_{\tilde{\tau}\varphi_F^*}^{+\infty} \frac{\varphi^\sigma}{(\varphi_F^*)^\sigma} g(\varphi)d\varphi > 0.$$

and the implicit function theorem yields:

$$\frac{\partial \varphi_F^*}{\partial \phi} = -\frac{\frac{\partial \mathcal{H}_M(\varphi_F^*)}{\partial \phi}}{\frac{\partial \mathcal{H}_M(\varphi_F^*)}{\partial \varphi_F^*}} > 0. \tag{55}$$

Thus, increasing trade openness leads to a larger φ_F^*.

Substituting $\varphi_F^* = \widetilde{\tau}^{-1}\varphi_X^*$ into (50), we obtain

$$\mathcal{H}_M(\varphi_X^*) \equiv \int_{\varphi_X^*/\widetilde{\tau}}^{+\infty} \left[\frac{\varphi^\sigma}{\phi(\varphi_X^*)^\sigma} - 1\right] g(\varphi)d\varphi + (R-1)\int_{\varphi_X^*}^{+\infty} \left[\frac{\varphi^\sigma}{(\varphi_X^*)^\sigma} - 1\right] g(\varphi)d\varphi - \frac{F_e}{F} = 0.$$

and

$$\begin{aligned}
\frac{\partial \mathcal{H}_M(\varphi_X^*)}{\partial \varphi_X^*} &= -\frac{\sigma}{\phi(\varphi_X^*)^{\sigma+1}}\int_{\varphi_X^*/\widetilde{\tau}}^{+\infty} \varphi^\sigma g(\varphi)d\varphi - (R-1)\frac{\sigma}{(\varphi_X^*)^{\sigma+1}}\int_{\varphi_X^*}^{+\infty} \varphi^\sigma g(\varphi)d\varphi < 0 \\
\frac{\partial \mathcal{H}_M(\varphi_X^*)}{\partial \phi} &= -\frac{1}{\phi^2(\varphi_X^*)^\sigma}\int_{\varphi_X^*/\widetilde{\tau}}^{+\infty} \varphi^\sigma g(\varphi)d\varphi < 0,
\end{aligned}$$

which implies that $\partial\varphi_X^*/\partial\phi < 0$. In other words, both online and offline firms who sell their products to external regions benefit from the decrease in transport costs. As a result, offline firms who serve only their home market suffer from the more integrated economy, resulting in an increase of φ_F^* and a decrease of φ_X^*.

Due to the cumbersome expression, we can only explore the impact of trade freeness on the threshold online firm $\underline{\varphi}^*$ in the case of $\alpha = 1$. As we obtained in the one-region model, it is readily verified that $\partial\underline{\varphi}^*/\partial\alpha > 0$. Thus, the impact on $\underline{\varphi}^*$ which we show below will be magnified in the case of $\alpha > 1$. Setting $\alpha = 1$ in (52) and using (55) yields

$$\left.\frac{d\mathcal{M}_M(\underline{\varphi}^*)}{d\phi}\right|_{\alpha=1} = \underbrace{\left.\frac{\partial\mathcal{M}_M(\underline{\varphi}^*)}{\partial\phi}\right|_{\alpha=1}}_{(-)} + \underbrace{\left.\frac{\partial\mathcal{M}_M(\underline{\varphi}^*)}{\partial\varphi_F^*}\right|_{\alpha=1}}_{(-)}\underbrace{\left.\frac{\partial\varphi_F^*}{\partial\phi}\right|_{\alpha=1}}_{(+)} < 0.^2$$

Thus, we obtain

$$\left.\frac{\partial\underline{\varphi}^*}{\partial\phi}\right|_{\alpha=1} = -\underbrace{\left.\frac{d\mathcal{M}_M(\underline{\varphi}^*)}{d\phi}\right|_{\alpha=1}}_{(-)}\left[\underbrace{\left.\frac{\partial\mathcal{M}_M(\underline{\varphi}^*)}{\partial\underline{\varphi}^*}\right|_{\alpha=1}}_{(+)}\right]^{-1} > 0.$$

The more integrated economy intensifies the competition among online firms and also improves the market access to external regions. The former dominates the latter, resulting in an increase of the threshold online firm $\underline{\varphi}^*$. Meanwhile, an increase of $\underline{\varphi}^*$ and φ_F^* leads

[2] It is readily verified that $\left.\frac{\partial\mathcal{M}_M(\underline{\varphi}^*)}{\partial\phi}\right|_{\alpha=1} < 0$ and $\left.\frac{\partial\mathcal{M}_M(\underline{\varphi}^*)}{\partial\varphi_F^*}\right|_{\alpha=1} < 0$ hold when L is sufficiently large such that $L < \underline{L}_M \equiv \frac{\sigma f}{R}\left\{1 + \int_{\widetilde{\tau}\varphi_F^*}^{+\infty}\varphi^\sigma g(\varphi)d\varphi\frac{\left[\int_{\underline{\varphi}^*}^{\varphi_F^*} g(\varphi)d\varphi\right]^{\sigma-1}}{\left[\int_{\underline{\varphi}^*}^{\varphi_F^*}\varphi g(\varphi)d\varphi\right]^{\sigma}}\right\}$ holds.

to an increase in the expected quality of online firms $(\mathbb{E}\varphi)^*$ in a more integrated economy, i.e., $\partial(\mathbb{E}\varphi)^*/\partial\phi > 0$ holds.

We now further consider the impact of the number of regions on the equilibrium. It is readily verified that $\partial\mathcal{H}_M(\varphi_F^*)/\partial R > 0$ and $\partial\mathcal{H}_M(\varphi_X^*)/\partial R > 0$ hold. Thus, we can obtain $\partial\varphi_F^*/\partial R > 0$ and $\partial\varphi_X^*/\partial R > 0$. The impact of becoming more integrated ($\phi \uparrow$) on the threshold quality of exporting offline firm φ_X^* is different from the impact of accessing to more external markets ($R \uparrow$). The intuition behind this is as follows: when a region accesses to more external markets, the exporting offline firm benefits from the improvement of market potential and suffers from more competition from external regions. The former is dominated by the latter, resulting in an increase of the threshold exporting offline firms φ_X^*. Similarly, it is readily verified that $\partial\underline{\varphi}^*/\partial R > 0$ holds when $\alpha = 1$. Thus, the threshold online firm $\underline{\varphi}^*$ increases when the number of regions increases. Through simple calculations, we can obtain that $\partial(\mathbb{E}\varphi)^*/\partial R > 0$ holds. Therefore, the expected quality of online firms increases when a region accesses to more external regions.

Finally, combining $\partial\varphi_F^*/\partial\phi > 0$, $\partial\varphi_F^*/\partial R > 0$, and (28) yields $d\mathcal{P}^*/d\phi < 0$ and $d\mathcal{P}^*/dR < 0$. Thus, lowering transport costs and/or increasing the number of regions lead to a decrease in the price index and improves the social welfare. Using (44), and $d\mathcal{P}^*/d\phi < 0$, $\partial(\mathbb{E}\varphi)^*/\partial\phi > 0$, $d\mathcal{P}^*/dR < 0$, $\partial(\mathbb{E}\varphi)^*/\partial R > 0$, we obtain $\partial n_x^*/\partial\phi < 0$ and $\partial n_x^*/\partial R < 0$. Thus, using (43) leads to $\partial f_x^*/\partial\phi > 0$ and $\partial f_x^*/\partial R > 0$. That is, the more integrated and/or the larger number of regions, the higher the average quality of products and the less mass of varieties in the online market. Meanwhile, fewer varieties lead to higher fixed costs, which result in a larger threshold quality of online firms and also a smaller number but the larger size of online firms.

Proposition 5 *As the economy is more integrated and/or the number of regions increases, both the threshold online firm $\underline{\varphi}^*$ and the threshold domestic offline firm φ_F^* increases, resulting in an increase in the expected quality of active online firms $\mathbb{E}\varphi$ and a smaller number of online firms n_x^*. Meanwhile, a more integrated economy decreases the threshold exporting offline firm φ_X^*. An increase of the number of regions leads to a hike of the threshold exporting offline firm φ_X^*. All in all, the price index decreases and thus the social welfare increases when the economy is more integrated and/or the number of* *regions increases.*

4 Conclusion

This paper analyzes the interplay between the cost-sharing and imperfect information among online firms which compete with offline firms within an industry. Our analysis clarifies the impact of improving electronic commerce platforms on the economy. We find that the improvement and the existence of the online firms do not affect social welfare, but the existence of the online market reduces the number of offline firms, which may show that a policy to mitigate the immediate change of the size of offline firms are needed. We also find that the improvement by the platform to intensify cost-sharing provide a higher level of expected quality in the online market but also results in a smaller number and larger size of online firms. This is because the impact of the cost-sharing on an online firm differs among online firms with different quality.

A key issue for further consideration is how to protect consumers in the online market and ensure a fair competitive environment for firms. Thus, additional analysis on the online market and between online and offline firms is needed in future studies.

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