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How do trade and communication costs shape the spatial organization of firms?*

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Abstract

We show how trade and communication costs interact to shape the way firms organize their activities across space. We consider the following three organizational types: (i) integrated firms in which all activities are conducted at the same location, (ii) horizontal firms, which operate several plants producing the same good at different locations, and (iii) vertical firms, which perform distinct activities at separated locations. We find necessary and sufficient conditions for the three types of organization to coexist within the same country, whereas firms located in the other country are all spatially integrated. We then study how trade and communication costs affect firms' organizational choices. First, lower trade costs lead fewer firms to go multinational. By contrast, less expensive communication flows leads to more investment abroad. The reason for this difference in results is that the two types of spatial frictions differ in nature: in the proximity-concentration trade-off, lower trade costs weaken the need for proximity, while lower communication costs foster deconcentration.

Keywords: trade costs, communication costs, spatial fragmentation of firms

JEL classification: F12; F21; R12

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1 Introduction

We observe a variety of organizational forms in the way firms conduct their activities in the space economy, as well as various models that aim to explain the spatial fragmentation of firms (Antràs and Yeaple, 2014). To a large extent, these models appeal, often indirectly and under different guises, to the concentration-proximity trade-off (Markusen, 1984; Brainard, 1997). The former term accounts for the various benefits associated with the concentration of means in a small number of units and the latter for the wide range of impediments to the mobility of goods, people and information. In this paper, we blend ingredients from economic geography and trade theory to investigate when and why *identical firms operating in the same environment choose simultaneously different spatial organizational forms*. To achieve our goal, we distinguish between trade and communication costs. This difference is critical because communication and trade costs play different roles in the way firms competing in the international marketplace organize their activities across locations. Communication costs stem from coordinating complementary and spatially separated specialized workers, whereas transport costs are a special case of production costs that are paid to make available at a particular location a good produced in another.

Even since the Industrial Revolution, trade costs have plummeted. Nevertheless, they remain a major impediment to trade and exchange, as shown by the many estimations of the gravity equation (Head and Mayer, 2014). Since trade costs stand for the costs of coordinating and connecting transactions between supplier and customer locations, it has long been recognized that many firms operate several plants that supply spatially separated markets (Beckenstein, 1975; Markusen, 1984). What is more, firms are packages of different functions, such as management, R&D, finance, marketing, and production. Due to the development of new information and communication technologies (ICT), firms are able to disperse these functions into geographically separated units in order to benefit from the attributes specific to different locations (Helpman, 2006; Aarland *et al.*, 2007). However, there must be powerful reasons for business people to meet despite the high opportunity cost associated with travelling.

For multi-plant US firms Giroud (2013) shows that the opening of new airline links that reduce the travel time between headquarters and plants has generated an increase of 7% in plants' productivity. Charnoz *et al.* (2018) use the development of the high-speed railway network in France to show how the decrease in passenger travel time between headquarters and affiliates has allowed a higher concentration of management functions in headquarters. In the same vein, Kalnins and Lafontaine (2013) observe that greater distance to headquarters is associated with shorter establishment longevity. Whereas the media steadily stress the globalization of finance, the empirical evidence reveals that a greater distance between lenders and borrowers tend to make loan contracts more restrictive (Hollander and Verriest, 2016). Why is it so? The transmission of knowledge via the new communication devices remains incomplete and imperfect (Leamer and Storper, 2001). In addition, face-to-face contacts are still needed between high-skilled workers operating in spatially separated plants and headquarters because such contacts allow for immediate feedbacks in non-routine activities (Battiston *et al.*, 2017). The list could go on much further. Thus, despite the ICT revolution, we may safely conclude that the communication curse is still with us.

Although the literature on multinational enterprises recognizes the existence of various types of spatial frictions, it typically assumes that trade cost associated with the shipment of the manufactured good is sufficient to reflect the impact of these frictions (Antràs and Yeaple, 2014). By establishing their plants in large markets, firms located in small countries save trade costs. But then, they must bear communication costs between plants and headquarters.

This points to the existence of a trade-off between these two types of spatial frictions. Therefore, the modeling strategy that consists in merging these two spatial frictions under the heading of trade costs is unwarranted in the study of multi-unit firms.

We consider the three main types of spatial organizational forms. A firm conducting all its activities under the same roof opts for what we call a spatially *integrated* structure. When firms are not spatially integrated, we follow the literature on FDIs and distinguish between the following two types of spatial organization (Caves, 1971). The firm adopts a *horizontal* structure when several plants produce the same good at different locations. The cost of being a horizontal firm is the loss in the returns to scale economies, while the benefit is direct access to each market with zero trade costs. By contrast, the firm selects a *vertical* structure when it organizes and performs discrete activities at distinct locations, which altogether form a supply chain. The vertical fragmentation of the firm aims to take advantage of differences across locations, but this involves communication costs between headquarters and plants, as well as trade costs from the foreign country to the domestic one. Thus, horizontal and vertical structures should not be viewed as competitors.

To the best of our knowledge, no paper has addressed the occurrence of the three types of spatial organizational forms in a trade setting involving firms established in different counties and competing in the same environment. While knowledge spillovers are key in urban economics (Carlino and Kerr, 2014), the costs of transmitting information and knowledge between headquarters and subsidiaries that are spatially separated are generally ignored in the trade literature.[1] This is where we hope to contribute by linking different strands of literature in a setting where firms are free to choose their number and locations of plants in the presence of trade and communication costs. Somewhat unexpectedly, we will see that horizontal and vertical firms may coexist under the same market and technological conditions. In addition, our setting is general enough to interpret communication costs as a "reduced form" for the various management and informational costs generated by spatial separation, such as those studied in the literature on the organization of multi-level enterprises (Antràs and Rossi-Hansberg, 2009; Antràs and Yeaple, 2014). Thus, very much like trade costs, communication costs may capture a wide range of effects.

What are our main findings? Assuming that firms are a priori identical, we show that *the three organizational forms may come together within the same country.*[2] Put differently, firms that are a priori homogeneous in productivity choose to become heterogeneous in their spatial organization.[3] For the coexistence of the three spatial organizational forms to arise, the following conditions are required. First, communication costs cannot be too large, for otherwise no firm chooses to be vertical. Second, trade costs cannot be too low, for otherwise all firms prefer to be integrated. Last, fixed costs cannot be too high, for otherwise no firm would be horizontal, nor too low, for otherwise all firms would avoid trade costs by being horizontal.

[1] Keller and Yeaple (2013) is a noticeable exception.

[2] In Japan, integrated firms account for more than 75 percent of the manufacturing sector and vertical firms for 10 percent. The remaining 15 percent are operated by horizontal firms. These shares remained very stable from 1992 to 2008. The census accounts for firms with more than four full-time employees, which probably explains the high share of integrated firms. We thank Toshihiro Okubo for these numbers.

[3] In a market with two identical firms, Mills and Smith (1996) show that a firm may invest in a new technology that has a lower marginal cost whereas its rival strategically chooses not to switch technology. Elberfeld (2003) extends this result to an oligopoly. This author also shows that under monopolistic competition all firms make the same technological choice. Note that those results are obtained in a closed and dimensionless economy.

Furthermore, while the smaller country accommodates the three types of organizational forms, the larger country's firms remain integrated. Hence, there is one-way offshoring. For this, the trading partners must differ in size but not too much. In this case, some of the smaller country's firms invest abroad to have a better access to the larger country, while other firms remain integrated and focus on the smaller country because the establishment of foreign plants strengthens competition in the larger country. The same holds for most of the other equilibria: the larger country's firms are integrated while it pays for the smaller country's firms to be different.

The coexistence of the three organizational forms is socially optimal under conditions similar to those that sustain the market equilibrium. Nevertheless, since a firm's production cost depends on its organizational choice, the cost distribution is endogenous, which implies that the numbers of firms adopting a specific structure in the equilibrium and optimal outcomes need not be the same, unlike the case where the cost distribution is exogenous (Dhingra and Morrow, 2018). To be precise, we show that too few firms are horizontal while too many firms are vertical. All in all, too few firms invest abroad.

We then study how trade and communication costs affect the pattern of organizational types. First, when shipping goods becomes cheaper, the number of plants operating in each country decreases. Unlike what economic geography tells us, a deeper integration makes competition softer in each country because firms change their organizational form in response to a drop in trade costs (Baldwin *et al.*, 2003). Our analysis confirms and extends a classical result in the theory of multinational enterprises, that is, fewer firms go multinational (Markusen, 2002). More specifically, lowering trade costs leads to a hike in the number of integrated firms, while reducing the number of horizontal firms but raising the number of vertical firms.

Falling communication costs generate the opposite results as more firms go multinational. Even though the total number of plants increases, the smaller country hosts fewer plants. In other words, *lowering trade costs or communication costs delivers contrasted spatial patterns of production*: in the former more firms are integrated, while more firms are fragmented in the latter. This should not come as a surprise since the two costs affect the proximity-concentration trade-off differently: lowering trade costs weakens the need for proximity, while lower communication costs weakens the benefits of concentration. In short, distance matters in different ways because distance means different things under trade and communication costs. These results concur with Baldwin (2016) who argues that drops in trade and communication costs are at the origin of two very different phases of globalization.[4]

When firms are a priori heterogeneous and differentiated by their own productivity, their incentives to choose a particular organizational structure are affected, so that it is not clear that firms may want to be differentiated in spatial organizational forms too. Therefore, we find it natural to investigate what our main findings become when firms are a priori cost-heterogeneous. As in the foregoing, we show that the smaller country hosts the three types of firms under conditions that are equivalent to those obtained when firms are homogeneous. The most efficient firms

[4] According to Baldwin (2016), the spatial organization of firms depend on three types of spatial frictions: the cost of moving goods, the cost of moving ideas and the cost of moving people when face-to-face contacts are required. For our purpose, there is no need to distinguish between the last two types of friction. It is, therefore, convenient to gather them under the heading of communication costs, which encompass here the cost of moving codified information, which is easily sent by using the new information and communication technologies, and tacit information, which often requires face-to-face contacts (Leamer and Storper, 2001). For our purpose, there is no need either to distinguish between communication technology and information technology (Bloom *et al.*, 2014). We refer to Baldwin for more details.

always choose to become horizontal because these firms are able to bear the higher fixed costs associated with the operation of two plants. On the other hand, the organizational form selected by the least efficient firms depends on the relative size of the two countries. When the asymmetry is strong, the medium efficient firms go vertical because their home market is too small. Otherwise, they go integrated because their domestic market offers a sufficiently big outlet. Last, we characterize and discuss the various spatial organizational forms that emerge in other equilibria.

Related literature. Our paper is obviously related to the huge literature on multinational enterprises (Markusen, 2002; Navaretti and Venables, 2004). The relationships with this literature will become clear as the paper develops. Our model is even more connected to the meager literature on multi-plant firms (see Beckenstein, 1975, for an early contribution). Following Markusen (1984), most of the contributions on multinational enterprises has focused on the concentration-proximity trade-off. Behrens and Picard (2007) use an economic geography setting to compare integrated and horizontal firms. These authors show that each country hosts both types of organizational forms when fixed production costs take neither high nor low values. Using a setting where all firms are established in a core region, Fujita and Thisse (2006) highlight the role of communication costs in firms' decisions to go vertical. They show that the core region may host both integrated and vertical firms. Fujita and Gokan (2005) extend this setting to the case where firms may be horizontal or vertical. By contrast, we focus on competition among domestic and foreign firms in the two countries, which leads to a richer set of results. For example, we show that the three types of firms may coexist in equilibrium. In this respect, Yeaple (2003) is closer to us in that he studies the simultaneous emergence of the three organizational forms. To do this, Yeaple considered a 3-country setting and shows that the same firm may choose to go horizontal in one country and vertical in the other. In a multi-country setting, Head and Mayer (2017) add two frictions, that is, headquarters services to the foreign affiliates and marketing costs between the headquarters and the markets, to trade costs. Head and Mayer highlight the empirical relevance of the relationships between headquarters and their foreign affiliates as a bilateral friction that comes on top of trade costs. In our two-country setting, both headquarters services and marketing costs are collected under the heading of trade costs. Our model also bears some resemblance with one of the workhorses of economic geography, that is, the footloose capital model (Baldwin *et al.*, 2003). In this model, firms run a single plant and are spatially integrated. By contrast, we allow firms to choose their organizational forms, that is, headquarters and plants may or may not collocate, while firms may operate one or several plants in each country. Therefore, our model can be viewed as the "footloose plant model." Finally, our setting is also related to the literature on the organization of firms with multiple layers (Antràs and Rossi-Hansberg, 2009). However, this literature focuses more on the micro underpinnings of the firm's production function and often ignores the product market feedback effects (see Chen, 2017, for a recent exception).

The paper is organized as follows. The model is described in Section 2. Section 3 deals with the equilibrium and welfare analyses when firms have the same productivity. The effects triggered by lower trade and communication costs are studied in Section 4. In Section 5, we briefly discuss what the other equilibrium patterns are. Section 6 discusses what our main findings become when firms differ in productivity, while Section 7 concludes.

2 The Model and Preliminary Results

2.1 The Economy

The economy features two countries - or any other spatial units such as regional trade blocks or subnational regions ($i = 1, 2$) -, a manufacturing sector and a sector producing a homogeneous good, and two production factors - skilled and unskilled labor. The mass of country i's consumers is $s_i > 0$ with $s_1 > s_2$ and $s_1 + s_2 = 1$.[5] The manufacturing sector supplies a differentiated good, which is produced under increasing returns and monopolistic competition using skilled and unskilled workers. Each variety is provided by a single firm and each firm supplies a single variety. The homogeneous good is produced under constant returns and perfect competition by using unskilled workers only. This good is costlessly traded, so that its price is the same in both countries. We choose it as the numéraire. Each consumer is endowed with one unit of skilled or unskilled labor, which is supplied inelastically. To rule out comparative advantage à la Heckscher-Ohlin, the share $\varphi \in (0, 1)$ of skilled workers is the same in both countries. Like in trade theory, both skilled and unskilled workers are spatially immobile.

A firm involves a *headquarters* (HQ) and one or two production *plants*. By convention, we refer to a firm's location as the location of its HQ. To operate, a HQ needs a given number of skilled workers only. A HQ provides the specialized pre- and post-fabrication services for the good to be processed and delivered to customers. For notational simplicity, we assume that a HQ needs φ units of skilled labor. Since the total supply of skilled labor is equal to φ, market clearing implies that the total mass of firms and varieties is equal to 1. By implication, country i hosts s_i firms. Unskilled labor is used in plants to produce the differentiated good. Each firm chooses to have a single production facility in one of the two countries or a production site in each country where the same variety is produced. Hence, the mass of plants is endogenous. More precisely, the total mass of plants varies from 1 to 2. The skilled's earnings are given by a firm's profits divided by the number of skilled working in the HQ.

Our main objective is to insulate the effects of two different spatial frictions on firms' organizational forms through the *number* and *location* of plants they operate. To achieve our goal, we consider two countries which share similar levels of economic and technological development. This does not strike us as an unrealistic context to investigate. Indeed, even though the peak of FDI inflows in OECD countries was reached in 2007 with 70% of all FDI inflows, these investments still account for 40% in 2015 (OECD, 2016). Another example is provided by two large regional economies of the same country, which are likely to share many common social and technological features.

More specifically, we assume that the wage of the unskilled is the same in both countries. This condition holds when the numéraire is costlessly traded. Furthermore, plants' productivity is the same in both countries, which implies that international productivity difference is not the reason for the geographical fragmentation of firms. In our setting the choice of different spatial organizational forms hinges on the interplay between trade, communication and fixed production costs. The gains from being integrated stem from saving communication costs, while the gains from being separated stem from saving transport costs by producing in the larger market.

[5] This normalization entails no loss of generality since the fixed labor requirement associated with the launching of a plant is an inverse measure of the size of the economy.

2.2 Consumers

Consumers share the same quasi-linear preferences given by

$$U = \ln\left[\left(\int_0^1 x_k^{\frac{\sigma-1}{\sigma}} \mathrm{d}k\right)^{\frac{\sigma}{\sigma-1}}\right] + z,$$

where x_k is the consumption of variety $k \in [0, 1]$, $\sigma > 1$ the elasticity of substitution between any two varieties, while z stands for the consumption of the composite good. A consumer's budget constraint on the differentiated good is thus given by

$$\int_0^1 x_k p_k \mathrm{d}k = 1, \tag{1}$$

where p_k is the consumer price of variety k. By implication, an increase in income generates the same increase in the consumption of the composite good. Therefore, the manufacturing sector operates as in a CES one-sector economy.

Since total profits are zero, most of the trade and economic geography literature focuses on a Cobb-Douglas upper-tier utility. Using such preferences makes our model especially hard to handle because skilled workers' incomes are endogenous and unequal across countries. As a result, the demand for a particular variety changes with consumers' incomes, which depend themselves on the overall demand system. Using quasi-linear preferences allows us to obviate this difficulty because the individual expenditure on the differentiated good is exogenous and equal between countries. Note that many, but not all, trade or economic geography models assumed that the homogeneous good is costlessly traded so that incomes are exogenous and the same in both countries. In this case, the individual expenditure on the manufactured good is also exogenous and the same in the two countries, like in (1). A noticeable exception is the footloose capital model with one sector in which individual expenditures are endogenous and different across countries (see, e.g., Takahashi *et al.*, 2013, for a complete solution of this problem).

It is well known that the individual demand for variety k is given by

$$x_k = \frac{p_k^{-\sigma}}{\Delta}, \tag{2}$$

where p_k is the consumer price of variety k while the market aggregate

$$\Delta \equiv \int_0^1 p_k^{-(\sigma-1)} \mathrm{d}k = P^{-(\sigma-1)} \tag{3}$$

is a monotone decreasing transformation of the CES-price index

$$P = \left[\int_0^1 p_k^{-(\sigma-1)} \mathrm{d}k\right]^{-1/(\sigma-1)}.$$

2.3 Producers

Firms are heterogeneous. More specifically, to operate a plant, a θ-firm needs a fixed requirement of $f > 0$ and a marginal requirement of c/θ units of unskilled labor where $\theta \in [1, \overline{\theta})$ is drawn from the cumulative distribution $G(\theta)$. In line with the literature, we assume that G is given by a truncated Pareto distribution $G(\theta) = \alpha \cdot [1 - (1/\theta)^{\kappa}]$ where $\alpha \equiv \overline{\theta}^{\kappa}/(\overline{\theta}^{\kappa} - 1) > 1$, while $\kappa > 2$ guarantees that the productivity distribution has a finite variance. A higher value of κ means a smaller variance in firms' heterogeneity. When firms are homogeneous ($\kappa \to \infty$), the marginal requirement of unskilled labor is the same across firms and equal to c.

In our model, the "distance" between countries is measured in two different ways. First, in line with the literature, when a firm ships one unit of its variety abroad it incurs an iceberg trade cost $\tau > 1$; it is costless to ship the variety to its local customers. Second, a firm's HQ provides various specialized inputs to its plant(s), while local managers require regularly pieces of information from their HQs related to specific tasks, unexpected issues, and more. This implies the existence of communication costs between the two units. Since distance affects productivity in a negative way, it is natural to assume that the plant's marginal cost is higher when the HQ and plant are located in different countries. In what follows, we also model communication costs as an iceberg cost $\gamma > 1$, while $\gamma = 1$ when plants and HQs are collocated. Our modeling strategy of communication costs may also be justified on the following grounds.

First, using an iceberg cost implies that communication costs are proportional to the plant output. This is in line with the literature on firms' organization where managers spend time solving sophisticated tasks arising, e.g., in distant plants while their working time is proportional to firms' output (Bolton and Dewatripont, 1994; Garicano, 2000; Gumpert, 2018). Second, since $\gamma > 1$ can take any arbitrary value our approach is consistent with communication costs that are unrelated to distance, as in the case of talks via communication devices or discriminatory trade policies (e.g., visa restrictions) and costs that vary with distance, as in the case of travel costs of business people. Third, since less efficient firms are likely to experience higher communication costs, the marginal cost of a c-firm may be expressed as $\gamma c/\theta$ when the plant is located in the foreign country. For example, a lower quality of internal resources makes firms more vulnerable when HQs and plants are spatially separated. Last, modeling both frictions in the same way makes it easier to compare their respective impact on firms' organizational forms.[6]

The choice of a specific organizational form affects a firm's production cost.[7] In what follows, we describe the cost functions associated with the three types of firms. We denote by q_{ij} the total consumption in country $j = 1, 2$ of a variety produced in country $i = 1, 2$.

(i) A θ-firm is said to be *integrated* (**I**) when it operates a single plant which is located together with its HQ; the plant supplies both markets. Hence, the cost function of a **I**-firm with productivity θ located in country $i = 1, 2$ is given by

$$C_i^n(\theta) = f + \frac{c}{\theta} \cdot (q_{ii} + \tau q_{ij}) \quad \text{with} \quad j \neq i. \tag{4}$$

The total output, or size, of this firm is thus equal to $q_i^n \equiv q_{ii} + \tau q_{ij}$.

(ii) A θ-firm is *vertical* (**V**) when it has a single plant, which operates abroad; the plant supplies both countries. A **V**-firm faces an additional cost associated with the operation of a plant set up away from its HQ. As discussed in the introduction, distance implies higher coordination and communication costs between the HQ and its plant. Therefore, the cost function of a **V**-firm located in country i is given by

$$C_i^v(\theta) = f + \frac{c}{\theta} \cdot (\tau\gamma q_{ii} + \gamma q_{ij}) \quad \text{with} \quad j \neq i. \tag{5}$$

This firm's total output is given by $q_i^v \equiv \tau\gamma q_{ii} + \gamma q_{ij}$.

(iii) Finally, a θ-firm is *horizontal* (**H**) when it has a plant in each country. When a firm splits its production between the two countries, it incurs an additional fixed cost f. Since the plant located abroad incurs communication costs γ to use the services supplied by its HQ, the marginal costs are, respectively, c/θ and $\gamma c/\theta$. Since both plants

[6] Duranton and Puga (2005) and Fujita and Thisse (2006) adopt the same modeling approach in different settings.

[7] In this respect, we differ from Melitz (2003) since the marginal costs change with the firms' organizational choices.

supply the same variety, the activity of a **H**-firm entails no trade between countries. The cost function of a **H**-firm located in country i is then given by the following expression:

$$C_i^h(\theta) = 2f + \frac{c}{\theta} \cdot (q_{ii} + \gamma q_{ij}) \quad \text{with} \;\; j \neq i, \tag{6}$$

while its total output is equal to $q_i^h \equiv q_{ii} + \gamma q_{ij}$.

Note that communication costs differ from a productivity differential between countries. Indeed, communication costs arise when a firm's HQ and its plant are spatially separated regardless of the country hosting the plant. By contrast, the plant of any type of firm produces at a lower cost only when it is located in the high productivity country.

The expressions (4)–(6) show that trade and communication costs affect firms' production costs in different ways according to their organizational form.[8]

2.4 Market Equilibrium

Since all country i-firms sharing the same productivity θ and the same organizational form $k = n, v, h$ choose the same equilibrium consumer price $p_{ii}^k(\theta)$ in country i ($p_{ij}^k(\theta)$ in country j), (2) implies that the profit function of a θ-firm is given by the following expression:

$$\pi_i^k(\theta) = s_i \cdot \frac{(p_{ii}^k(\theta))^{1-\sigma}}{\Delta_i} + s_j \cdot \frac{(p_{ij}^k(\theta))^{1-\sigma}}{\Delta_j} - C_i^k(\theta) \quad \text{with } k = n, v, h, \quad i, j = 1, 2 \text{ and } j \neq i.$$

The timing of the game is as follows. First, firms choose their organizational forms and, then, their prices and quantities sold in each country.

For notational simplicity, we choose the unit of output for $c = (\sigma - 1)/\sigma < 1$ to hold. Using (2), profit-maximization yields the equilibrium consumer price of a variety produced in country $i = 1, 2$ by a **I**-firm and sold in countries i and j:

$$p_{ii}^n(\theta) = \frac{1}{\theta} \qquad p_{ij}^n(\theta) = \frac{\tau}{\theta} > p_{ii}^n \quad \text{with } j \neq i. \tag{7}$$

A **V**-firm located in country i charges prices equal to

$$p_{ii}^v(\theta) = \frac{\gamma\tau}{\theta} > p_{ii}^n(\theta) \qquad p_{ij}^v(\theta) = \frac{\gamma}{\theta} < p_{ii}^v(\theta) \quad \text{with} \;\; j \neq i, \tag{8}$$

while a **H**-firm in i sets prices given by

$$p_{ii}^h(\theta) = \frac{1}{\theta} \qquad p_{ij}^h(\theta) = \frac{\gamma}{\theta} > p_{ii}^h(\theta) \quad \text{with} \;\; j \neq i. \tag{9}$$

In this case, we have the following ranking of consumer prices:

$$p_{ii}^n(\theta) = p_{ii}^h(\theta) < p_{ij}^v(\theta) = p_{ij}^h(\theta) < p_{ij}^n(\theta) < p_{ii}^v(\theta).$$

In equilibrium, firms sharing the same productivity choose the same organizational form. Then, we denote by N_i (or V_i or H_i) the set of firms in country i, which are integrated (or vertical or horizontal). Using (7)–(9), the market

[8] Note that the communication cost γ in (5) cannot be interpreted as a wage wedge between the two countries. Indeed, this interpretation would mean that producing in i is more expensive than in j. However, as C_i^v and C_j^v have the same functional form, this would imply that producing in i would be cheaper than in j, a contradiction.

aggregate Δ_i is given by the following expression:

$$\Delta_i = A \cdot (n_i + n_j\phi + v_i\phi\omega + v_j\omega + h_i + h_j\omega),$$

where $0 < \phi \equiv \tau^{-(\sigma-1)} < 1$ and $0 < \omega \equiv \gamma^{-(\sigma-1)} < 1$ whose values measure, respectively, the freeness of trade and the freeness of communication, while

$$n_i \equiv \frac{s_i}{A}\int_{N_i} \theta^{\sigma-1}\mathrm{d}G \qquad v_i \equiv \frac{s_i}{A}\int_{V_i} \theta^{\sigma-1}\mathrm{d}G \qquad h_i \equiv \frac{s_i}{A}\int_{H_i} \theta^{\sigma-1}\mathrm{d}G, \tag{10}$$

and

$$A \equiv \frac{\kappa}{\kappa-\sigma+1} \cdot \frac{\left(\overline{\theta}\right)^{\kappa} - \left(\overline{\theta}\right)^{\sigma-1}}{\left(\overline{\theta}\right)^{\kappa} - 1} > 0. \tag{11}$$

The constant A is a normalization parameter which guarantees that $s_i + s_j = 1$; it converges to 1 when firms are homogeneous ($\kappa \to \infty$).

Computing the above integrals and summing yields

$$n_i + v_i + h_i = s_i, \tag{12}$$

It follows from (12) that n_i (or v_i or h_i) is the actual mass of integrated (or vertical or horizontal) firms in country i. Consequently, Δ_i can be interpreted as the *effective* mass of plants competing in country i, that is, the mass of plants discounted by the corresponding friction factors ϕ and ω. Indeed, everything works as if the mass of plants located in country i were equal to Δ_i. As Δ_i rises through lower trade or communication costs, the price index P_i decreases because the effective mass of plants is higher. In other words, when the organizational structure of firms is given, lower communication and/or trade costs render both markets more competitive. On the contrary, when trade and communication costs are prohibitively high ($\phi = \omega = 0$), $\Delta_i = s_i$. When there is no spatial friction ($\phi = \omega = 1$), $\Delta_i = 1$, which means that all plants compete symmetrically in each country regardless of their locations. Note also that the price index in country i depends on the spatial structure chosen by firms located in *both* countries.

Using (12), we can rewrite Δ_i as follows:

$$\Delta_i = A \cdot [s_i + \omega s_j - (\omega - \phi)n_j - (1 - \phi\omega)v_i], \qquad i = 1, 2. \tag{13}$$

Measuring the intensity of competition in a market by the inverse of the corresponding price index, we may conclude as follows. If all country i-firms are integrated ($n_i = s_i$), competition becomes tougher in i and softer in country j because all i-firms produce home, which protects j-firms. If all firms are vertical ($v_i = s_i$), competition becomes tougher in country j, and softer in country i because all varieties are imported from j. Last, if all i-firms are horizontal ($h_i = s_i$), competition gets tougher in both countries because each country hosts a larger mass of plants. In short, the organizational structure of firms affects the intensity of competition in both countries.

Using (2) and (7)–(9), the profits made by a **I**-firm, a **V**-firm and a **H**-firm are, respectively, given by the following expressions:

$$\pi_i^n(\theta) = \frac{\theta^{\sigma-1}}{\sigma}\left(\frac{s_i}{\Delta_i} + \phi\frac{s_j}{\Delta_j}\right) - f, \tag{14}$$

$$\pi_i^v(\theta) = \frac{\theta^{\sigma-1}}{\sigma}\left(\phi\omega\frac{s_i}{\Delta_i} + \omega\frac{s_j}{\Delta_j}\right) - f, \tag{15}$$

$$\pi_i^h(\theta) = \frac{\theta^{\sigma-1}}{\sigma}\left(\frac{s_i}{\Delta_i} + \omega\frac{s_j}{\Delta_j}\right) - 2f. \tag{16}$$

An *equilibrium* is such that consumer maximizes utility, each firm maximizes its profits, markets clear, and profits are positive in both countries. Since firms are free to choose the organizational form across space, the equilibrium profits in country $i = 1, 2$ are such that

$$\pi_i^*(\theta) = \max\{\pi_i^n(\theta), \pi_i^v(\theta), \pi_i^h(\theta)\} > 0.$$

The following remarks are in order. First, **I**-firms' profits decrease with communication costs because the price indices P_1 and P_2 fall, while **H**-firms' profits fall for the same reason when trade costs decrease. Profits of **V**-firms change with ϕ and ω in more complex ways. Note already the importance of communication costs for the difference between integrated and multinational firms. If communication costs are prohibitive ($\omega = 0$), all firms are integrated. Second, when communication costs are negligible ($\omega = 1$), the model has a continuum of equilibrium distributions of organizational types (see Appendix 1). This is reminiscent of Krugman (1980) where there is a continuum of firm distributions when $\phi = 1$. In order to eliminate such extreme cases, we assume that $0 < \omega < 1$. If $\phi = \omega = 1$, no firm seeks to become horizontal while integrated and vertical firms face the same profit function and, therefore, remain identical. More generally, when identical firms that face the option of investing in new technologies to produce at a lower marginal cost, they all choose to invest or not to invest, which implies that they are always identical (Elberfeld, 2003).

Third, a straightforward comparison of (14) and (15) implies that $\pi_i^n(\theta) > \pi_i^v(\theta)$ when communication costs are higher than trade costs ($\omega < \phi$). In other words, *when communication costs are high, no firm is vertical.* Similarly, if trade costs are very low ($\phi \approx 1$), (14) and (16) imply that $\pi_i^n(\theta) > \pi_i^h(\theta)$ when $s_j > s_i$. Put differently, *when trade costs are low, no firm is horizontal.* Since our focus is on the coexistence of the three organizational forms within the same country, we assume from now on that

$$0 < \phi < \omega < 1$$

holds. This describes well the on-going situation because the recent drop in communication costs associated with the rapid development of ICTs has been sharp, while the supply of high-speed railway and airline links has drastically expanded. Trade costs also came down, but at a slower pace.

In this case, (13) becomes easy to interpret. The term $s_i + \omega s_j$ in the right hand-side of (13) is the effective mass of plants in country i when all domestic firms are integrated or horizontal. When some foreign firms choose to be integrated, the price of their varieties is affected by the gap $\omega - \phi > 0$ between communication and trade costs. Similarly, the term $(1 - \phi\omega)v_i$ accounts for the i-firm that choose to go vertical, which generates a price gap equal to $1 - \phi\omega$. Since communication costs are lower than trade costs, everything else equal this renders market in country i more competitive because more j-firms locate their plants in country i.

Last, it follows from (14) and (15) that $s_j/\Delta_j > s_i/\Delta_i$ must hold for some i-firms to go vertical. Since $s_j/\Delta_j < s_i/\Delta_i$ must also hold for some j-firms to be vertical, **V**-firms can exist at most in one country.

3 Homogeneous Firms

Although we recognize that firms are differentiated by their productivity in the real world, working with heterogeneous firms would blur the sheer effects that drive firms in their organizational choices in the space-economy. This is why we start with the case of homogeneous firms. In other words, we assume that $\kappa \to \infty$, so that θ and A converge to 1. A comprehensive analysis of all possible patterns would be very burdensome. Rather, we focus on the telling example in which the three types of organizational forms emerge in equilibrium. We define a *mixed* equilibrium as an equilibrium outcome in which at least one country hosts the three types of firms. Since **V**-firms cannot coexist in both countries, only one country, say j, can accommodate the three organizational forms. In this case, the equilibrium condition in country j is as follows:

$$\pi_j^n = \pi_j^v = \pi_j^h > 0. \tag{17}$$

More specifically, we determine necessary and sufficient conditions for homogeneous firms located in country j, to become heterogeneous in the way they organize their production activities between countries, which shows that *competition alone is sufficient for identical firms to operate under the three organizational forms.*

As shown in Appendix 1, at any mixed equilibrium one country, say i, hosts only integrated firms ($n_i = s_i$). In what follows, we find the mass of j-firms which choose each organizational form and show that $i = 1$ and $j = 2$, meaning that diversification arises among the smaller country's firms. Furthermore, we determine the necessary and sufficient conditions for the candidate mixed equilibrium to exist.

3.1 Organizational Forms

When $n_i = s_i$, we may use (17) to determine the corresponding equilibrium values of Δ_i and Δ_j.

1. Using (14) and (16), the condition $\pi_j^h = \pi_j^n$ implies

$$\Delta_i^* = \frac{\omega - \phi}{\sigma f} s_i. \tag{18}$$

Observe that (3) and (18) imply that P_i^* decreases with the size of country i. Similarly, P_i^* decreases when σ and/or f falls because more plants settle in country i when varieties are less differentiated and/or fixed costs are lower.

2. Using (15) and (16), the condition $\pi_j^h = \pi_j^v$ implies

$$\Delta_j^* = \frac{1 - \phi\omega}{\sigma f} s_j. \tag{19}$$

For the three firm-types to coexist in a country, the national indices Δ_i^* and Δ_j^* must be given by (18) and (19).

3. The last condition $\pi_i^n = \pi_i^v$ yields

$$\frac{\Delta_i^*}{\Delta_j^*} = \frac{s_i}{s_j} \cdot \frac{\omega - \phi}{1 - \phi\omega}, \tag{20}$$

which follows immediately from (18) and (19). The expression (20) highlights how communication and trade costs interact in j-firms' spatial choices through the price indices of the two markets. Furthermore, if $\omega = 1$, that is, there are no communication costs, (20) becomes

$$\frac{\Delta_i^*}{\Delta_j^*} = \frac{s_i}{s_j},$$

which is identical to the equilibrium condition obtained by Helpman *et al.* (2004) and Baldwin and Forslid (2010) when firms have the same productivity. In this case, the price index ratio is determined by the relative size of countries.

3.2 Mixed Equilibrium

We now study the configuration where all firms located in the larger country are integrated ($n_1^* = s_1$), while the smaller country accommodates integrated, vertical and horizontal firms.

Denote by $S \equiv s_2/s_1$ the relative size of the two countries, with $S \in (0, 1)$. We show in Appendix 2 that profits are equal across types when the 2-firms are split into the following three groups:

$$n_2^* = \frac{1}{1+S} \cdot \left(\frac{1+\omega S}{\omega - \phi} - \frac{1}{\sigma f} \right), \tag{21}$$

$$v_2^* = \frac{1}{1+S} \cdot \left(\frac{\phi + S}{1 - \phi\omega} - \frac{S}{\sigma f} \right), \tag{22}$$

$$h_2^* = \frac{1}{1+S} \cdot \left[\frac{1+S}{\sigma f} - \frac{(1-\phi^2)(1+\omega S)}{(1-\phi\omega)(\omega - \phi)} \right]. \tag{23}$$

But does a mixed equilibrium exist and is it unique? Inspecting n_2^* and v_2^* shows immediately that σf must be bounded below for n_2^* and v_2^* to be positive. Otherwise competition is too soft, or fixed costs are too low, to prevent all 2-firms to be horizontal. Likewise, it follows from h_2^* that σf must be bounded above from h_2^* to be positive. Otherwise competition is too tough, or fixed costs are too high, for some 2-firms to be able to cover the fixed cost associated with the launching of a second plant. In short, varieties cannot be very poor or very close substitutes, fixed costs cannot be very small or very large, or both.

Using (21)-(23) yields necessary and sufficient conditions for $n_2^* > 0$, $v_2^* > 0$, and $h_2^* > 0$ to hold. Putting these conditions together shows that country 2 hosts the three types of organizational forms if and only if the following condition holds:

$$B_L < \sigma f < B_R, \tag{24}$$

where B_L and B_R are bundles of the parameters S, ω, and ϕ defined as follows:

$$B_L \equiv \max\left\{ \frac{\omega - \phi}{1+\omega S}, \frac{(1-\phi\omega)S}{\phi + S} \right\}, \qquad B_R \equiv \frac{(\omega - \phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}.$$

Furthermore, for (24) to be feasible, B_R must exceed B_L. We show in Appendix 2 that there exists a unique value $\overline{S}$ such that $B_L < B_R$ if and only if the size ratio S satisfies the following inequalities:

$$\frac{\phi}{K} < S < \overline{S} < \frac{1}{K}, \tag{25}$$

where

$$K \equiv \frac{1 - \omega\phi}{\omega - \phi} > 1.$$

Since S must be smaller than 1 for (24) to be satisfied, Appendix 1 implies that country 1 hosts only **I**-firms.

Finally, it can be shown that the equilibrium (21)-(23) is unique under (24) and (25).[9]

To sum up, we have:

Proposition 1. *Assume that* $0 < \phi < \omega < 1$. *Then, there exists a mixed equilibrium if and only if* (24) *and* (25) *hold. This equilibrium is unique and given by* $n_1^* = s_1$ *and* (21)-(23).

Without productivity differences across firms and international wage differences, the 2-firms are at a disadvantage in accessing the larger market. It is, therefore, no surprise that some of these firms choose to invest in country 1. What

[9] This is done by showing that some configurations are never an equilibrium while the remaining configurations are not an equilibrium under (24)-(25). Details can be found in the Supplementary Material, which is available from the authors upon request.

is less straight forward is that the three organizational forms coexist even when there is no exogenous heterogeneity across firms and countries but their relative size.[10]

Yet, the intuition behind Proposition 1 is easy to grasp. Since the 1-firms have a direct access to the larger market, they are not incited to differentiate their spatial structures. In other words, the larger country has no **V**-firms and **H**-firms. By contrast, the smaller country accommodates both **V**-firms and **H**-firms in order to have a better access to the larger market. However, for this to happen, the mass of plants established in country 1 cannot be too large relative to the size of this country. Moreover, since the 1-firms always choose to be integrated while (21)-(23) is the unique equilibrium configuration that prevails in country 2 under (24) and (25), the equilibrium described in Proposition 1 is the unique mixed equilibrium.

Furthermore, what matters for a mixed equilibrium to arise is the relative size S of the two countries. If they have similar sizes, the 2-firms have a strong incentive to focus on their domestic market, making **V**-firms unprofitable. By contrast, owing to the fixed cost they have to bear, these firms have little incentive to invest home when country 2 is not big enough, making **H**-firms unprofitable. As a result, the size of country 1 must take on intermediate values for a mixed configuration to arise in equilibrium. In the same vein, the fixed cost associated with the construction of a second plant cannot be very low, for otherwise all the 2-firms would undertake horizontal investments, neither very large, for otherwise no 2-firms would undertake such investments. This is precisely what (24) says. In addition, fixed production costs relative to country sizes cannot be too different for horizontal firms to emerge, while they cannot be similar either, for otherwise no firm would be integrated. In short, *full diversification requires trade between countries which differ in size but not too much.*

In addition, we can use the demand (2) and the equilibrium prices (7)–(9) to find the equilibrium size of 1-firms and the different types of 2-firms:

$$\begin{aligned} q_1^n &= \left(\frac{\phi}{1-\phi\omega} + \frac{1}{\omega-\phi}\right)\sigma f, \\ q_2^n &= \left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi}\right)\sigma f = q_2^v = \left(\frac{\phi\omega}{1-\phi\omega} + \frac{\omega}{\omega-\phi}\right)\sigma f < q_2^h = \left(\frac{1}{1-\phi\omega} + \frac{\omega}{\omega-\phi}\right)\sigma f. \end{aligned} \tag{26}$$

Hence, the **I**- and **V**-firms have the same size, which is smaller than that of the **H**-firms. However, the **I**- and **V**-firms sell different quantities in each country because they set different consumer prices. Moreover, the integrated 1-firms are bigger than the integrated 2-firms. This is because the market size effect ($s_1 > s_2$) dominates the competition effect triggered by the higher mass of plants located in country 1.

Finally, the equilibrium profits are given by

$$\begin{aligned} \pi_1^* = \pi_1^n &= \left(\frac{1}{\omega-\phi} + \frac{\phi}{1-\phi\omega} - 1\right) f, \\ \pi_2^* = \pi_2^n = \pi_2^v = \pi_2^h &= \left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi} - 1\right) f. \end{aligned} \tag{27}$$

We have $\pi_1^* > \pi_2^* > 0$, where the second inequality holds because $\omega > \phi$. In other words, the skilled workers earn more in the larger country than in the smaller one. This agrees with the empirical literature that stresses the existence of a robust relationship between the wage of (skilled) workers and market size (Redding, 2011).

[10] When (24)-(25) do not hold, the market equilibrium typically involves *partial diversification*. See Section 5 for further discussion.

3.3 Welfare

Does the multiplicity of spatial organizations entail a waste of resources? The benefit of using quasi-linear preferences are reap in the welfare analysis because we have four groups of individuals, that is, the skilled and unskilled workers in countries 1 and 2, whose utilities can be added. More specifically, the planner chooses the consumption level of each variety and the mass of firm-types in each country so as to maximize the sum of individual utilities net of all costs:

$$W \equiv \sum_{i=1}^{2} s_i U_i - \sum_{i=1}^{2} \left(n_i C_i^n + v_i C_i^v + h_i C_i^h \right) \tag{28}$$

subject to (12), where we have set:

$$U_i \equiv \frac{\sigma}{\sigma - 1} \ln \left[n_i (x_{ii}^n)^{\frac{\sigma-1}{\sigma}} + v_i (x_{ii}^v)^{\frac{\sigma-1}{\sigma}} + h_i (x_{ii}^{hi})^{\frac{\sigma-1}{\sigma}} + n_j (x_{ji}^n)^{\frac{\sigma-1}{\sigma}} + v_j (x_{ji}^v)^{\frac{\sigma-1}{\sigma}} + h_j (x_{ii}^{hj})^{\frac{\sigma-1}{\sigma}} \right] + z,$$

while the cost functions are given by (4)-(6) where $q_{ij} = s_j x_{ij}$. Varieties are priced at marginal cost at the first best outcome.

The next proposition is proven in Appendix 3.

Proposition 2. *Assume that* $0 < \phi < \omega < 1$. *If*

$$B_L < (\sigma - 1) f < B_R, \tag{29}$$

then the social optimum is such that all firms in the larger country are integrated, while the smaller country hosts the three types of organizational forms:

$$n_2^* > n_2^o = \frac{1}{1+S} \cdot \left(\frac{1 + \omega S}{\omega - \phi} - \frac{1}{f(\sigma - 1)} \right) \tag{30}$$

$$v_2^* > v_2^o = \frac{1}{1+S} \cdot \left(\frac{\phi + S}{1 - \phi\omega} - \frac{S}{f(\sigma - 1)} \right), \tag{31}$$

$$h_2^* < h_2^o = \frac{1}{1+S} \cdot \left[\frac{1+S}{f(\sigma - 1)} - \frac{(1-\phi^2)(1+\omega S)}{(\omega - \phi)(1 - \phi\omega)} \right]. \tag{32}$$

Following the same approach as in 3.2, it is readily verified that $n_2^o > 0$, $v_2^o > 0$ and $h_2^o > 0$ if and only if (29) holds. Here too, communication costs must be lower than trade costs ($\omega > \phi$) for this condition to be satisfied.

Under CES preferences, the equilibrium and optimum of a one-sector economy coincide even when firms are heterogeneous (Dhingra and Morrow, 2018). Therefore, it is no surprise that the coexistence of different organizational forms is not socially wasteful. Indeed, comparing (24) and (29) shows that both the market equilibrium and the social optimum involve the coexistence of all organizational forms when $B_L/(\sigma - 1) < f < B_R/\sigma$. However, the numbers of firm-types in the smaller country need not be the same at the two outcomes because the cost distribution is now endogenous through the organizational choices made by firms.

Propositions 1 and 2 have the following implication: *the social optimum involves fewer integrated and vertical firms and more horizontal firms than the market equilibrium.* Since $n_2^* > n_2^o$, too few country 2-firms become multinational when firms compete. Indeed, the 2-firms hold back their investments in the larger market to soften competition therein. As a result, competition in the larger country becomes weak enough for this market to host too many **V**-firms. This in turn implies that too many 2-firms do not invest in their home country by delocalizing their production activities in the larger country. Hence, *each country accommodates too few plants at the market outcome.* To put it differently, there is an excessive geographical concentration of production. Note also that Proposition 2 shows that the diversity of organizational forms allows minimizing the total trade and communication costs associated with the first-best flows of varieties.

4 Market Size and Spatial Frictions

In this section, we study the effects of market size, trade and communication costs on the mass of plants and the numbers of each firm-type. In particular, we will see that trade and communication costs have very different impacts on the market outcome and its welfare properties.

4.1 The Home Market Effect

Our set-up allows us to determine the total mass of plants in the whole economy and their distribution between the two countries. In this section, we show how these masses vary with the absolute and relative sizes of the two countries.

First of all, Proposition 1 implies that the mass of plants located in the larger country is equal to $s_1 + v_2^* + h_2^* > s_1$, while the mass of plants established in the smaller country is $n_2^* + h_2^* = s_2 - v_2^* < s_2$. Consequently, the larger country hosts a disproportionately higher mass of plants. This result echoes the home market effect (HME), which states that the larger country hosts a more than proportionate share of firms, which are by assumption spatially integrated (Baldwin *et al.*, 2003).

We now study the impact of the relative size of the two countries on the mass of plants located in country 1 by differentiating $n_1^* + v_2^* + h_2^*$ with respect to $S = s_2/s_1$. First, we have:

$$\frac{\mathrm{d}n_1^*}{\mathrm{d}S} = -\frac{1}{(1+S)^2}. \tag{33}$$

Second, some tedious calculations show that the following expression holds:

$$\frac{\mathrm{d}v_2^*}{\mathrm{d}S} + \frac{\mathrm{d}h_2^*}{\mathrm{d}S} = \frac{1}{(1+S)^2}\left(\frac{1-\phi}{\omega-\phi} - \frac{1}{\sigma f}\right). \tag{34}$$

By implication of (24), we have

$$\sigma f < B_R = \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)} < \frac{\omega-\phi}{1-\phi} \Leftrightarrow \frac{1-\phi}{\omega-\phi} - \frac{1}{\sigma f} < 0,$$

because $(1+S)/(1+\omega S)$ is an increasing function of S while the inequality holds at $S = 1/K$. Therefore, we have:

$$\frac{\mathrm{d}v_2^*}{\mathrm{d}S} + \frac{\mathrm{d}h_2^*}{\mathrm{d}S} < 0. \tag{35}$$

Combining (33) and (34) yields

$$\frac{\mathrm{d}(n_1^* + v_2^* + h_2^*)}{\mathrm{d}S} = \frac{1}{(1+S)^2}\left(\frac{1-\phi}{\omega-\phi} - \frac{1}{\sigma f} - 1\right) < -1.$$

Since an increase in s_1 amounts to a decrease in S, the share of plants located in the larger country grows disproportionately with the size of this country. More specifically, a relatively higher number of workers in country 1 triggers an even stronger flow of foreign investments through a higher mass of **V**-firms. This corresponds to a drop in the mass of **I**-firms established in the smaller country.

Furthermore, we have:

$$\frac{\mathrm{d}(n_2^* + h_2^*)}{\mathrm{d}S} = \frac{1}{(1+S)^2}\left[\frac{\phi(1-\omega)}{1-\phi\omega} + \frac{1}{\sigma f}\right] > 0.$$

Combining this expression with (35) implies

$$\frac{\mathrm{d}(v_2^* + h_2^*)}{\mathrm{d}S} = -\frac{\mathrm{d}n_2^*}{\mathrm{d}S} < 0 < \frac{\mathrm{d}(n_2^* + h_2^*)}{\mathrm{d}S}.$$

Hence, when the relative size of the smaller country decreases, it hosts fewer integrated firms. Moreover, the mass of country 2's **H**-firms decreases, but this drop is more than compensated by the hike in the mass of **V**-firms generated by the larger size of country 1. In other words, country 1 hosts more foreign plants.

Finally, since

$$\frac{\mathrm{d}(n_1^* + v_2^* + h_2^*)}{\mathrm{d}S} + \frac{\mathrm{d}(n_2^* + h_2^*)}{\mathrm{d}S} = \frac{1}{(1+S)^2}\frac{(1-\omega)(1-\phi^2)}{(\omega-\phi)(1-\phi\omega)} > 0,$$

the increase in the mass of country 1's plants is smaller than the decrease in the mass of plants operating in country 2. By implication, the total mass of plants in the economy falls when countries become more dissimilar in size.

The following proposition comprises a summary.

Proposition 3. *Assume that* $0 < \phi < \omega < 1$. *At a mixed equilibrium, the larger country hosts a more than proportionate share of plants. Furthermore, the mass of plants established in this country increases more than proportionally with its size, while the total mass of plants operating in the economy decreases.*

This proposition suggests the gradual hollowing out of the smaller country as its relative size shrinks.

4.2 Trade Costs

The most popular thought experiment in the literature deals with the impact of trade costs on firms' locational decisions. Using (18) and (19) where $i = 1$ and $j = 2$ shows that both Δ_1^* and Δ_2^* decrease when ϕ rises. In other words, lowering trade costs is associated with a smaller effective mass of plants on each market. Therefore, *competition is softened in each country*, as reflected by a higher price index in each country (P_1^* and P_2^* increase).

To shed more light on the various effects at work, we differentiate n_2^*, v_2^* and h_2^*:

$$0 < \frac{\mathrm{d}v_2^*}{\mathrm{d}\phi} < \frac{\mathrm{d}n_2^*}{\mathrm{d}\phi} < -\frac{\mathrm{d}h_2^*}{\mathrm{d}\phi}.$$

Hence, *fewer firms go multinational when market integration becomes deeper*, so that the mass of **I**-firms rises. However, the impact on **H-** and **V**-firms are opposite. While a decrease in trade costs leads to a smaller mass of **H**-firms since the access to country 1 becomes easier from country 2, the mass of **V**-firms rises because reimporting goods from country 1 to the country 2 is cheaper. In addition, when trade costs fall, both markets become less competitive (Δ_1^* and Δ_2^* decrease, hence P_1^* and P_2^* increase). Since more 2-firms become vertical, fewer 2-firms invest home, which renders market 2 less competitive. Similarly, market 1 becomes less competitive since the drop in the mass of **H**-firms is stronger than the hike in the mass of **V**-firms.

Furthermore, it is well known that a deeper market integration induces the relocation of firms from the smaller to the larger country when firms are spatially integrated and mobile (Baldwin *et al.*, 2003). Here, the total mass of plants operating in the larger country decreases faster than in smaller country when trade costs fall. In other words, a deeper market integration makes the HME weaker rather than stronger. However, the result that production is concentrated in a smaller mass of plants when trade costs decrease concurs with the main message of economic geography, that is, lowering trade costs fosters the agglomeration of activities. This shows that the phenomenon of agglomeration may take different concrete forms.

4.3 Communication Costs

It follows immediately from (18) and (19) that lowering communication costs have a different impact on the two markets. Indeed, as ω increases, the effective mass of plants competing in the larger country rises, whereas the effective mass of plants competing in the smaller country falls. Consequently, competition is intensified in country 1 and weakened in country 2.

More specifically, since making the transfer of information cheaper facilitates the spatial fragmentation of firms, it is readily verified that

$$\frac{\mathrm{d}n_2^*}{\mathrm{d}\omega} < 0 \qquad \frac{\mathrm{d}v_2^*}{\mathrm{d}\omega} > 0 \qquad \frac{\mathrm{d}h_2^*}{\mathrm{d}\omega} > 0.$$

In other words, *lowering communication costs leads more 2-firms to go multinational*, which increases the mass of plants hosted by the larger market, while the mass of plants established in the smaller country decreases. Observe the difference with the impact of lower trade costs which lead to a drop in the mass of multinational firms. Furthermore, whereas lower trade costs weakens the HME, the total mass of plants located in the larger country increases with ω, hence there is magnification of the HME. That is to say, communication costs play here the same role as trade costs in the footloose capital model (Baldwin *et al.*, 2003). Since country 2 hosts fewer firms, decreasing communication costs also triggers the hollowing out of the smaller country through the relocation of manual jobs toward the larger country.

How does the size of each type of firm reacts a drop in trade and communication costs? Differentiating (26) with respect ϕ and ω yields the following inequalities:

$$\begin{aligned}
\frac{\partial q_1^n}{\partial \phi} &> \frac{\partial q_2^n}{\partial \phi} = \frac{\partial q_2^v}{\partial \phi} = \frac{\partial q_2^h}{\partial \phi} = \left[\frac{\omega}{(1-\phi\omega)^2} + \frac{\omega}{(\omega-\phi)^2}\right]\sigma f > 0, \\
\frac{\partial q_1^n}{\partial \omega} &< \frac{\partial q_2^n}{\partial \omega} = \frac{\partial q_2^v}{\partial \omega} = \frac{\partial q_2^h}{\partial \omega} = \left[\frac{\phi}{(1-\phi\omega)^2} - \frac{\phi}{(\omega-\phi)^2}\right]\sigma f < 0.
\end{aligned}$$

Therefore, trade liberalization makes *all* firms bigger, regardless of their type and location, while the ICT revolution generates the reverse. Again, trade and communication costs have opposite effects.

Finally, the diverging impact of trade and communication costs may also be illustrated by studying how these costs affect firms' profits. First, since market integration leads to fewer plants in each country, competition is relaxed in both countries, which leads firms to make higher profits. Indeed, differentiating the equilibrium profits (27) with respect to ϕ yields:

$$\frac{\mathrm{d}\pi_2^*}{\mathrm{d}\phi} = \omega\frac{\mathrm{d}\pi_1^*}{\mathrm{d}\phi} > 0.$$

Therefore, a deeper market integration allows all the skilled to earn higher incomes in both countries. However, the income divergence is exacerbated as the two countries become more integrated.

Second, differentiating (27) with respect to ω, it is readily verified that

$$\frac{\mathrm{d}\pi_1^*}{\mathrm{d}\omega} < \frac{\mathrm{d}\pi_2^*}{\mathrm{d}\omega} < 0.$$

Since the 1-firms are integrated, they do not benefit from the drop in communication costs while facing a higher mass of foreign competitors on their domestic market. Consequently, the 1-firms and the 2-vertical and horizontal firms make lower profits in the larger market. Although the smaller market is less competitive because fewer 2-firms invest home, the difference in market sizes is sufficiently big ($s_1 > \bar{s} > s_2$) for the losses incurred in country 1 to overcome the gains made in country 2. Consequently, in both countries the skilled end up with lower incomes when communication costs fall. Moreover, the income gap shrinks when communication costs fall.

The main predictions of our model are summarized in the following proposition.

Proposition 4. *Assume that* $0 < \phi < \omega < 1$. *At a mixed equilibrium, lowering trade costs makes all firms bigger and leads to a smaller mass of plants, while lower communication costs have the opposite impact. Furthermore, trade liberalization raises profits while the adoption of new ICTs yields lower profits.*

Hence, as suggested by several empirical studies, exports and FDI are indeed substitutes (Head and Ries, 2003). Note also that Propositions 1, 2 and 4 imply that the optimal and equilibrium masses of firms respond in the same way to shocks on trade or communication costs.

5 What Are the Other Equilibrium Patterns of Organization?

When (24) and/or (25) do not hold, the market outcome differs from (21)-(23). The following result extends the existence and uniqueness result of Proposition 1 (see the Supplemental Material for a proof).[11] The properties of the equilibrium are discussed further down.

Proposition 5. *Assume that* $0 < \phi < \omega < 1$. *Then, there exists a unique organizational equilibrium almost everywhere in* $X = \{(S, \sigma f) \mid 0 < S < 1, 0 < \sigma f\}$.

One of the main thought experiments in the economics of multinational enterprises is to study how firms' organizational forms vary with the level of fixed costs and the relative size of markets (Markusen, 2002). In other words, we want to determine the market outcome when the value of σf does not belong to the interval (24). In what follows, we briefly describe the various equilibria and refer the reader to the Supplemental Material for more details. To achieve our goal, we assume that σf steadily decreases from very high to very low values or, equivalently, the size of the global economy rises.

There are two polar cases. When σf is very high, the horizontal organizational form is ruled out regardless of the value of S. The market outcome depends only upon the relative size S of countries. If the two countries do not differ much in size ($S > \bar{S}$), the equilibrium is **I - I**. Put differently, *there are no FDIs* and the mass of plants is minimized. This configuration corresponds to the canonical model of intraindustry trade. As S decreases below the threshold $\bar{S}$, some 2-firms become vertical because country 1 is relatively bigger (**I - IV**). When the two countries have very different sizes ($S < \phi/K$), all 2-firms find it profitable to establish their plants in the larger country (**I - V),** so that there is *one-way trade* from country 1 to country 2. In these three cases, (25) does not hold.

At the other extreme of the spectrum, when σf is very low all firms are horizontal (**H - H**). However, as S decreases the corresponding domain shrinks because country 2 becomes too small for 1-firms to invest there. More

[11] There are 49 possible configurations. Depending on the values of the parameters σf, S, ϕ and ω, 10 of them are market equilibria.

specifically, the configuration **H - H** emerges if and only if

$$\sigma f < \frac{(\omega - \phi)S}{S + \omega}.$$

In this case, there is no trade because the whole range of varieties is produced in each country. In other words, *FDI is a perfect substitute for trade*, while the mass of plants is maximized.

When $S > \bar{S}$, the two countries are similar enough to host the same types of organizational form, while the number of horizontal firms keeps growing as σf decreases. Assume now that $S < \overline{S}$ holds. All configurations but one involve asymmetric organizational forms between or within countries, that is, trade and FDI are imperfect substitutes. If σf exceeds B_R, some 2-firms invest abroad when country 1 is sufficiently large. As seen above, the equilibrium is given by **I - IV** if and only if $\sigma f > B_R$ and

$$\frac{\phi}{K} < S < \overline{S}.$$

As σf falls below B_R, the economy displays the mixed equilibrium (**I - IVH**) described in Proposition 1. What happens when σf falls below B_L? The equilibrium configuration depends on the relative size of the two countries. More specifically, two cases may arise, that is, country 2 hosts either no **V**-firms or no **I**-firms.

(i) The configuration **I - IH** becomes the equilibrium outcome if and only if country 2 remains big enough, that is, $S > \hat{S}$ where $\hat{S}$ is a bundle of ϕ and ω defined in the Supplemental Material, while

$$(\omega - \phi) \max \left\{ \frac{S}{\phi + S}, \frac{1}{1 + \omega S} \right\} < \sigma f < B_L.$$

Indeed, the relative size of country 2 must be large enough for some 2-firms to remain integrated, while the others are horizontal because fixed costs are sufficiently low.

When σf decreases further, two subcases may arise according to the value of S.

(i.a) **IH - IH** when $S > \tilde{S}$ ($\tilde{S}$ is a bundle of ϕ and ω defined in the Supplemental Material), that is, country 2 is large enough for a few 1-firms to produce abroad;

(i.b) **I - H** when $S < \tilde{S}$ because country 2 is small, so that no 1-firm invests abroad and no 2-firm remains integrated.

(ii) The configuration **I - VH** becomes the equilibrium outcome if and only if $\phi/K < S < \hat{S}$ and

$$\frac{(1 - \phi\omega)S}{\phi + S} < \sigma f < B_L.$$

Indeed, some 2-firms choose to be either **V**-firms because country 2 is small or **H**-firms because fixed costs are low. Next, when σf decreases further, **I - VH** becomes **I - H** because fixed costs are low enough for all 2-firms to be horizontal.

Finally, as σf keeps falling we have, first, **IH - H** and, then, **H - H**, that is, the solution mentioned above.

In short, when fixed costs are not too high or too small, all 1-firms are integrated and the 2-firms choose at least two different organizational forms. Although producer prices are the same regardless of how firms organize their activities across space, *it pays for the* 2*-firms to be different* (except in the limit case of very small fixed costs). Thus, working with a single spatial friction, e.g., trade costs, leads to a narrow set of equilibrium outcomes. On the other hand, the 1-firms are integrated because they benefit from a direct access to the larger market. These firms choose to be horizontal only when fixed costs are very low.

Assume that the market outcome is given by the mixed equilibrium (**I - IVH**). When the drop in trade costs is strong enough, the market outcome shifts to (**I - IV**) because investing in both countries ceases to be profitable. On the other hand, when communication costs decrease, being integrated is no longer attractive for the 2-firms because producing in the larger country is less expensive. As a result, the equilibrium becomes (**I - VH**). Thus, starting from the same initial outcome, a gradual decrease in trade or communication costs leads to different outcomes. This concurs with Proposition 4.

Likewise, if (**I - VH**) is the initial equilibrium outcome, when trade costs steadily decrease the economy moves to (**I - IV**) through the mixed equilibrium. By contrast, for the same path to arise, communication costs must rise. Hence, trade and communication costs have contrasted effects on the distribution of plants in the global economy. To illustrate even further this, the trade-off between increasing returns and trade costs implies that the economy moves from (**IH - IH**) to (**I - IH**) with a strong drop in trade costs. In contrast, the economy moves from (**I - IH**) to (**IH - IH**) when communication costs fall sharply.

Combining this discussion with what we saw in Section 4, we may safely conclude that *decreases in trade or communication costs do not affect the geographical distribution of production in the same way.*

6 Heterogeneous Firms

In this section, we study what Proposition 1 becomes in the case where firms differ a priori in productivity regardless of the organizational form they choose. As in the homogeneous firm case, we focus on the configuration where country 2 hosts the three types of firms. It then follows from Appendix 1 that all 1-firms are integrated when $\overline{\theta}$ is not too large. We assume perfect sorting, i.e., firms sharing the same productivity choose the same organizational form.[12]

Only the most productive firms can afford to invest in two plants. Hence, the horizontal firms (if any) are always the most productive. Consequently, it remains to investigate the following two cases. In the first one, the least productive 2-firms are integrated: $1 < \theta_2^v < \theta_2^h < \overline{\theta}$, where θ_2^v and θ_2^h are the productivity thresholds such that a **I**-firms has a productivity $\theta_2 < \theta_2^v$, a **V**-firm has a productivity $\theta_2^v < \theta_2 < \theta_2^h$, while a **H**-firm has a productivity $\theta_2 > \theta_2^h$. In the second case, the least productive 2-firms are vertical, i.e., $1 < \theta_2^n < \theta_2^h < \overline{\theta}$. In the former case, the equilibrium conditions are given by $\pi_2^n(\theta_2^v) = \pi_2^v(\theta_2^v)$ and $\pi_2^v(\theta_2^h) = \pi_2^h(\theta_2^h)$ while they are $\pi_2^n(\theta_2^n) = \pi_2^v(\theta_2^n)$ and $\pi_2^n(\theta_2^h) = \pi_2^h(\theta_2^h)$ in the latter.

In either case, the equilibrium conditions are equivalent to

$$\Delta_1^*(\theta_2^h) = \frac{\omega - \phi}{\sigma f} s_1 \cdot \left(\theta_2^h\right)^{\sigma - 1}, \tag{36}$$

$$\Delta_2^*(\theta_2^h) = \frac{1 - \phi\omega}{\sigma f} s_2 \cdot \left(\theta_2^h\right)^{\sigma - 1}. \tag{37}$$

Note that (36) ((37)) is identical (18) ((19)) when firms are homogeneous since $\theta_2^h = 1$.

Using (12), we may rewrite (36)-(37) as follows:

[12] Note that the **I**- and **V**-firms that have the same productivity earn the same profits. However, assuming that **I**- and **V**-firms have different fixed labor requirement implies that the mid-productive firms always adopt the organizational form associated with the higher fixed requirement. As a result, there is perfect sorting.

$$\Delta_1^*(\theta_2^h) = A \cdot [s_1 + \omega s_2 - (\omega - \phi) n_2], \tag{38}$$

$$\Delta_2^*(\theta_2^h) = A \cdot [\phi s_1 + s_2 - (1 - \phi\omega) v_2], \tag{39}$$

where A is given by (11).

Following the same approach as in the homogeneous firm case, we find that (12) and (36)-(39) yields the following expressions:

$$n_2^*(\theta_2^h) = \frac{1}{1+S} \cdot \left(\frac{1+\omega S}{\omega - \phi} - \frac{\left(\theta_2^h\right)^{\sigma-1}}{A} \cdot \frac{1}{\sigma f} \right), \tag{40}$$

$$v_2^*(\theta_2^h) = \frac{1}{1+S} \cdot \left(\frac{\phi + S}{1 - \phi\omega} - \frac{\left(\theta_2^h\right)^{\sigma-1}}{A} \cdot \frac{S}{\sigma f} \right), \tag{41}$$

$$h_2^*(\theta_2^h) = \frac{s_2}{A} \cdot \int_{\theta_2^h}^{\bar{\theta}} \theta^{\sigma-1} \mathrm{d}G = \frac{1}{1+S} \cdot \left[\frac{\left(\theta_2^h\right)^{\sigma-1}}{A} \cdot \frac{1+S}{\sigma f} - \frac{(1+\omega S)(1-\phi^2)}{(\omega-\phi)(1-\phi\omega)} \right]. \tag{42}$$

Since the left-hand side of (42) is decreasing and positive at $\theta_2^h = 1$ while the right-hand side is increasing and negative at $\theta_2^h = 1$, (42) has a unique solution. Furthermore, this solution exceeds 1 and is smaller than $\overline{\theta}$. Plugging this solution in (40) and (41) yields the corresponding equilibrium masses of **I**- and **V**-firms. As consequence, there exists at most one equilibrium and the equilibrium value θ_2^h is independent of the respective masses of integrated and vertical firms.

Similar to the homogenous firm case, it can be shown that (40)-(42) imply that country 2 hosts the three types of firms if and only if the following condition holds:

$$B_L < \frac{\left(\theta_2^h\right)^{\sigma-1}}{A} \cdot \sigma f < B_R. \tag{43}$$

Similarly, a mixed equilibrium with heterogeneous firms exists when

$$0 < \frac{\phi}{K} < S < \overline{S} < \frac{1}{K} < 1 \tag{44}$$

holds.

Note that the conditions (40)-(42) boil down to (21)-(23), while (43)-(44) reduces to (24)-(25) when firms are homogeneous because $A/\left(\theta_2^h\right)^{\sigma-1} = 1$.

It remains to determine whether the least productive 2-firms are integrated or vertical.

Case 1. Assume that the least productive firms are integrated: $1 < \theta_2^v < \theta_2^h < \overline{\theta}$. Computing the integrals in (10) for the truncated Pareto distribution yields the following expressions:

$$n_2^* = \frac{S}{1+S} \cdot \frac{1 - (\theta_2^v)^{-(\kappa-\sigma+1)}}{1 - \left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}, \tag{45}$$

$$v_2^* = \frac{S}{1+S} \cdot \frac{(\theta_2^v)^{-(\kappa-\sigma+1)} - \left(\theta_2^h\right)^{-(\kappa-\sigma+1)}}{1 - \left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}, \tag{46}$$

and

$$h_2^* = \frac{S}{1+S} \cdot \frac{\left(\theta_2^h\right)^{-(\kappa-\sigma+1)} - \left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}{1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}. \tag{47}$$

For the assumed configuration to be an equilibrium, the equations (40)-(42) and (45)-(47) must be consistent. In particular, (46)-(47) and (41)-(42) must be equal. Using (36)-(37), we then obtain the equilibrium conditions corresponding to the configuration $1 < \theta_2^v < \theta_2^h$:

$$\left(\theta_2^h\right)^{-(\kappa-\sigma+1)} = \frac{\phi K S^2 + (K-1)S - \phi}{(1-\omega\phi)S} \cdot \left[1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}\right] + (1+S)\left(\theta_2^v\right)^{-(\kappa-\sigma+1)}, \tag{48}$$

and

$$\frac{\left(\theta_2^h\right)^{\sigma-1}}{A\sigma f} - \frac{S\left(\theta_2^h\right)^{-(\kappa-\sigma+1)}}{1+S} \cdot \frac{1}{\left(\overline{\theta}\right)^{\kappa-\sigma+1}-1} = \frac{(1+\phi K)S+\phi+K}{(1-\phi\omega)(1+S)} - \frac{S}{1+S} \cdot \frac{1}{\left(\overline{\theta}\right)^{\kappa-\sigma+1}-1}. \tag{49}$$

It remains to determine under which conditions the inequalities $1 < \theta_2^v < \theta_2^h$ hold. We show in Appendix 4 that this configuration is an equilibrium when $S \in [\phi/K, \underline{S}]$, where the constant $\underline{S}$ is defined in the same appendix.

Case 2. Assume now that the least productive firms are vertical: $1 < \theta_2^n < \theta_2^h < \overline{\theta}$. Hence, n_2^* and v_2^* are given by

$$n_2^* = \frac{S}{1+S} \cdot \frac{\left(\theta_2^n\right)^{-(\kappa-\sigma+1)} - \left(\theta_2^h\right)^{-(\kappa-\sigma+1)}}{1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}, \tag{50}$$

$$v_2^* = \frac{S}{1+S} \cdot \frac{1-\left(\theta_2^n\right)^{-(\kappa-\sigma+1)}}{1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}, \tag{51}$$

while h_2^* is still given by (47).

Following the same approach as in the case above, we obtain the equilibrium conditions corresponding to the configuration $1 < \theta_2^n < \theta_2^h$:

$$S\left(\theta_2^h\right)^{-(\kappa-\sigma+1)} = (1+S)\left(\theta_2^n\right)^{-(\kappa-\sigma+1)} - \frac{\omega K S^2 + (K-1)S - \phi}{(1-\phi\omega)S}\left[1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}\right] - 1, \tag{52}$$

$$\frac{\left(\theta_2^h\right)^{\sigma-1}}{A\sigma f} - \frac{S\left(\theta_2^h\right)^{-(\kappa-\sigma+1)}}{1+S} \cdot \frac{1}{\left(\overline{\theta}\right)^{\kappa-\sigma+1}-1} = \frac{(1+\phi K)S+\phi+K}{(1-\phi\omega)(1+S)} - \frac{S}{1+S} \cdot \frac{1}{\left(\overline{\theta}\right)^{\kappa-\sigma+1}-1}. \tag{53}$$

Observe that (49) and (53) are the same. In other words, the equilibrium mass of **H**-firms is the same in the two configurations. However, the equilibrium masses of **I**- and **V**-firms are not the same because (48) and (52) differ.

It remains to determine under which conditions $1 < \theta_2^n < \theta_2^h < \overline{\theta}$ holds. We show in Appendix 5 that this configuration is an equilibrium when $S \in [\underline{S}, \overline{S}]$.

Our main findings may be summarized as follows.

Proposition 6. *Assume that firms are cost-heterogeneous. Then, a mixed equilibrium exists if and only if* (43) *and* (44) *hold. This equilibrium is such that all* 1*-firms are integrated while the most productive* 2*-firms are horizontal. Furthermore, when (i)* $S \in [\phi/K, \underline{S}]$ *the least productive* 2*-firms are integrated, and (ii)* $S \in [\underline{S}, \overline{S}]$ *the least productive* 2*-firms are vertical.*

The intuition behind Proposition 6 is easy to grasp. The most productive firms choose to be horizontal because this allows them to avoid paying trade costs which exceed communication costs. Which organizational form choose the mid-productive firms depends on the relative size of countries. When the asymmetry is relatively high (bullet (i) in Proposition 6) the mid-productive firms go vertical because they are able to provide the large market at lower prices than under the **I**-form. However, if the asymmetry is mild (bullet (ii)), the local market matters more, which leads the mid-productive firms to be integrated because they can supply the local market at lower prices than under the **V**-form.

The effect of lowering trade and communication costs on the equilibrium configurations is more involved than in the homogeneous firm case. Nevertheless, a few neat results hold true. First of all, we show in Appendix 6 that θ_2^h always increases with ϕ and decreases with ω. Therefore, as in the homogeneous firm case, the mass of H-firms decreases (increases) when trade costs (communication costs) fall.

Furthermore, for the configuration where the least productive firms are integrated, the first term in the right-hand side of (48) decreases with ω, hence θ_2^v also decreases. Consequently, a drop in communication costs leads to fewer **I**-firms, like in the homogeneous firm case, while the change in the mass of **V**-firms depend on the shape parameter κ of productivity distribution. Similarly, decreasing trade costs leads to hike in θ_2^h, so that the left-hand side of (48) decreases. Since the first term in the right-hand side of (48) increases when $\omega < \overline{\omega}$, with $\overline{\omega} = 2\phi/(1+\phi^2)$, θ_2^v increases, we may conclude that trade liberalization makes **I**-firms more profitable.[13] Under these circumstances, communication and trade costs have the same impacts on **I**-firms as in the homogeneous firm case. However, the impact on the mass of **V**-firms is ambiguous.

Finally, regarding the configuration where the least productive firms are vertical, it can be shown that the second term in the right-hand side of (52) increases with ω, so that the impact of ω on θ_2^n is ambiguous. However, trade liberalization leads to an increase in θ_2^n. Similarly to homogeneous firm case, more firms thus choose to become vertical when trade costs decrease if (i) communication costs are low enough, i.e., $\omega > \omega^*$ where $\omega^* > \overline{\omega}$, and (ii) countries are sufficiently asymmetric, i.e., $S \in (S^*, 1/K)$, where $S^* > \underline{S}$.

7 Concluding Remarks

Our analysis has shown that neglecting communication costs as a specific determinant of firms' spatial structure is unwarranted in the geography and trade literature. On the contrary, understanding how firms organize their activities across space requires a clear distinction between communication and trade costs because these costs affect firms' choices differently. More specifically, both costs often have opposite impacts on the geography of production. Since the social optimum also involves diversification under conditions similar to those obtained at the market equilibrium, the diversification of organizational forms is driven by the fundamentals of the economy, especially trade and communication costs.

Furthermore, identical firms may choose to become heterogeneous by choosing the whole range of organizational forms. For this to arise, communication costs must be sufficiently low while trade costs cannot be too low. Under similar conditions, the same holds for heterogeneous firms. In both the optimum and the market equilibrium, when

[13] The proof is given in Appendix 6.

communication costs are lower than trade costs, the smaller country's firms display three types of organizational forms when the foreign market is sufficient large, but not too much, to permit some firms to go vertical or horizontal whereas the others remain integrated. By contrast, the larger country's firms choose to be spatial integrated since they supply the large market without bearing any spatial friction.

How to measure of communication costs remains a difficult issue. Keller and Yeaple (2013) propose to solve this problem by using knowledge-intensive inputs as a substitute for direct communication costs, while Giroux (2013) focusses on airline connections. Though ingenious, both approaches remain incomplete.

Appendix 1

Step 1. (i) Assume that $\phi < \omega < 1$. We show that one country hosts only one type of firms at any mixed equilibrium.

Since only the most productive firms can afford to invest in two plants, the horizontal firms (if any) are the most efficient ones. Consequently, the least productive firms are integrated or vertical. In the first case, the least productive j-firms are integrated: $1 < \theta_j^v < \theta_j^h < \overline{\theta}$, where θ_j^v and θ_j^h are the productivity thresholds such that a **I**-firms has a productivity $\theta_j < \theta_j^v$, a **V**-firm has a productivity $\theta_j^v < \theta_j < \theta_j^h$, while a **H**-firm has a productivity $\theta_j > \theta_j^h$; the equilibrium conditions are given by $\pi_j^n(\theta_j^v) = \pi_j^v(\theta_j^v)$. In the second, the least productive j-firms are vertical, i.e., $1 < \theta_j^n < \theta_j^h < \overline{\theta}$. In the former case, and $\pi_j^v(\theta_j^h) = \pi_j^h(\theta_j^h)$; the equilibrium conditions are $\pi_j^n(\theta_j^n) = \pi_j^v(\theta_j^n)$ and $\pi_j^n(\theta_j^h) = \pi_j^h(\theta_j^h)$. Using (14)-(16), the equilibrium conditions are in both cases equivalent to

$$\Delta_i^* = \left(\theta_j^h\right)^{\sigma-1} \cdot \frac{\omega - \phi}{\sigma f} s_i, \tag{A.1}$$

$$\Delta_j^* = \left(\theta_j^h\right)^{\sigma-1} \cdot \frac{1 - \phi\omega}{\sigma f} s_j, \tag{A.2}$$

while the mirror-image equations for country i are as follows:

$$\Delta_j^{**} = \left(\theta_i^h\right)^{\sigma-1} \cdot \frac{\omega - \phi}{\sigma f} s_j, \tag{A.3}$$

$$\Delta_i^{**} = \left(\theta_i^h\right)^{\sigma-1} \cdot \frac{1 - \phi\omega}{\sigma f} s_i. \tag{A.4}$$

At any equilibrium in which one country hosts the three types of firms and the other two or three types, at least two of the following conditions must hold: (i) $\Delta_i^* = \Delta_i^{**}$ and (ii) $\Delta_j^* = \Delta_j^{**}$. However, $\omega - \phi \neq 1 - \phi\omega$ because $\omega < 1$. This implies $\Delta_i^* \neq \Delta_i^{**}$ and $\Delta_j^* \neq \Delta_j^{**}$. Hence, we have: (a) $\pi_j^v(\theta_j^h) \neq \pi_j^h(\theta_j^h)$ must hold when $\pi_i^n(\theta_i^h) = \pi_i^h(\theta_i^h)$; (b) $\pi_j^n(\theta_j^h) \neq \pi_j^h(\theta_j^h)$ when $\pi_i^v(\theta_i^h) = \pi_i^h(\theta_i^h)$; and (c) $\pi_j^n(\theta_j^h) \neq \pi_j^v(\theta_j^h)$ when $\pi_i^n(\theta_i^h) = \pi_i^v(\theta_i^h)$. As a result, country i can host only one type of firms when j-firms are fully diversified.

(ii) Assume now that $\omega = 1$. It follows from (A.1)-(A.4) that

$$\frac{\Delta_i^*}{\Delta_j^*} = \frac{\Delta_i^{**}}{\Delta_j^{**}} = \frac{s_i}{s_j}, \tag{A.5}$$

which implies $\theta^h \equiv \theta_i^h = \theta_j^h$. Therefore, using (13) and the equilibrium conditions $\pi_i^h(\theta_i^h) = \pi_i^v(\theta_i^h)$ and $\pi_i^v(\theta_i^v) = \pi_i^n(\theta_i^v)$ for $i = 1, 2$, as well as $n_i + v_i + h_i = s_i$ for $i = 1, 2$, we obtain:

$$\Delta_i^* = A \cdot \left[s_i + s_j - (1 - \phi)(n_j^* + v_i^*)\right] = \left(\theta^h\right)^{\sigma-1} \cdot \frac{1 - \phi}{\sigma f} s_i, \tag{A.6}$$

$$\Delta_j^* = A \cdot \left[\phi s_i + s_j - (1 - \phi)(n_i^* + v_j^*)\right] = \left(\theta^h\right)^{\sigma-1} \cdot \frac{1 - \phi}{\sigma f} s_j. \tag{A.7}$$

Hence, we have three equations (A.5)-(A.7) and four unknowns, n_j, v_i, n_i, v_j. As a result, there is a continuum of solutions to the equilibrium conditions.

Step 2. Assume a mixed equilibrium where the three types of firms coexist in country j. Then, all i-firms are integrated when the productivity range of these firms is not "too" large: $\pi_i^n(\theta) > \pi_i^v(\theta)$ and $\pi_i^n(\theta) > \pi_i^h(\theta)$ for all $\theta \in [1, \overline{\theta}]$ if $\overline{\theta}$ does not exceed some threshold.

Plugging (25) and (24) into (14)-(16) yields

$$\pi_i^n(\theta) - \pi_i^v(\theta) = \frac{\theta^{\sigma-1} f}{\left(\theta_i^h\right)^{\sigma-1}} \left(\frac{1-\phi\omega}{\omega-\phi} - \frac{\omega-\phi}{1-\phi\omega} \right),$$

$$\pi_i^n(\theta) - \pi_i^h(\theta) = f \cdot \left[1 - \left(\frac{\theta}{\theta_i^h} \right)^{\sigma-1} \cdot \frac{\omega-\phi}{1-\phi\omega} \right].$$

First, $\pi_i^n(\theta) > \pi_i^v(\theta)$ for all θ since $1-\phi\omega > \omega-\phi$. Second, since $\pi_i^n(\theta) - \pi_i^h(\theta)$ is decreasing in θ, $\pi_i^n(\theta) - \pi_i^h(\theta) > 0$ if

$$\pi_i^n(\overline{\theta}) - \pi_i^h(\overline{\theta}) > 0 \Leftrightarrow \overline{\theta} \leq K^{\frac{1}{\sigma-1}} \cdot \theta_i^h, \tag{A.8}$$

In the worst case, $\theta_i^h \approx 1$ so that the desired inequality holds if $\overline{\theta} \leq K^{\frac{1}{\sigma-1}}$.

When firms are homogeneous ($\theta_i^h = \bar{\theta}$), (A.8) reduces to $\omega - \phi < 1 - \phi\omega$, which always holds. Q.E.D.

Appendix 2

We first determine the candidate equilibrium values n_j^*, v_j^*, h_j^* when $n_i^* = s_i$ and $v_i^* = h_i^* = 0$ and, then, find the conditions for (21)–(23) to be positive. Finally, we show that $i = 1$ and $j = 2$.

Step 1. Substituting $v_i^* = 0$ into (13) and setting $A = 1$ leads to $\Delta_i^* = s_i + \omega s_j - (\omega - \phi)n_j^*$. Using (18) thus yields (21) for $j = 2$. Substituting $n_i^* = s_i$ and Δ_j^* into (13) yields (22) for $j = 2$. Substituting v_j^* and n_j^* into the condition $n_j + v_j + h_j = s_j$, we obtain (23) for $j = 2$.

Step 2. Set $S = s_j/s_i$. Using (21)-(23), the inequalities $n_j^* > 0$, $v_j^* > 0$ and $h_j^* > 0$ are, respectively, equivalent to the following conditions:

$$\sigma f > \frac{\omega-\phi}{1+\omega S} \qquad \sigma f > \frac{(1-\phi\omega)S}{\phi+S} \qquad \sigma f < \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)},$$

which amounts to (24) where

$$B_L \equiv \max\left\{ \frac{\omega-\phi}{1+\omega S}, \frac{(1-\phi\omega)S}{\phi+S} \right\} \quad \text{and} \quad B_R \equiv \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}.$$

Step 3. Observe first that the inequality

$$\frac{\omega-\phi}{1+\omega S} < \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}$$

may be rewritten as follows:

$$S > \frac{\phi}{K},$$

Furthermore, the inequality

$$\frac{(1-\phi\omega)S}{\phi+S} < \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}$$

is equivalent to

$$F(S) \equiv \phi K S^2 + (K-1)S - \phi < 0. \tag{B.1}$$

Let $\overline{S}$ be the positive root of $F(S) = 0$. Since $F(\phi/K) < 0$ and $F(1/K) > 0$, the condition (24) holds if and only if

$$\frac{\phi}{K} < S < \overline{S} < \frac{1}{K},$$

which implies $S < 1$. Therefore, it must be that $i = 1$ and $j = 2$.

Step 4. Since $\frac{1}{1+\omega S}$ is decreasing in S while $\frac{S}{\phi+S}$ is increasing, the latter is smaller than the former if and only if this inequality holds when S takes on its lowest value, that is, $S = \phi/K$. Therefore, (24) and (25) are necessary and sufficient for Proposition 1 to hold. Q.E.D.

Appendix 3

The proof involves several steps. First, we show that the solutions to the first-order conditions for, say, country i cannot be all positive and determine the optimal values of n_i, v_i and h_i under the assumption that the solutions to the first-order conditions for country j are strictly positive (Steps 1 and 2). Then, we determine the necessary and sufficient conditions for country j's solutions to be strictly positive (Step 3), while Step 4 shows that the so-obtained solutions maximize the total welfare W.

The first letter in the subscript of a variable stands for the firm's HQ location while the second letter denotes the supplied market. We use the constraint $h_j = s_j - n_j - v_j > 0$ to replace h_j in U_i and W.

Step 1. Assume that the optimal solution is such that all three variables are strictly positive in country j. Differentiating (28) yields the following system of equations:

$$\begin{aligned} \frac{\partial W}{\partial n_j} &= s_j \frac{\partial U_j}{\partial n_j} + s_i \frac{\partial U_i}{\partial n_j} - C_j^n + C_j^h = 0, \\ \frac{\partial W}{\partial v_j} &= s_j \frac{\partial U_j}{\partial v_j} + s_i \frac{\partial U_i}{\partial v_j} - C_j^v + C_j^h = 0, \end{aligned} \tag{C.1}$$

and

$$\begin{aligned} \frac{\partial W}{\partial x_{jj}^n} &= s_j \frac{\partial U_j}{\partial x_{jj}^n} - n_j \frac{\partial C_j^n}{\partial x_{jj}^n} = 0 \Leftrightarrow x_{jj}^n = \left(\frac{1}{c\Omega_j}\right)^\sigma, \\ \frac{\partial W}{\partial x_{jj}^v} &= s_j \frac{\partial U_j}{\partial x_{jj}^v} - v_j \frac{\partial C_j^v}{\partial x_{jj}^v} = 0 \Leftrightarrow x_{jj}^v = \left(\frac{1}{\tau\gamma c\Omega_j}\right)^\sigma, \\ \frac{\partial W}{\partial x_{jj}^h} &= s_j \frac{\partial U_j}{\partial x_{jj}^{hj}} - h_j \frac{\partial C_j^h}{\partial x_{jj}^{hj}} = 0 \Leftrightarrow x_{jj}^h = \left(\frac{1}{c\Omega_j}\right)^\sigma, \\ \frac{\partial W}{\partial x_{ij}^n} &= s_j \frac{\partial U_j}{\partial x_{ij}^n} - n_i \frac{\partial C_j^n}{\partial x_{ij}^n} = 0 \Leftrightarrow x_{ij}^n = \left(\frac{1}{\tau c\Omega_j}\right)^\sigma, \\ \frac{\partial W}{\partial x_{ij}^v} &= s_j \frac{\partial U_j}{\partial x_{ij}^v} - v_i \frac{\partial C_j^v}{\partial x_{ij}^v} = 0 \Leftrightarrow x_{ij}^v = \left(\frac{1}{\gamma c\Omega_j}\right)^\sigma, \\ \frac{\partial W}{\partial x_{ij}^h} &= s_j \frac{\partial U_j}{\partial x_{ij}^h} - h_i \frac{\partial C_j^h}{\partial x_{ij}^h} = 0 \Leftrightarrow x_{ij}^h = \left(\frac{1}{\gamma c\Omega_j}\right)^\sigma, \end{aligned} \tag{C.2}$$

where

$$\Omega_j \equiv n_j (x_{jj}^n)^{\frac{\sigma-1}{\sigma}} + v_j (x_{jj}^v)^{\frac{\sigma-1}{\sigma}} + h_j (x_{jj}^h)^{\frac{\sigma-1}{\sigma}} + n_i (x_{ij}^n)^{\frac{\sigma-1}{\sigma}} + v_i (x_{ij}^v)^{\frac{\sigma-1}{\sigma}} + h_i (x_{ij}^h)^{\frac{\sigma-1}{\sigma}}.$$

Substituting (C.2) into Ω_j, we obtain

$$\Omega_j^\sigma = \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\Lambda_j,$$

where

$$\Lambda_j \equiv s_j + \omega s_i - (\omega-\phi)n_i - (1-\phi\omega)v_j \tag{C.3}$$

Furthermore, plugging (C.2) into the cost functions, we obtain:

$$C_j^n = f + \frac{s_j}{\Lambda_j} + \frac{s_i\phi}{\Lambda_i}, \tag{C.4}$$

$$C_j^v = f + \frac{s_j\phi\omega}{\Lambda_j} + \frac{s_i\omega}{\Lambda_i}, \tag{C.5}$$

$$C_j^h = 2f + \frac{s_j}{\Lambda_j} + \frac{s_i\omega}{\Lambda_i}. \tag{C.6}$$

Differentiating U_j and U_i with respect to n_j and v_j and plugging (C.2) in the resulting expressions, we obtain the following system of 4 equations:

$$\frac{\partial U_j}{\partial n_j} = \frac{\partial U_i}{\partial v_j} = 0, \tag{C.7}$$

$$\frac{\partial U_i}{\partial n_j} = \frac{\sigma}{\sigma-1}(\phi-\omega)\frac{1}{\Lambda_i} < 0, \tag{C.8}$$

$$\frac{\partial U_j}{\partial v_j} = \frac{\sigma}{\sigma-1}(\phi\omega-1)\frac{1}{\Lambda_j} < 0. \tag{C.9}$$

Substituting (C.4)–(C.6) and (C.7)–(C.9) into (C.1) and solving for Λ_i and Λ_j yields the following expressions:

$$\Lambda_j = \frac{s_j(1-\phi\omega)}{(\sigma-1)f} \qquad \Lambda_i = \frac{s_i(\omega-\phi)}{(\sigma-1)f}, \tag{C.10}$$

which must hold at any interior optimal solution.

Step 2. Differentiating W with respect to n_i, using (C.4), (C.6), (C.7) and (C.8) in terms of i instead of j, and plugging (C.10) in the resulting expression yields:

$$\frac{\partial W}{\partial n_i} = \frac{(1-\omega)(1+\phi)}{1-\phi\omega}f > 0. \tag{C.11}$$

Therefore, the optimal solution cannot be interior. Moreover, it follows from (C.11) that $n_i^o = s_i$, hence $v_i^o = h_i^o = 0$, always maximize W when country $j \neq i$ accommodates the three types of firms at the optimum.

Step 3. We now show when the first-order conditions for country j yield a strictly positive solution when $n_i^o = s_i$ and $v_i^o = h_i^o = 0$. Setting $n_i = s_i$ and $v_i = h_i = 0$ into Λ_i and Λ_j defined in (C.3) yields the following two expressions:

$$\Lambda_j = s_j + \phi s_i - (1-\phi\omega)v_j \qquad \Lambda_i = s_i + \omega s_j - (\omega-\phi)n_j. \tag{C.12}$$

Equalizing (C.10) and (C.12) leads to two equations in n_j and v_j, which have a unique solution given by (30) and (31). As for (32), it is given by $h_j^o = s_j - n_j^o - v_j^o$. These *three solutions* are positive if and only if the following conditions hold:

$$(\sigma-1)f > \frac{(\omega-\phi)s_i}{s_i+\omega s_j} \qquad (\sigma-1)f > \frac{s_j(1-\phi\omega)}{s_j+\phi s_i} \qquad (\sigma-1)f < \frac{(\omega-\phi)(1-\phi\omega)}{(1-\phi^2)(s_i+\omega s_j)},$$

which are equivalent to (29). Given $n_i^o = s_i$ and $v_i^o = h_i^o = 0$, (30)–(32) are, therefore, positive and the unique solution to the first-order conditions $\partial W/\partial n_j = \partial W/\partial v_j = \partial W/\partial h_j = 0$. If (29) holds, it must be $n_i^o = s_i$ and $v_i^o = h_i^o = 0$ because the solutions to the first-order conditions for country j are strictly positive.

Step 4. We now check that (30) and (31) maximize $W(n_j, v_j, s_j - n_j - v_j, n_i^o, v_i^o, h_i^o)$. Substituting the cost functions (C.4)–(C.6) and the first-order conditions (C.7)–(C.9) into (C.1), we obtain the following two expressions:

$$\frac{\partial W}{\partial n_j} = f - \frac{s_i(\omega - \phi)}{\Lambda_i}\frac{1}{\sigma - 1} \qquad \frac{\partial W}{\partial v_j} = f - \frac{s_j(1 - \phi\omega)}{\Lambda_j}\frac{1}{\sigma - 1}.$$

Differentiating (C.12) yields:

$$\frac{\partial \Lambda_i}{\partial n_j} = -(\omega - \phi) \qquad \frac{\partial \Lambda_j}{\partial v_j} = -(1 - \phi\omega) \qquad \frac{\partial \Lambda_i}{\partial v_j} = \frac{\partial \Lambda_j}{\partial n_j} = 0.$$

It is thus readily verified that the Hessian

$$\begin{pmatrix} \frac{\partial^2 W}{\partial n_j^2} & \frac{\partial^2 W}{\partial n_j \partial v_j} \\ \frac{\partial^2 W}{\partial v_j \partial n_j} & \frac{\partial^2 W}{\partial v_j^2} \end{pmatrix} = \begin{pmatrix} -\frac{s_i(\omega-\phi)^2}{\Lambda_i^2}\frac{1}{\sigma-1} & 0 \\ 0 & -\frac{s_j(1-\phi\omega)^2}{\Lambda_j^2}\frac{1}{\sigma-1} \end{pmatrix}$$

has the following characteristic equation:

$$\lambda^2 + \frac{1}{\sigma - 1}\left[\frac{s_i(\omega - \phi)^2}{\Lambda_i^2} + \frac{s_j(1 - \phi\omega)^2}{\Lambda_j^2}\right]\lambda + \left(\frac{1}{\sigma - 1}\right)^2 \frac{s_i(\omega - \phi)^2}{\Lambda_i^2}\frac{s_j(1 - \phi\omega)^2}{\Lambda_j^2} = 0,$$

which has two negative eigenvalues. Therefore, when (29) holds (30) and (31) maximize $W(n_j, v_j, h_j, n_i, v_i, h_i)$.

Step 5. Finally, for $B_L < B_R$, we know from Appendix 2 that S must be smaller than 1. This implies that $i = 1$ and $j = 2$. Q.E.D.

Appendix 4

We determine the conditions on S for $1 < \theta_2^v < \theta_2^h < \bar{\theta}$ to hold.

Step 1. $\theta_2^v < \theta_2^h$. This inequality holds if and only if the first term in the right-hand side of (48) is negative. Since this inequality must hold for any value of $\bar{\theta}$, it boils down to (B.1) when $\bar{\theta}$ becomes arbitrarily large. Therefore, we have $S < \bar{S}$.

Step 2. $\theta_2^v > 1$. Since $\theta_2^h > 1$ and the right-hand side of (48) decreases with θ_2^v, $\theta_2^v > 1$ holds if and only if the right-hand side of (48) is smaller than 1 at $\theta_2^v = 1$:

$$\frac{\phi K S^2 + (K - 1)S - \phi}{(\omega - \phi)KS} \cdot \left[1 - \left(\overline{\theta}\right)^{-(\kappa - \sigma + 1)}\right] + S < 0. \tag{D.1}$$

Since (D.1) must hold for any value of $\bar{\theta}$, it boils down to

$$G_2(S) \equiv \omega K S^2 + (K - 1)S - \phi < 0 \tag{D.2}$$

when $\bar{\theta}$ grows indefinitely. Denoting by $\underline{S}$ the positive root of $G_2(S) = 0$, (D.2) holds if and only if $S < \underline{S}$. It is readily verified that $\underline{S} < \bar{S}$. Thus, combining (44) and (D.2), we have $1 < \theta_2^v < \theta_2^h < \bar{\theta}$ if and only if $\phi/K < S < \underline{S}$. Note also that these inequalities imply $\theta_2^h > 1$. Q.E.D.

Appendix 5

We determine the conditions on S for $1 < \theta_2^h < \theta_2^v < \bar{\theta}$ to hold.

Step 1. $\theta_2^n < \theta_2^v$. This inequality holds if and only if the first term in the right-hand side (52) is negative:

$$\frac{\omega K S^2 + (K-1)S - \phi}{(1-\phi\omega)S}\left[1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}\right] + 1 > 0,$$

which reduces to

$$G_3(S) \equiv \omega K S^2 + (K - \phi\omega)S - \phi > 0 \tag{E.1}$$

when $\bar{\theta}$ becomes arbitrarily large. The positive root of $G_3(S) = 0$ being given by $S = \phi/K$, (E.1) holds if and only if $S > \phi/K$.

Step 2. $\theta_2^n > 1$. This holds if and only if the right-hand side of (52) is smaller than S at $\theta_2^n = 1$:

$$-\frac{\omega K S^2 + (K-1)S - \phi}{(1-\phi\omega)S}\left[1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}\right] < 0,$$

which is equivalent to

$$G_2(S) > 0. \tag{E.2}$$

when $\bar{\theta}$ becomes arbitrarily large.

Observe that (E.2) is the opposite of (D.2) and holds if and only if $S > \underline{S}$. Summing up, we have $1 < \theta_2^n < \theta_2^v < \bar{\theta}$ if and only if $\underline{S} < S < \overline{S}$. Q.E.D.

Appendix 6

First, we study the impact of trade and commuting costs on the mass of **H**-firms. The left-hand side of (49) is an increasing function of θ_2^h and does not depend on both ϕ and ω. The impact of changes in ϕ and ω on the right-hand side of (49) is captured by the first term, which can be rewritten as follows:

$$\frac{(1+\phi K)S + \phi + K}{(1-\phi\omega)(1+S)} \cdot \left[1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}\right] = \frac{1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}{1+S} \cdot \left[\left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi}\right)S + \frac{\phi}{1-\phi\omega} + \frac{1}{\omega-\phi}\right]$$

By differentiating this expression with respect to ϕ and ω, we obtain:

$$\left[\left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi}\right)S + \frac{\phi}{1-\phi\omega} + \frac{1}{\omega-\phi}\right]'_{\phi} = \left(\frac{\omega}{(1-\phi\omega)^2} + \frac{\omega}{(\omega-\phi)^2}\right)S + \frac{1}{(1-\phi\omega)^2} + \frac{1}{(\omega-\phi)^2} > 0,$$

$$\left[\left(\frac{1}{1-\phi\omega} + \frac{\phi}{\omega-\phi}\right)S + \frac{\phi}{1-\phi\omega} + \frac{1}{\omega-\phi}\right]'_{\omega} = \left(\frac{\phi}{(1-\phi\omega)^2} - \frac{\phi}{(\omega-\phi)^2}\right)S + \frac{\phi^2}{(1-\phi\omega)^2} - \frac{1}{(\omega-\phi)^2} < 0.$$

Therefore, θ_2^h increases with ϕ and decreases with ω, which implies that the mass of **H**-firms decreases (increases) when trade costs (communication costs) fall.

Second, the left-hand side of (48) increases with ω, while the first term of the right-hand side

$$\frac{\phi K S^2 + (K-1)S - \phi}{(\omega-\phi)KS} \cdot \left[1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}\right] = \frac{1-\left(\overline{\theta}\right)^{-(\kappa-\sigma+1)}}{S} \cdot \left[\frac{\phi}{\omega-\phi}S^2 + \left(\frac{1}{\omega-\phi} - \frac{1}{1-\phi\omega}\right)S - \frac{\phi}{1-\phi\omega}\right]$$

decreases with ω:

$$-\frac{\phi}{(\omega-\phi)^2}S^2 - \left(\frac{1}{(\omega-\phi)^2} + \frac{\phi}{(1-\phi\omega)^2}\right)S - \frac{\phi^2}{(1-\phi\omega)^2} < 0.$$

Therefore, θ_2^v decreases with ω, which leads to fewer **I**-firms.

Third, the left-hand side of (48) decreases with ϕ, while the behavior of first term in the right-hand side of (48) is a priori undetermined:

$$\left[\frac{\phi}{\omega-\phi}S^2+\left(\frac{1}{\omega-\phi}-\frac{1}{1-\phi\omega}\right)S-\frac{\phi}{1-\phi\omega}\right]'_\phi=\frac{\omega K^2S^2+(K^2-\omega)S-1}{(1-\phi\omega)^2}.$$

The right-hand side of this expression has a unique positive root smaller than 1. Since the range of countries' asymmetry we work with is $\phi/K<S<\overline{S}<1/K$, the derivative is positive at $S=1/K$:

$$\frac{\omega K^2S^2+(K^2-\omega)S-1}{(1-\phi\omega)^2}|_{S=\frac{1}{K}}=\frac{\omega K^2\frac{1}{K^2}+(K^2-\omega)\frac{1}{K}-1}{(1-\phi\omega)^2}=\frac{(K-1)(\frac{\omega}{K}+1)}{(1-\phi\omega)^2}>0.$$

When $S=\phi/K$, the derivative

$$\frac{\omega K^2S^2+(K^2-\omega)S-1}{(1-\phi\omega)^2}|_{S=\frac{\phi}{K}}=\frac{\omega K^2\frac{\phi^2}{K^2}+(K^2-\omega)\frac{\phi}{K}-1}{(1-\phi\omega)^2}=\frac{\left(\frac{\omega\phi}{K}+1\right)(\phi K-1)}{(1-\phi\omega)^2},$$

is also positive if $\phi K-1>0$, which is equivalent to

$$\omega<\overline{\omega}=\frac{2\phi}{1+\phi^2}.$$

In sum, θ_2^v decreases with ϕ for all admissible countries' degrees of asymmetry when communication costs are not too large, i.e., $\omega<\overline{\omega}$.

Last, the left-hand side of (52) decreases with ϕ. In the right-hand side, only the second term given by

$$-\frac{\omega KS^2+(K-1)S-\phi}{(1-\omega\phi)S}\left[1-(\overline{\theta})^{-(\kappa-\sigma+1)}\right]=-\frac{1-(\overline{\theta})^{-(\kappa-\sigma+1)}}{S}\left[\frac{\omega}{\omega-\phi}S^2+\left(\frac{1}{\omega-\phi}-\frac{1}{1-\phi\omega}\right)S-\frac{\phi}{1-\phi\omega}\right].$$

is affected by ϕ. By differentiating the above expression, we obtain:

$$-\left[\frac{\omega}{\omega-\phi}S^2+\left(\frac{1}{\omega-\phi}-\frac{1}{1-\phi\omega}\right)S-\frac{\phi}{1-\phi\omega}\right]'_\phi=-\frac{\omega K^2S^2+(K^2-\omega)S-1}{(1-\phi\omega)^2},$$

which is negative when $S=1/K$ and positive at $S=\phi/K$ if $\omega>\overline{\omega}$. Moreover, when $\omega=1$ the derivative is positive for $S=\underline{S}$. Therefore, $\omega^*>\overline{\omega}$ exists such that for $\omega>\omega^*$, there is a threshold value S^* such that the derivative is positive for $S\in(S^*,\,1/K)$. Hence, θ_2^n increases with ϕ.

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Supplemental Material

Proof of Proposition 5.

There are 49 possible configurations. Proposition 1 shows that (**I - IVH**) is an equilibrium. Besides, it also rules out the 6 configurations in which the smaller country hots **I**-firms, **V**-firms and **H**-firms and the larger country hosts **H**-firms only, **V**-firms only, two or three types of firms, as well as the 6 configurations in which the larger country hosts the three types of firms regardless of the configurations with one or two types of firms in the smaller country. So, we have to consider the remaining 36 configurations.

Step A. The following 21 configurations are never an equilibrium.

(A.1) (**V - V**) *is not an equilibrium.*

Assume that (**V - V**) is an equilibrium. In this case, $\pi_i^v > \pi_i^n$ and $\pi_j^v > \pi_j^n$. Using (14)-(16), we obtain:

$$\pi_i^v > \pi_i^n \Leftrightarrow \frac{\Delta_i^*}{\Delta_j^*} > \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}, \qquad \pi_j^v > \pi_j^n \Leftrightarrow \frac{\Delta_j^*}{\Delta_i^*} > \frac{1-\phi\omega}{\omega-\phi}\frac{s_j}{s_i}.$$

Combining these two inequalities yields $1-\phi\omega < \omega-\phi$, a contradiction since $\omega < 1$. Q.E.D.

(A.2) (**V - H**) *and* (**H - V**) *are not equilibria.*

If all i-firm choose to be vertical, then $\pi_i^v > \pi_i^h$. If all j-firms choose to be horizontal, then $\pi_j^h > \pi_j^n$. Using (14)-(16), we obtain:

$$\pi_i^v > \pi_i^h \Leftrightarrow \Delta_i^* > (1-\phi\omega)\frac{s_i}{\sigma f} \qquad \pi_j^h > \pi_j^n \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f} > \Delta_i^*.$$

Note that $(\omega-\phi)\frac{s_i}{\sigma f} > (1-\phi\omega)\frac{s_i}{\sigma f}$ is equivalent to $1-\phi\omega < \omega-\phi$, a contradiction. The mirror-image equations lead to a contradiction for **H - V**. Similar arguments are applicable to the subsequent cases. Q.E.D.

(A.3) (**IV - H**) *and* (**H - IV**) *are not equilibria.*

If no i-firms chooses to be horizontal, then $\pi_i^v > \pi_i^h$. If all j-firm choose to be horizontal, then $\pi_j^h > \pi_j^n$. Using (14)-(16), we obtain:

$$\pi_i^v > \pi_i^h \Leftrightarrow \Delta_i^* > (1-\phi\omega)\frac{s_i}{\sigma f}, \qquad \pi_j^h > \pi_j^n \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f} > \Delta_i^*.$$

As in (A.2), these inequalities lead to a contradiction. Q.E.D.

(A.4) (**H - VH**) *and* (**VH - H**) *are not equilibria.*

Using (15) and (16), it is readily verified that $\pi_j^v = \pi_j^h \Leftrightarrow \Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}$. If all i-firms choose to be horizontal, then $\pi_i^h > \pi_i^n \Leftrightarrow (\omega-\phi)\frac{s_j}{\sigma f} > \Delta_j^*$. Substituting $\Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}$ into $(\omega-\phi)\frac{s_j}{\sigma f} > \Delta_j^*$ yields $1-\phi\omega < \omega-\phi$, a contradiction. Q.E.D.

(A.5) (**V - IH**) *and* (**IH - V**) *are not equilibria.*

Using (14) and (16), we have: $\pi_j^n = \pi_j^h \Leftrightarrow \Delta_i^* = (\omega-\phi)\frac{s_i}{\sigma f}$. If all i-firms choose to be vertical, then $\pi_i^v > \pi_i^h \Leftrightarrow \Delta_i^* > (1-\phi\omega)\frac{s_i}{\sigma f}$. Substituting $\Delta_i^* = (\omega-\phi)\frac{s_i}{\sigma f}$ into $\Delta_i^* > (1-\phi\omega)\frac{s_i}{\sigma f} \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f}$ yields $1-\phi\omega < \omega-\phi$, a contradiction. Q.E.D.

(A.6) (**VH - V**) *and* (**V - VH**) *are not equilibria.*

If i-firms are horizontal or vertical, then $\pi_i^h > \pi_i^n$. If all j-firms choose to be vertical, then $\pi_j^v > \pi_j^h$. Using (14)-(16), we obtain:

$$\pi_i^h > \pi_i^n \Leftrightarrow (\omega-\phi)\frac{s_j}{\sigma f} > \Delta_j^*, \qquad \pi_j^v > \pi_j^h \Leftrightarrow \Delta_j^* > (1-\phi\omega)\frac{s_j}{\sigma f}.$$

Since $(\omega-\phi)\frac{s_j}{\sigma f} > (1-\phi\omega)\frac{s_j}{\sigma f}$ is equivalent to $1-\phi\omega < \omega-\phi$, we get a contradiction. Q.E.D.

(A.7) (**IV - V**) *and* (**V - IH**) *are not equilibria.*

If i-firms are integrated or vertical, then $\pi_i^n = \pi_i^v$. If all j-firm choose to be vertical, then $\pi_j^v > \pi_j^n$. Using (14)-(16) leads to

$$\pi_i^n = \pi_i^v \Leftrightarrow \frac{\Delta_i^*}{\Delta_j^*} = \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}, \qquad \pi_j^v > \pi_j^n \Leftrightarrow \frac{\Delta_j^*}{\Delta_i^*} > \frac{1-\phi\omega}{\omega-\phi}\frac{s_j}{s_i}.$$

Combining these inequalities yields $1-\phi\omega < \omega-\phi$, a contradiction. Q.E.D.

(A.8) **(VH - IH)** *and* **(IH - VH)** *are not equilibria.*

Using (14)-(16), we obtain:

$$\pi_j^n = \pi_j^h \Leftrightarrow \Delta_i^* = \frac{\omega-\phi}{\sigma f}s_i, \qquad \pi_i^v = \pi_i^h \Leftrightarrow \Delta_i^{**} = \frac{1-\phi\omega}{\sigma f}s_i.$$

If country i hosts **V**-firms and **H**-firms and country j hosts **I**-firms and **H**-firms, then $\Delta_i^* = \Delta_i^{**}$, i.e. a contradiction. A similar argument applies to **(IH - VH)**. Q.E.D.

(A.9) **(IV - IV)** *is not an equilibrium.*

Using (14)-(16), we obtain:

$$\pi_j^n = \pi_j^v \Leftrightarrow \Delta_j^*/\Delta_i^* = \frac{1-\phi\omega}{\omega-\phi}\frac{s_j}{s_i}, \qquad \pi_i^n = \pi_i^v \Leftrightarrow \Delta_i^{**}/\Delta_j^{**} = \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}.$$

If each country hosts **I**-firms and **V**-firms, the condition $\Delta_i^*/\Delta_j^* = \Delta_i^{**}/\Delta_j^{**}$ must hold. However, $s_i \neq s_j$ implies $\Delta_i^*/\Delta_j^* \neq \Delta_i^{**}/\Delta_j^{**}$, a contradiction. Q.E.D.

(A.10) (**IV - VH**) *and* (**VH - IV**) *are not equilibria.*

Using (14)-(16), we obtain:

$$\pi_j^v = \pi_j^h \Leftrightarrow \Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}, \qquad \pi_i^n = \pi_i^v \Leftrightarrow \Delta_i^*/\Delta_j^* = \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}.$$

Combining $\Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}$ and $\Delta_i^*/\Delta_j^* = \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}$ yields $\Delta_i^* = \frac{(1-\phi\omega)^2}{\omega-\phi}\frac{s_i}{\sigma f}$.

If j-firms are horizontal or vertical, then $\pi_j^h > \pi_j^n$. Using (14) and (16), we obtain $\pi_j^h > \pi_j^n \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f} > \Delta_i^*$. Substituting $\Delta_i^* = \frac{(1-\phi\omega)^2}{(\omega-\phi)}\frac{s_i}{\sigma f}$ into $(\omega-\phi)\frac{s_i}{\sigma f} > \Delta_i^*$ yields $1-\phi\omega < \omega-\phi$, a contradiction. Q.E.D.

(A.11) (**IV - IH**) *and* (**IH - IV**) *are not equilibria.*

Using (14)-(16), we find:

$$\pi_j^n = \pi_j^h \Leftrightarrow \Delta_i^* = (\omega-\phi)\frac{s_i}{\sigma f}, \qquad \pi_i^n = \pi_i^v \Leftrightarrow \Delta_i^*/\Delta_j^* = \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}.$$

Combining $\Delta_i^* = (\omega-\phi)\frac{s_i}{\sigma f}$ and $\Delta_i^*/\Delta_j^* = \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}$ yields $\Delta_j^* = \frac{(\omega-\phi)^2}{1-\phi\omega}\frac{s_j}{\sigma f}$. If i-firms are integrated or vertical, then $\pi_i^v > \pi_i^h \Leftrightarrow \Delta_i^* > (1-\phi\omega)\frac{s_i}{\sigma f}$. Substituting $\Delta_i^* = (\omega-\phi)\frac{s_i}{\sigma f}$ into $\Delta_i^* > (1-\phi\omega)\frac{s_i}{\sigma f}$ yields $1-\phi\omega < \omega-\phi$, a contradiction. Q.E.D.

(A.12) (**VH - VH**) *is not an equilibrium.*

Using (14)-(16), we get:

$$\pi_i^v = \pi_i^h \Leftrightarrow \Delta_i^* = (1-\phi\omega)\frac{s_i}{\sigma f}, \qquad \pi_j^v = \pi_j^h \Leftrightarrow \Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}.$$

If i-firms choose to be vertical or horizontal, then $\pi_i^h > \pi_i^n \Leftrightarrow (\omega-\phi)\frac{s_j}{\sigma f} > \Delta_j^*$. Substituting $\Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}$ into $(\omega-\phi)\frac{s_j}{\sigma f} > \Delta_j^*$ yields $1-\phi\omega < \omega-\phi$, a contradiction. Q.E.D.

Step B. Besides the mixed equilibrium, there exist 9 equilibria defined over specific domains of parameters, while the remaining 6 configurations are not equilibria.

(B.1) (**H - H**) *is an equilibrium.*

Setting $n_i^* = v_i^* = n_j^* = v_j^* = 0$ in (13) yields $\Delta_i^* = s_i + \omega s_j$ and $\Delta_j^* = s_j + \omega s_i$. If firms choose to be neither integrated nor vertical, it must be that $\pi_i^v < \pi_i^h$, $\pi_i^n < \pi_i^h$, $\pi_j^n < \pi_j^h$, and $\pi_j^v < \pi_j^h$. Using (14)-(16), we obtain:

$$\pi_i^v < \pi_i^h \Leftrightarrow \Delta_i^* < (1-\phi\omega)\frac{s_i}{\sigma f}, \qquad \pi_i^n < \pi_i^h \Leftrightarrow \Delta_j^* < (\omega - \phi)\frac{s_j}{\sigma f},$$

$$\pi_j^v < \pi_j^h \Leftrightarrow \Delta_j^* < (1-\phi\omega)\frac{s_j}{\sigma f}, \qquad \pi_j^n < \pi_j^h \Leftrightarrow \Delta_i^* < (\omega - \phi)\frac{s_i}{\sigma f},$$

which amounts to

$$s_i + \omega s_j < (1-\phi\omega)\frac{s_i}{\sigma f}.$$

Then, $\pi_i^v < \pi_i^h$, $\pi_i^n < \pi_i^h$, $\pi_j^n < \pi_j^h$, and $\pi_j^v < \pi_j^h$ if and only if

$$\sigma f < \min\left\{\frac{(1-\phi\omega)s_i}{s_i+\omega s_j}, \frac{(\omega-\phi)\,s_j}{s_j+\omega s_i}, \frac{(1-\phi\omega)s_j}{s_j+\omega s_i}, \frac{(\omega-\phi)\,s_i}{s_i+\omega s_j}\right\}.$$

Consequently, (**H - H**) is an equilibrium if and only if

$$\sigma f < \frac{(\omega-\phi)\,S}{S+\omega}.$$

Q.E.D.

(B.2) (**IH - H**) *is an equilibrium and* (**H - IH**) *is not an equilibrium.*

We determine n_j^* and h_j^* when $h_i^* = s_i$ and $v_j^* = n_i^* = v_i^* = 0$. Using (14) and (16), we obtain $\pi_j^n = \pi_j^h \Leftrightarrow \Delta_i^* = (\omega-\phi)\frac{s_i}{\sigma f}$. Substituting $v_i^* = 0$ and Δ_i^* into (13) yields

$$n_j^* = \frac{s_i + \omega s_j}{\omega - \phi} - \frac{s_i}{\sigma f}, \qquad h_j^* = -\frac{s_i + s_j\phi}{\omega-\phi} + \frac{s_i}{\sigma f}.$$

Substituting $n_i^* = v_j^* = 0$ into (13) yields $\Delta_j^* = s_j + \omega s_i$. Using (14)-(16) and substituting Δ_i^* and Δ_j^* leads to

$$\pi_j^n > \pi_j^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i} > \frac{\Delta_j^*}{\Delta_i^*}, \qquad \pi_j^h > \pi_j^v \Leftrightarrow \Delta_j^* < (1-\phi\omega)\frac{s_j}{\sigma f},$$

$$\pi_i^h > \pi_i^n \Leftrightarrow \Delta_j^* < (\omega-\phi)\frac{s_j}{\sigma f}, \qquad \pi_i^h > \pi_i^v \Leftrightarrow \Delta_i^* < (1-\phi\omega)\frac{s_j}{\sigma f}.$$

Therefore, we have $\pi_i^h > \pi_i^n$, $\pi_i^h > \pi_i^v$, $\pi_j^n > \pi_j^v$, and $\pi_j^h > \pi_j^v$ if and only if

$$\sigma f < \frac{s_j\,(\omega-\phi)}{s_j+\omega s_i} \tag{S.1}$$

holds.

The inequalities $n_j^* > 0$ and $h_j^* > 0$ are, respectively, equivalent to:

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \sigma f < \frac{(\omega-\phi)s_i}{s_j\phi + s_i}. \tag{S.2}$$

It follows from (S.1) and (S.2) that

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \sigma f < \min\left\{\frac{(\omega-\phi)s_i}{s_j\phi+s_i}, \frac{(\omega-\phi)\,s_j}{s_j+\omega s_i}\right\}.$$

Observe that

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \frac{(\omega-\phi)s_i}{\phi s_j+s_i}$$

is equivalent to $\phi < \omega$.

Finally,

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \frac{(\omega-\phi)\,s_j}{s_j+\omega s_i}$$

is equivalent to $s_i < s_j$. Therefore, j is the larger country, that is, $j = 1$. This implies that (**H - IH**) is not an equilibrium, for otherwise we would have $s_1 < s_2$, a contradiction.

Using $s_1 = 1/(1+S)$, the necessary and sufficient conditions for (**IH - H**) to be an equilibrium may be rewritten as follows:

$$\frac{(\omega-\phi)S}{\omega+S} < \sigma f < \min\left\{\frac{(\omega-\phi)S}{\phi+S}, \frac{\omega-\phi}{1+\omega S}\right\}.$$

Q.E.D.

(B.3) (**I - H**) *is an equilibrium and* (**H - I**) *is not an equilibrium.*

Setting $n_i^* = s_i$, $h_j^* = s_j$, and $v_i^* = n_j^* = v_j^* = 0$ into (13), we obtain $\Delta_i^* = s_i + \omega s_j$ and $\Delta_j^* = s_j + \phi s_i$.

If all i-firms choose to be integrated and all j-firms choose to be horizontal, we have $\pi_i^v < \pi_i^n$, $\pi_i^h < \pi_i^n$, $\pi_j^v < \pi_j^h$, and $\pi_j^n < \pi_j^h$. Using (14)-(16), we obtain:

$$\pi_i^n > \pi_i^v \Leftrightarrow \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j} > \frac{\Delta_i^*}{\Delta_j^*}, \qquad \pi_i^h < \pi_i^n \Leftrightarrow (\omega-\phi)\,\frac{s_j}{\sigma f} < \Delta_j^*,$$

$$\pi_j^v < \pi_j^h \Leftrightarrow \Delta_j^* < (1-\phi\omega)\frac{s_j}{\sigma f}, \qquad \pi_j^h > \pi_j^n \Leftrightarrow (\omega-\phi)\,\frac{s_i}{\sigma f} > \Delta_i^*.$$

Thus, $\pi_i^v < \pi_i^n$, $\pi_i^h < \pi_i^n$, $\pi_j^v < \pi_j^h$, and $\pi_j^n < \pi_j^h$ hold if and only if

$$\frac{(\omega-\phi)\,s_j}{s_j+\phi s_i} < \sigma f < \min\left\{\frac{(1-\phi\omega)s_j}{s_j+\phi s_i}, \frac{(\omega-\phi)\,s_i}{s_i+\omega s_j}\right\}$$

and

$$\frac{s_i+\omega s_j}{s_j+\phi s_i} < \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}.$$

Since $\omega < 1$, it is readily verified that

$$\frac{(\omega-\phi)\,s_j}{s_j+\phi s_i} < \frac{(1-\phi\omega)s_j}{s_j+\phi s_i}$$

holds. Observe that

$$\frac{(\omega-\phi)\,s_j}{s_j+\phi s_i} < \frac{(\omega-\phi)\,s_i}{s_i+\omega s_j} \tag{S.3}$$

is equivalent to $f_{B3}(s_i) > 0$ where

$$f_{B3}(s_i) \equiv -(\omega-\phi)s_i^2 + 2\omega s_i - \omega.$$

Since $f_{B3}(1/2) < 0$ and $f_{B3}(1) > 0$, it must be that $s_i > 1/2$, that is, $i = 1$.

As $s_1 = 1/(1+S)$, we have $f_{B3}(s_1)/s_1^2 = \phi - \omega S^2 \equiv H(S)$. Since $H(0) > 0$ and $H(1) < 0$, (S.3) holds if and only if

$$0 < S < \widetilde{S} < 1,$$

where $\widetilde{S} \equiv \sqrt{\phi/\omega}$ is the positive root of $H(S) = 0$.

Finally, observe that $\frac{s_i+\omega s_j}{s_j+\phi s_i} < \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}$ is equivalent to $\frac{(\omega-\phi)s_j}{s_j+\phi s_i} < \frac{(1-\phi\omega)s_i}{s_i+\omega s_j}$, which follows from (S.3).

Summing up, (**I - H**) is an equilibrium if and only if

$$\frac{(\omega-\phi)\,S}{\phi+S}<\sigma f<\min\left\{\frac{(1-\phi\omega)S}{\phi+S},\frac{\omega-\phi}{1+\omega S}\right\}$$

and

$$0<S<\widetilde{S}<1$$

hold. Q.E.D.

(B.4) (**IH - IH**) *is an equilibrium.*

We determine n_i^*, h_i^*, n_j^* when h_j^* and $v_i^*=v_j^*=0$. Using (14) and (16), we obtain:

$$\pi_j^h=\pi_j^n \Leftrightarrow \Delta_i^*=(\omega-\phi)\,\frac{s_i}{\sigma f} \qquad \pi_i^h=\pi_i^n \Leftrightarrow \Delta_j^*=(\omega-\phi)\,\frac{s_j}{\sigma f}.$$

Substituting $v_i^*=0$ and Δ_i^* into (13) yields

$$n_j^*=\frac{s_i+\omega s_j}{\omega-\phi}-\frac{s_i}{\sigma f}.$$

Likewise, substituting $v_j^*=0$ and Δ_j^* into (13) yields

$$n_i^*=\frac{s_j+\omega s_i}{\omega-\phi}-\frac{s_j}{\sigma f}.$$

Therefore,

$$h_i^*=-\frac{\phi s_i+s_j}{\omega-\phi}+\frac{s_j}{\sigma f} \qquad h_j^*=-\frac{\phi s_j+s_i}{\omega-\phi}+\frac{s_i}{\sigma f}.$$

We show that no firm chooses to be vertical. Using (14)-(16) and substituting $\Delta_i^*=(\omega-\phi)\,\frac{s_i}{\sigma f}$ and $\Delta_j^*=(\omega-\phi)\,\frac{s_j}{\sigma f}$ into $\pi_i^n>\pi_i^v$, $\pi_i^h>\pi_i^v$, $\pi_j^n>\pi_j^v$, and $\pi_j^h>\pi_j^v$, we obtain:

$$\begin{aligned}\pi_i^n &> \pi_i^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_i}{s_j}>\frac{\Delta_i^*}{\Delta_j^*}, \qquad \pi_i^h>\pi_i^v \Leftrightarrow \Delta_i^*<(1-\phi\omega)\frac{s_i}{\sigma f},\\ \pi_j^n &> \pi_j^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i}>\frac{\Delta_j^*}{\Delta_i^*}, \qquad \pi_j^h>\pi_j^v \Leftrightarrow \Delta_j^*<(1-\phi\omega)\frac{s_j}{\sigma f},\end{aligned}$$

which hold since $\phi<\omega<1$.

The inequalities $n_i^*>0$, $h_i^*>0$, $n_j^*>0$, and $h_j^*>0$ are, respectively, equivalent to the following conditions:

$$\sigma f>\frac{(\omega-\phi)s_j}{s_j+\omega s_i}, \qquad \sigma f<\frac{(\omega-\phi)s_j}{\phi s_i+s_j}, \qquad \sigma f>\frac{(\omega-\phi)s_i}{s_i+\omega s_j}, \qquad \sigma f<\frac{(\omega-\phi)s_i}{\phi s_j+s_i},$$

which amount to

$$\max\left\{\frac{(\omega-\phi)s_i}{s_i+\omega s_j},\frac{(\omega-\phi)s_j}{s_j+\omega s_i}\right\}<\sigma f<\min\left\{\frac{(\omega-\phi)s_i}{\phi s_j+s_i},\frac{(\omega-\phi)s_j}{\phi s_i+s_j}\right\}. \tag{S.4}$$

Observe that

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j}<\frac{(\omega-\phi)s_i}{\phi s_j+s_i}$$

and

$$\frac{(\omega-\phi)s_j}{s_j+\omega s_i}<\frac{(\omega-\phi)s_j}{\phi s_i+s_j}$$

hold since $\phi<\omega$.

Furthermore,

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j}<\frac{(\omega-\phi)s_j}{\phi s_i+s_j} \tag{S.5}$$

is equivalent to $f_{B4}(s_i) > 0$ where

$$f_{B4}(s_i) \equiv (\omega - \phi)s_i^2 - 2\omega s_i + \omega = -f_{B3}(s_i).$$

Likewise,

$$\frac{(\omega-\phi)s_j}{s_j+\omega s_i} < \frac{(\omega-\phi)s_i}{\phi s_j + s_i} \tag{S.6}$$

is equivalent to $g_{B4}(s_i) > 0$ where

$$g_{B4}(s_i) \equiv (\omega - \phi)s_i^2 + 2\phi s_i - \phi.$$

Since $f_{B4}(s_i) = -f_{B3}(s_i)$, $f_{B4}(0) < 0$, $f_{B4}(1/2) < 0$, and $f_{B4}(1) > 0$, while $g_{B4}(0) < 0$, $g_{B4}(1/2) > 0$, and $g_{B4}(1) > 0$, (S.5) and (S.6) hold if and only if

$$0 < \widetilde{S} < S < 1$$

$$\frac{\omega-\phi}{1+\omega S} < \sigma f < \frac{(\omega-\phi)S}{\phi+S}, \tag{S.7}$$

where (S.7) comes from (S.4). Since $f_{B4}(s_i) = g_{B4}(1-s_i)$, the above conditions holds for both $i = 1$ and $i = 2$. Q.E.D.

(B.5**)** (**I - IV**) *is an equilibrium and* (**IV - I**) *is not an equilibrium.*

We determine n_j^* and v_j^* when $n_i^* = s_i$ and $h_j^* = v_i^* = h_i^* = 0$. Using (14) and (15), we obtain $\pi_j^n = \pi_j^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i} = \frac{\Delta_j^*}{\Delta_i^*}$.

Substituting $n_i^* = s_i$, $v_i^* = 0$ into (13) yields

$$\Delta_i^* = s_i + \omega s_j - (\omega-\phi)n_j \qquad \Delta_j^* = s_j + \phi s_i - (1-\phi\omega)v_j.$$

Combining $\frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i} = \frac{\Delta_j^*}{\Delta_i^*}$, $v_j = s_j - n_j$, Δ_i^*, and Δ_i^* yields

$$n_j^* = \frac{-(1-\phi^2)(1-\omega)s_i^2 + [(1-\phi\omega)(1-\phi-\omega) - (\omega-\phi)]s_i + (1-\phi\omega)\,\omega}{(1-\phi\omega)(\omega-\phi)}$$

and

$$v_j^* = \frac{(1-\phi^2)(1-\omega)s_i^2 - (1-\phi)(1-\omega-2\omega\phi)s_i - (1-\phi\omega)\,\phi}{(1-\phi\omega)(\omega-\phi)}.$$

The inequalities $n_j^* > 0$ and $v_j^* > 0$ are, respectively, equivalent to the conditions:

$$f_{B5}(s_i) > 0 \qquad g_{B5}(s_i) > 0$$

where

$$f_{B5}(s_i) \equiv -(1-\phi^2)(1-\omega)s_i^2 + [(1-\phi\omega)(1-\phi-\omega) - (\omega-\phi)]s_i + (1-\phi\omega)\,\omega$$

and

$$g_{B5}(s_i) \equiv (1-\phi^2)(1-\omega)s_i^2 - (1-\phi)(1-\omega-2\omega\phi)s_i - (1-\phi\omega)\,\phi.$$

Since $f_{B5}(1/2) > 0$, $f_{B5}\left(\frac{1-\phi\omega}{1-\phi^2}\right) = 0$, and $f_{B5}(1) < 0$, while $g_{B5}(1/2) < 0$, $g_{B5}(1/2) < 0$, $g_{B5}\left(\frac{1-\phi\omega}{1-\phi^2}\right) > 0$, and $g_{B5}(1) > 0$, $n_j^* > 0$ and $h_j^* > 0$ if and only if

$$0 < \overline{s}_i < s_i < \frac{1-\phi\omega}{1-\phi^2},$$

where $\overline{s}_i$ is the positive root of $g_{B5}(s_i) = 0$. Since $\overline{s}_i > 1/2$, i is the larger country, that is, $i = 1$.

We now show that all i-firms choose to be integrated and no j-firm chooses to be horizontal. Substituting n_j^* into Δ_i^* leads to $\Delta_i^* = (1-\phi^2)s_i \frac{\omega s_j + s_i}{1-\phi\omega}$. Since $\pi_j^n = \pi_j^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i} = \frac{\Delta_j^*}{\Delta_i^*}$, we obtain $\Delta_j^* = (1-\phi^2)s_j \frac{\omega s_j + s_i}{\omega - \phi}$. Using (14)-(16) and substituting Δ_i^* and Δ_j^* yields:

$$\pi_i^v < \pi_i^n \Leftrightarrow \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j} > \frac{\Delta_i^*}{\Delta_j^*}, \qquad \pi_i^h < \pi_i^n \Leftrightarrow (\omega-\phi)\frac{s_j}{\sigma f} < \Delta_j^*,$$
$$\pi_j^h < \pi_j^v \Leftrightarrow \Delta_j^* > (1-\phi\omega)\frac{s_j}{\sigma f}, \qquad \pi_j^h < \pi_j^n \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f} < \Delta_i^*.$$

Hence, $\pi_i^n > \pi_i^v$, $\pi_i^n > \pi_i^h$, $\pi_j^n > \pi_j^h$ and $\pi_j^v > \pi_j^h$ hold if and only if

$$\frac{(\omega-\phi)(1-\phi\omega)}{(1-\phi^2)(\omega s_j + s_i)} < \sigma f.$$

Using (B.1) in Appendix 2 shows that $F(S) = -g_{B5}(s_1)/[s_1^2(\omega-\phi)]$. Therefore, (**I - IV**) is an equilibrium if and only if

$$\frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)} < \sigma f$$

and

$$\frac{\phi}{K} < S < \overline{S},$$

where $\overline{S}$ is the positive root of $F(S) = 0$. Q.E.D.

(B.6) (**I - VH**) *is an equilibrium and* (**VH - I**) *is not an equilibrium.*

We determine v_j^* and h_j^* when $n_i^* = s_i$ and $n_j^* = v_i^* = h_i^* = 0$. Using (15) and (16) leads to $\pi_j^v = \pi_j^h \Leftrightarrow \Delta_j^* = (1-\phi\omega)\frac{s_j}{\sigma f}$. Substituting $n_j^* = 0$ and Δ_j^* into (13) yields

$$v_j^* = \frac{s_j + \phi s_i}{1-\phi\omega} - \frac{s_j}{\sigma f}.$$

Then, we obtain

$$h_j^* = \frac{s_j}{\sigma f} - \frac{\phi s_i + \phi\omega s_j}{1-\phi\omega}.$$

We now show that all i-firms choose to be integrated and no j-firms chooses to be integrated. Substituting $v_i^* = n_j^* = 0$ into (13) leads to $\Delta_i^* = s_i + \omega s_j$. Using (14)-(16) and substituting Δ_i^* and Δ_j^* leads to

$$\pi_i^v < \pi_i^n \Leftrightarrow \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j} > \frac{\Delta_i^*}{\Delta_j^*}, \qquad \pi_i^h < \pi_i^n \Leftrightarrow (\omega-\phi)\frac{s_j}{\sigma f} < \Delta_j^*,$$

$$\pi_j^n < \pi_j^h \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f} > \Delta_i^*, \qquad \pi_j^n < \pi_j^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i} < \frac{\Delta_j^*}{\Delta_i^*}.$$

Hence, $\pi_i^n > \pi_i^v$, $\pi_i^n > \pi_i^h$, $\pi_j^h > \pi_j^n$, and $\pi_j^v > \pi_j^n$ hold if and only if

$$\sigma f < \frac{(\omega-\phi)\, s_i}{s_i + \omega s_j}. \tag{S.8}$$

The inequalities $v_j^* > 0$ and $h_j^* > 0$ are, respectively, equivalent to the following conditions:

$$\frac{(1-\phi\omega)s_j}{s_j + \phi s_i} < \sigma f < \frac{(1-\phi\omega)s_j}{\phi\omega s_j + \phi s_i}. \tag{S.9}$$

Combining (S.8) and (S.9) leads to

$$\frac{(1-\phi\omega)s_j}{s_j+\phi s_i} < \sigma f < \min\left\{\frac{(\omega-\phi)\,s_i}{s_i+\omega s_j}, \frac{(1-\phi\omega)s_j}{\phi\omega s_j+\phi s_i}\right\}.$$

Observe that the expression

$$\frac{(1-\phi\omega)s_j}{s_j+\phi s_i} < \frac{(\omega-\phi)\,s_i}{s_i+\omega s_j}$$

is equivalent to $f_{B6}(s_i) > 0$ where

$$f_{B6}(s_i) \equiv [(1-\phi\omega)(1-\omega)-(\omega-\phi)\,(1-\phi)]s_i^2 + [(\omega-\phi)-(1-\phi\omega)(1-2\omega)]s_i - \omega(1-\phi\omega).$$

Since $f_{B6}(1/2) < 0$ and $f_{B6}(1) > 0$, we have

$$1/2 < \widehat{s}_i < s_i < 1,$$

where $\widehat{s}_i$ is the positive root of $f_{B6}(s_i) = 0$. Therefore, i must be the larger country, that is, $i = 1$.

Since $\phi\omega < 1$, we have:

$$\frac{(1-\phi\omega)s_j}{s_j+\phi s_i} < \frac{(1-\phi\omega)s_j}{\phi\omega s_j+\phi s_i}.$$

Since

$$-f_{B6}(s_1)/[s_1^2\,(\omega-\phi)] = \omega K S^2 + (K-1)S - \phi \equiv J(S),$$

the necessary and sufficient conditions (**I - VH**) to be an equilibrium are:

$$\frac{(1-\phi\omega)S}{\phi+S} < \sigma f < \min\left\{\frac{\omega-\phi}{1+\omega S}, \frac{(1-\phi\omega)S}{\phi+\phi\omega S}\right\}$$

and

$$0 < S < \widehat{S} < 1,$$

where $\widehat{S}$ is the positive root of $J(S) = 0$. Q.E.D.

(B.7) (**I - IH**) *is an equilibrium and* (**IH - I**) *is not an equilibrium.*

We determine n_j^* and h_j^* when $n_i^* = s_i$ and $v_j^* = h_j^* = h_i^* = 0$. Using (14) and (15), we obtain $\pi_j^h = \pi_j^n \Leftrightarrow \Delta_i^* = (\omega-\phi)\,\frac{s_i}{\sigma f}$. Substituting $v_i^* = 0$ and Δ_i^* into (13) yields

$$n_j^* = \frac{s_i+\omega s_j}{\omega-\phi} - \frac{s_i}{\sigma f} \qquad h_j^* = -\frac{\phi s_j+s_i}{\omega-\phi} + \frac{s_i}{\sigma f}.$$

The inequalities $n_j^* > 0$ and $h_j^* > 0$ are, respectively, equivalent to the following conditions:

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \sigma f < \frac{(\omega-\phi)s_i}{s_j\phi+s_i}.$$

Since $\phi < \omega$, it is readily verified that

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \frac{(\omega-\phi)s_i}{s_j\phi+s_i}.$$

We now show that all i-firms choose to be integrated and no j-firm chooses to be vertical. Substituting $n_i^* = s_i$ and $v_j^* = 0$ into (13) leads to $\Delta_j^* = s_j + \phi s_i$. Using (14)-(16) and substituting Δ_i^* and Δ_j^*,

$$\pi_j^n > \pi_j^v \Leftrightarrow \frac{(1-\phi\omega)}{(\omega-\phi)}\frac{s_j}{s_i} > \frac{\Delta_j^*}{\Delta_i^*}, \qquad \pi_j^v < \pi_j^h \Leftrightarrow \Delta_j^* < (1-\phi\omega)\frac{s_j}{\sigma f},$$

$$\pi_i^n < \pi_i^h \Leftrightarrow (\omega-\phi)\,\frac{s_j}{\sigma f} > \Delta_j^*, \qquad \pi_i^v < \pi_i^h \Leftrightarrow \Delta_i^* < (1-\phi\omega)\frac{s_i}{\sigma f},$$

we find that $\pi_i^h > \pi_i^n$, $\pi_i^h > \pi_i^v$, $\pi_j^n > \pi_j^v$, and $\pi_j^h > \pi_j^v$ hold if and only if

$$\frac{(\omega-\phi)\,s_j}{s_j+\phi s_i} < \sigma f < \frac{(1-\phi\omega)s_j}{s_j+\phi s_i}.$$

Note that $\frac{(\omega-\phi)s_j}{s_j+\phi s_i} < \frac{(1-\phi\omega)s_j}{s_j+\phi s_i}$ holds since $\omega < 1$.

Observe that the expression

$$\frac{(\omega-\phi)\,s_j}{s_j+\phi s_i} < \frac{(\omega-\phi)s_i}{s_j\phi+s_i}$$

is equivalent to $s_j < s_i$. Thus, i must be the larger country, that is, $i = 1$.

Furthermore,

$$\frac{(\omega-\phi)s_i}{s_i+\omega s_j} < \frac{(1-\phi\omega)s_j}{s_j+\phi s_i} \Leftrightarrow \frac{\omega-\phi}{1+\omega S} < \frac{(1-\phi\omega)S}{\phi+S},$$

which is equivalent to $f_{B7}(s_i) > 0$ where

$$f_{B7}(s_i) \equiv -[(1-\phi\omega)(1-\omega)-(\omega-\phi)\,(1-\phi)]s_i^2 - [(\omega-\phi)-(1-\phi\omega)(1-2\omega)]s_i + \omega(1-\phi\omega) = -f_{B6}(s_i).$$

Since $f_{B7}(0) > 0$, $f_{B7}(1/2) > 0$, and $f_{B7}(1) < 0$, we have:

$$1/2 < s_1 < \widehat{s}_1 < 1$$

where $\widehat{s}_1$ is the positive root of $f_{B8}(s_1) = 0$.

In short, the conditions for (**I - IH**) to be an equilibrium are as follows:

$$\max\left\{\frac{(\omega-\phi)\,S}{\phi+S}, \frac{\omega-\phi}{1+\omega S}\right\} < \sigma f < \min\left\{\frac{(1-\phi\omega)S}{\phi+S}, \frac{\omega-\phi}{1+\phi S}\right\}$$

and

$$0 < \widehat{S} < S < 1,$$

where $\widehat{S}$ is the solution to $J(S) = 0$. Q.E.D.

(B.8) (**I - V**) *is an equilibrium and* (**V - I**) *is not an equilibrium.*

Setting $n_i^* = s_i$, $v_j^* = s_j$, and $v_i^* = n_j^* = 0$ in (13), we obtain $\Delta_i^* = s_i + \omega s_j$ and $\Delta_j^* = \phi s_i + \phi\omega s_j$.

If all i-firms choose to be integrated and all j-firms choose to be vertical, it must be that $\pi_i^v < \pi_i^n$, $\pi_i^h < \pi_i^n$, $\pi_j^n < \pi_j^v$, and $\pi_j^h < \pi_j^v$. Using (14)-(16), we get:

$$\pi_i^n > \pi_i^v \Leftrightarrow \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j} > \frac{\Delta_i^*}{\Delta_j^*}, \qquad \pi_i^h < \pi_i^n \Leftrightarrow \Delta_j^* > (\omega-\phi)\,\frac{s_j}{\sigma f},$$

$$\pi_j^n < \pi_j^v \Leftrightarrow \frac{1-\phi\omega}{\omega-\phi}\frac{s_j}{s_i} < \frac{\Delta_j^*}{\Delta_i^*}, \qquad \pi_j^v > \pi_j^h \Leftrightarrow \Delta_j^* > (1-\phi\omega)\frac{s_j}{\sigma f}.$$

Hence, $\pi_i^v < \pi_i^n$, $\pi_i^h < \pi_i^n$, $\pi_j^n < \pi_j^v$, and $\pi_j^h < \pi_j^v$ if and only if

$$\frac{1}{\phi} < \frac{\omega-\phi}{1-\phi\omega}\frac{s_i}{s_j} \qquad \text{(S.10)}$$

and

$$\frac{(1-\phi\omega)s_j}{\phi s_i + \phi\omega s_j} < \sigma f.$$

Rewriting (S.10) yields

$$1 < \frac{1}{\phi}\frac{1-\phi\omega}{\omega-\phi} < \frac{s_i}{s_j},$$

so that i must be the larger country, that is, $i = 1$.

To sum up, the conditions for the larger country to host only **I**-firms and the smaller country to host only **V**-firms are:

$$0 < S < \frac{\phi}{K}$$

and

$$\frac{(1-\phi\omega)S}{\phi + \phi\omega S} < \sigma f.$$

Q.E.D.

(B.9) (**I - I**) *is an equilibrium.*

Setting $n_i^* = s_i$, $n_j^* = s_j$ and $v_i^* = v_j^* = 0$ in (13), we obtain $\Delta_i^* = s_i + \phi s_j$ and $\Delta_j^* = s_j + \phi s_i$.

When no firm chooses to be vertical or horizontal, we have $\pi_i^v < \pi_i^n$, $\pi_i^h < \pi_i^n$, $\pi_j^v < \pi_j^n$, and $\pi_j^h < \pi_j^n$. Using (14)-(16), we obtain:

$$\pi_i^v < \pi_i^n \Leftrightarrow \frac{\Delta_i^*}{\Delta_j^*} < \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}, \qquad \pi_i^h < \pi_i^n \Leftrightarrow (\omega-\phi)\frac{s_j}{\sigma f} < \Delta_j^*,$$

$$\pi_j^v < \pi_j^n \Leftrightarrow \frac{\Delta_j^*}{\Delta_i^*} < \frac{1-\phi\omega}{\omega-\phi}\frac{s_j}{s_i}, \qquad \pi_j^h < \pi_j^n \Leftrightarrow (\omega-\phi)\frac{s_i}{\sigma f} < \Delta_i^*.$$

Hence, $\pi_i^v < \pi_i^n$, $\pi_i^h < \pi_i^n$, $\pi_j^v < \pi_j^n$, and $\pi_j^h < \pi_j^n$ if and only if

$$\frac{\omega-\phi}{1-\phi\omega}\frac{s_i}{s_j} < \frac{s_i+\phi s_j}{s_j+\phi s_i} < \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j}$$

and

$$\max\left\{\frac{(\omega-\phi)\,s_i}{s_i+\phi s_j}, \frac{(\omega-\phi)\,s_j}{s_j+\phi s_i}\right\} < \sigma f. \tag{S.11}$$

Note that

$$\frac{\omega-\phi}{1-\phi\omega}\frac{s_i}{s_j} < \frac{s_i+\phi s_j}{s_j+\phi s_i} \tag{S.12}$$

is equivalent to $f_{B9}(s_i) > 0$ where

$$f_{B9}(s_i) \equiv -(1-\omega)(1-\phi^2)s_i^2 + (1-\phi)(1-\omega-2\omega\phi)s_i + \phi(1-\phi\omega) = -g_{B5}(s_i).$$

Since $f_{B9}(1/2) > 0$ and $f_{B9}(1) < 0$, (S.12) holds if and only if

$$0 < s_i < \overline{s} < 1 \tag{S.13}$$

where $\overline{s}$ is the positive root of $f_{B9}(s_i) = 0$.

Likewise, the expression

$$\frac{s_i+\phi s_j}{s_j+\phi s_i} < \frac{1-\phi\omega}{\omega-\phi}\frac{s_i}{s_j} \tag{S.14}$$

is equivalent to

$$g_{B9}(s_i) \equiv -(1-\omega)(1-\phi^2)s_i^2 + [(1-\phi\omega) - (\omega-\phi)(1-2\phi)]s_i - (\omega-\phi)\phi = f_{B9}(s_j) > 0.$$

Since $g_{B9}(1/2) > 0$ and $g_{B9}(1) > 0$, (S.14) holds if and only if

$$0 < \underline{s} < s_i < 1, \tag{S.15}$$

where $\underline{s}$ is the positive root of $g_{B9}(s_i) = 0$. As $f_{B9}(1/2) > 0$ and $g_{B9}(1/2) > 0$, i can be either the larger country or the smaller country.

Since $g_{B9}(1-s_1) = f_{B9}(s_1)$ and $g_{B9}(1-s_2) = f_{B9}(s_2)$, it follows from (S.11), (S.13), and (S.15) that each country to host only **I**-firms if and only i

$$0 < \overline{S} < S < 1 \tag{S.16}$$

and

$$\frac{\omega - \phi}{1 + \phi S} < \sigma f, \tag{S.17}$$

hold. Q.E.D.

Step C. We study the ordering of the thresholds $\widehat{S}$, $\widetilde{S}$, and $\overline{S}$. First, since $J(S)$ is increasing on $[0, 1]$ and $J(\phi/K) < 0$, $J(\widehat{S}) = 0$, and $J(\widetilde{S}) > 0$, it must be that $\phi/K < \widehat{S} < \widetilde{S}$. Second, we have:

$$F(\widetilde{S}) \gtreqless 0 \Leftrightarrow \varrho(\omega, \phi) \equiv \omega\sqrt{\omega} + \sqrt{\phi}\omega + \left(\phi^2 - \phi - 1\right)\sqrt{\omega} - \phi\sqrt{\phi} \gtreqless 0 \Leftrightarrow \widetilde{S} \gtreqless \overline{S}.$$

Since $F(0) = J(0)$ and $J(S) > F(S)$ for any $S > 0$, it must be that $\widehat{S} < \overline{S}$. Therefore, (i) $\phi/K < \widehat{S} < \widetilde{S} < \overline{S}$ if and only if $\varrho(\omega, \phi) > 0$ and (ii) $\phi/K < \widehat{S} < \overline{S} < \widetilde{S}$ if and only if $\varrho(\omega, \phi) < 0$. Q.E.D.

Step D. It remains to show that, up to a zero-measure set, the above parameter domains form a partition of $X = \{(S, \sigma f) \mid 0 < S < 1, 0 < \sigma f\}$. Hence, there exists a unique organizational equilibrium almost everywhere in X.

(D.1) The equilibrium conditions for the 10 equilibrium configurations obtained in Appendix 2 and in Step B can be rewritten as follows.

(1) (**H - H**) is an equilibrium in the set

$$A_1 \equiv \left\{(S, \sigma f) \mid 0 < \sigma f < \frac{(\omega - \phi)\, S}{\omega + S}, 0 < S < 1\right\}.$$

(2) (**IH - H**) is an equilibrium in $A_2 \cup A_3$ where

$$\begin{aligned} A_2 &\equiv \left\{(S, \sigma f) \mid \frac{(\omega - \phi) S}{\omega + S} < \sigma f < \frac{(\omega - \phi) S}{\phi + S}, 0 < S < \widetilde{S}\right\}, \\ A_3 &\equiv \left\{(S, \sigma f) \mid \frac{(\omega - \phi) S}{\omega + S} < \sigma f < \frac{\omega - \phi}{1 + \omega S}, \widetilde{S} < S < 1\right\}. \end{aligned}$$

(3) (**I - H**) is an equilibrium in $A_4 \cup A_5$ where

$$\begin{aligned} A_4 &\equiv \left\{(S, \sigma f) \mid \frac{(\omega - \phi)\, S}{\phi + S} < \sigma f < \frac{(1 - \phi\omega) S}{\phi + S}, 0 < S < \widehat{S}\right\}, \\ A_5 &\equiv \left\{(S, \sigma f) \mid \frac{(\omega - \phi)\, S}{\phi + S} < \sigma f < \frac{\omega - \phi}{1 + \omega S}, \widehat{S} < S < \widetilde{S}\right\}. \end{aligned}$$

(4) (**IH - IH**) is an equilibrium in A_6 where

$$A_6 \equiv \left\{(S, \sigma f) \mid \frac{\omega - \phi}{1 + \omega S} < \sigma f < \frac{(\omega - \phi) S}{\phi + S}, \widetilde{S} < S < 1\right\}.$$

(5) (**I - IV**) is an equilibrium in A_7 where

$$A_7 \equiv \left\{(S, \sigma f) \mid \frac{(\omega - \phi)(1 - \phi\omega)(1 + S)}{(1 - \phi^2)(1 + \omega S)} < \sigma f, \frac{\phi}{K} < S < \overline{S}\right\}.$$

(6) (**I - VH**) is an equilibrium in $A_8 \cup A_9$ where

$$\begin{aligned} A_8 &\equiv \left\{(S,\sigma f) \mid \frac{(1-\phi\omega)S}{\phi+S} < \sigma f < \frac{(1-\phi\omega)S}{\phi+\phi\omega S}, 0 < S < \frac{\phi}{K}\right\}, \\ A_9 &\equiv \left\{(S,\sigma f) \mid \frac{(1-\phi\omega)S}{\phi+S} < \sigma f < \frac{\omega-\phi}{1+\omega S}, \frac{\phi}{K} < S < \widehat{S}\right\}. \end{aligned}$$

(7) (**I - IVH**) is an equilibrium in $A_{10} \cup A_{11}$ where

$$\begin{aligned} A_{10} &\equiv \left\{(S,\sigma f) \mid \frac{\omega-\phi}{1+\omega S} < \sigma f < \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}, \frac{\phi}{K} < S < \widehat{S}\right\}, \\ A_{11} &\equiv \left\{(S,\sigma f) \mid \frac{(1-\phi\omega)S}{\phi+S} < \sigma f < \frac{(\omega-\phi)(1-\phi\omega)(1+S)}{(1-\phi^2)(1+\omega S)}, \widehat{S} < S < \overline{S}\right\}. \end{aligned}$$

(8) (**I - IH**) is an equilibrium in $B_1 \cup B_2 \cup B_3$ if and only if $\widetilde{S} < \overline{S}$ where

$$\begin{aligned} B_1 &\equiv \left\{(S,\sigma f) \mid \frac{\omega-\phi}{1+\omega S} < \sigma f < \frac{(1-\phi\omega)S}{\phi+S}, \widehat{S} < S < \widetilde{S}\right\}, \\ B_2 &\equiv \left\{(S,\sigma f) \mid \frac{(\omega-\phi)\,S}{\phi+S} < \sigma f < \frac{(1-\phi\omega)S}{\phi+S}, \widetilde{S} < S < \overline{S}\right\}, \\ B_3 &\equiv \left\{(S,\sigma f) \mid \frac{(\omega-\phi)\,S}{\phi+S} < \sigma f < \frac{\omega-\phi}{1+\phi S}, \overline{S} < S < 1\right\}; \end{aligned}$$

and in the set $C_1 \cup C_2 \cup C_3$ if and only if $\overline{S} < \widetilde{S}$ where

$$\begin{aligned} C_1 &\equiv \left\{(S,\sigma f) \mid \frac{\omega-\phi}{1+\omega S} < \sigma f < \frac{(1-\phi\omega)S}{\phi+S}, \widehat{S} < S < \overline{S}\right\}, \\ C_2 &\equiv \left\{(S,\sigma f) \mid \frac{\omega-\phi}{1+\omega S} < \sigma f < \frac{\omega-\phi}{1+\phi S}, \overline{S} < S < \widetilde{S}\right\}, \\ C_3 &\equiv \left\{(S,\sigma f) \mid \frac{(\omega-\phi)\,S}{\phi+S} < \sigma f < \frac{\omega-\phi}{1+S\phi}, \widetilde{S} < S < 1\right\}. \end{aligned}$$

(9) (**I - V**) is an equilibrium in A_{12} where

$$A_{12} \equiv \left\{(S,\sigma f) \mid \frac{(1-\phi\omega)S}{\phi+\phi\omega S} < \sigma f, 0 < S < \frac{\phi}{K}\right\}.$$

(10) (**I - I**) is an equilibrium in A_{13} where

$$A_{13} \equiv \left\{(S,\sigma f) \mid \frac{\omega-\phi}{1+\phi S} < \sigma f, \overline{S} < S < 1\right\}.$$

In the next step, we show that, up to a zero-measure set, the sets A_i $(i = 1, ..., 13)$, B_j $(j = 1,2,3)$, and C_k $(k = 1,2,3)$ form a partition of X.

(D.2) Set

$$X_1 = \{(S,\sigma f) : 0 < \sigma f, 0 < S < \phi/K\}, \qquad X_2 = \left\{(S,\sigma f) : 0 < \sigma f, \phi/K < S < \widehat{S}\right\},$$

$$X_3 = \left\{(S,\sigma f) : 0 < \sigma f, \widehat{S} < S < \widetilde{S}\right\}, \quad X_4 = \left\{(S,\sigma f) : 0 < \sigma f, \widetilde{S} < S < \overline{S}\right\}, \quad X_5 = \left\{(S,\sigma f) : 0 < \sigma f, \overline{S} < S\right\},$$

$$X_6 = \left\{(S,\sigma f) : 0 < \sigma f, \widehat{S} < S < \overline{S}\right\}, \quad X_7 = \left\{(S,\sigma f) : 0 < \sigma f, \overline{S} < S < \widetilde{S}\right\}, \quad X_8 = \left\{(S,\sigma f) : 0 < \sigma f, \widetilde{S} < S\right\}.$$

Then, we have:

$$X = X_1 \cup X_2 \cup X_3 \cup X_4 \cup X_5 \Leftrightarrow \widetilde{S} < \overline{S} \quad \text{up to a zero-measure set,} \tag{S.18}$$

$$X = X_1 \cup X_2 \cup X_6 \cup X_7 \cup X_8 \Leftrightarrow \overline{S} < \widetilde{S} \quad \text{up to a zero-measure set,} \tag{S.19}$$

Furthermore, it is readily verified that the sets X_1, X_2, X_3, X_4, and X_5 are pairwise disjoint. Consequently, up to a zero-measure set, $\{X_1, X_2, X_3, X_4, X_5\}$ is a partition of X. The same holds for $\{X_1, X_2, X_6, X_7, X_8\}$.

Using the definition of A_i, B_i, and C_i in (S.18), the sets X_1 and X_2 are as follows:

$$X_1 = ((A_1 \cup A_2 \cup A_4) \cap X_1) \cup A_8 \cup A_{12}, \tag{S.20}$$

$$X_2 = ((A_1 \cup A_2 \cup A_4) \cap X_2) \cup A_9 \cup A_{10} \cup (A_7 \cap X_2). \tag{S.21}$$

Furthermore, the sets $((A_1 \cup A_2 \cup A_4) \cap X_1)$, A_8, and A_{12} are mutually disjoint. The same holds for $(A_1 \cup A_2 \cup A_4) \cap X_2$, A_9, A_{10}, $A_7 \cap X_2$.

When $\widetilde{S} < \overline{S}$, the sets X_3, X_4, and X_5 are such that

$$\begin{aligned} X_3 &= ((A_1 \cup A_2) \cap X_3) \cup A_5 \cup B_1 \cup ((A_7 \cup A_{11}) \cap X_3), \\ X_4 &= ((A_1 \cup A_3 \cup A_6) \cap X_4) \cup B_2 \cup ((A_7 \cup A_{11}) \cap X_4), \\ X_5 &= ((A_1 \cup A_3 \cup A_6) \cap X_5) \cup B_3 \cup A_{13}. \end{aligned} \tag{S.22}$$

Furthermore, the sets $(A_1 \cup A_2) \cap X_3, A_5$, B_1, and $(A_7 \cup A_{11}) \cap X_3$ are mutually disjoint. The same holds for $(A_1 \cup A_3 \cup A_6) \cap X_4$, B_2, and $(A_7 \cup A_{11}) \cap X_4$, and for $(A_1 \cup A_3 \cup A_6) \cap X_5$, B_3, and A_{13}.

When $\overline{S} < \widetilde{S}$, the sets X_6, X_7, and X_8 are as follows:

$$\begin{aligned} X_6 &= ((A_1 \cup A_2 \cup A_5) \cap X_6) \cup C_1 \cup A_{11} \cup (A_7 \cap X_6), \\ X_7 &= ((A_1 \cup A_2 \cup A_5) \cap X_7) \cup C_2 \cup (A_{13} \cap X_7), \\ X_8 &= (A_1 \cap X_8) \cup A_3 \cup A_6 \cup C_3 \cup (A_{13} \cap X_8). \end{aligned} \tag{S.23}$$

Furthermore, the sets $(A_1 \cup A_2 \cup A_5) \cap X_6$, C_1, A_{11}, and $A_7 \cap X_6$ are mutually disjoint. The same holds for $(A_1 \cup A_2 \cup A_5) \cap X_7$, C_2, and $A_{13} \cap X_7$, and for $A_1 \cap X_8$, A_3, A_6, and $C_3, A_{13} \cap X_8$.

Thus, up to a zero-measure set, all subsets of X_i $(i = 1, ..., 8)$ is a partition of X_i.

(D.3) Set $I = \{1, ..., 13\}$ and $J = K = \{1, 2, 3\}$.

(1) It is readily verified that $A_i \neq \emptyset$ for all $i \in I$; $B_j \neq \emptyset$ for all $j \in J$ if and only if $\widetilde{S} < \overline{S}$; and $C_k \neq \emptyset$ for all $k \in K$ if and only if $\overline{S} < \widetilde{S}$.

(2a) Substituting (S.20)-(S.21) into (S.18) yields $(\cup_{i \in I} A_i) \cup (\cup_{j \in J} B_j) = X$ almost everywhere when $\widetilde{S} < \overline{S}$.

(2b) Substituting (S.20)-(S.21) and (S.23) into (S.19) yields $(\cup_{i \in I} A_i) \cup (\cup_{k \in K} C_k) = X$ almost everywhere when $\overline{S} < \widetilde{S}$.

(3a) If $i \in I$, $l \in I$, and $i \neq l$, then $A_i \cap A_l = \emptyset$.

(3b) If $j \in J$, $m \in J$, and $j \neq m$, then $B_j \cap B_m = \emptyset$; if $i \in I$ and $n \in J$, then $A_i \cap B_n = \emptyset$ when $\widetilde{S} < \overline{S}$.

(3c) If $k \in K$, $r \in K$, and $k \neq r$, then $C_k \cap C_r = \emptyset$; if $i \in I$ and $t \in K$, then $A_i \cap C_t = \emptyset$ when $\overline{S} < \widetilde{S}$. Q.E.D.