Economies of Transport Density and Industrial Agglomeration[*]

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Abstract

This paper develops a model of a spatial economy in which interregional trade patterns and the structure of the transport network are determined endogenously as a result of the interaction between industrial location behavior and increasing returns in transportation, in particular, *economies of transport density*. The traditional models assume either the structure of the transport network or industrial location patterns, and hence, they are unable to explain the interdependence of the two. It is shown that economies of transport density can be the primary source of industrial localization.

Keywords: Economies of transport density; Formation of a transport hub; Agglomeration economies; Industrial localization

JEL classification: F12; O14; R12; R49

1 Introduction

Industrial agglomerations often appear in association with major traffic nodes. Obvious examples are those in cities which are usually seen near key junctions of highway networks or large railroad stations. At a more aggregated level, the unprecedented growth in Asian industries in the 1980s took place around the three largest ports in the world: Hong Kong, Singapore, and Kaohsiung.

The coincidence of industrial agglomeration and transport nodes results from the process of recipro-cal reinforcements between them. Of the two reinforcement forces, firms' motivation to save transport costs attracts these firms to locate around transport nodes. Indeed, the total transport costs paid by major manufacturing firms in Japan amount to 8.69 per cent of their total sales value (Japan Logistic Systems Association, 1996). In addition to these pecuniary costs, firms bear significant time costs for transportation. In particular, they often need business contacts with their customers and material suppliers in other regions. Even within a firm, local managers must regularly meet to discuss business decisions. All these things, of course, require frequent business trips across regions which incur a lot of time and money. For another example, assembly firms of electronic products in Asia are constantly subject to uncertain changes in market demand and production technologies. They are thus forced to frequently alter the amount and variety of components to be assembled. If the transportation of components takes time, they need to order them much earlier without knowing the exact type and amount of necessary components. To avoid this sort of risk, they prefer to operate at locations with good transport access, such as large international ports.

The other reinforcement force is that the efficiency of transport nodes is improved by the increase in transport demand stemming from the growth of industrial agglomeration. The basic mechanism originates from scale economies in transportation which have been realized by the development of large-sized and high-speed carriers, such as container ships, bullet trains, and jumbo jets. The scale economies provide an incentive for collective transportation and hence stimulate the development of trunk routes and the hub-spoke structure of transportation. The process of the trunk route formation exhibits the following circular causation. Suppose there are frequent transport services on a given link, such that these are available on demand. As a result, a large number of shippers are attracted to use the link, which in turn supports even more frequent transport services on the link. This positive feedback mechanism eventually leads to *the endogenous formation of trunk links and transport hubs*. When scale economies in transportation rule the transport advantage of each location, a major transport node can spontaneously emerge at *any* place having large transport demand like the location of industrial agglomeration. We call the above mechanism of circular causation *economies of transport density*.

Several studies have shown evidence that economies of density are significant in air and railroad transportation, which mainly carry passengers.[1] Yet, economies of density appear to be significant in freight transportation as well. In the case of maritime transportation, the transport cost per container decreases by 0.31 per cent given one per cent increase in ship size (Journal of Commerce, 1997). Also, the fare for shipping a container from Japan to each of the Southeast Asian ports decreases by 0.12 per

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[1] See Brueckner and Spiller (1991), Brueckner, Dyer and Spiller (1992), Caves, Christensen and Tretheway (1984), for the case of air transportation, and Braeutigam, Daughety and Turnquist (1982, 1984) for the case of rail transportation.

cent given one per cent increase in the number of ships on a given transport route, where the number of ships tends to increase with the transport density. Thus, the concentration of traffic demand and transport services exhibits a positive correlation with the efficiency of transportation. The effect can be clearly seen if we compare a pair of transport routes – say, Singapore-Japan and Jakarta-Japan – which are similar distance-wise but different with respect to transport density. Whereas Jakarta and Singapore may be equidistant vis-a-vis Japan, travel time from Jakarta is twice (100 per cent) longer than that from Singapore which has a large hub port linked to international trunk routes. (Shipping Gazette, 1997). A similar relation can be found between Hong Kong (hub) and Manila (non-hub). On average, transport costs from Japan to a non-hub port in Southeast Asia is 22.6 per cent higher than to a hub port in the same region.[2]

There are two groups in the existing literature on the causal relationship between industrial location patterns and the transport network structure. The works in one group depict the design of a transport network as a problem for a planner in a transport sector when economies of density exist (e.g., Campbell, 1996; Hendricks, Piccione and Tan, 1995). However, origin-destination flows between each pair of locations are assumed to be given in their models. As a result, they do not explain how the structure of the transport network affects the industrial location pattern. On the other hand, the works in the other group focus on deriving industrial location patterns under a given structure of the transport network (e.g., Fujita and Mori, 1996; Konishi, 2000; Krugman, 1993a; Mills, 1972). However, they do not explain how the spatial distribution of industries affects the structure of the transport network. Unlike the previous studies, this paper offers a simple general equilibrium model which analyzes *the interdependence between industrial location behavior and the transport network structure* when economies of transport density exist.

Now, let us describe the model informally. We consider transport and specialization patterns of two regions, East and West, which produce two types of homogeneous consumption goods, agricultural and manufactured. In both regions, the agricultural good can be produced by using only domestic factors, typically labor, while the manufactured good requires import of intermediate inputs from a third region (say, North) in addition to the domestic factors.[3] Assume that the production technologies in the three industries are all linear and that East and West are completely identical in terms of geographical advantage and factor endowment. In this context, if the transport technology is also linear (in distance and volume), there is no reason for industrial agglomeration to take place in either East or West.

Here, however, let us suppose that the transportation of intermediate goods is subject to economies of density. Then, if a large number of manufacturing firms locate in one region and yield sufficient demand for interregional transport, it will be followed by a decrease in the transport rate for intermediate goods, due to density economies. This in turn attracts more manufacturing firms to the region.

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[2] Transport rates are based on data given in Usui (1997). The regression results are available upon request.

[3] In the international context, North corresponds to advanced regions such as the EU, Japan and the US, producing high-tech components, while East and West correspond to developing countries in Southeast Asia and South America which assemble these components and export them back to North. In the context of a domestic economy, North may represent the core region such as Tokyo in Japan or New England/Calfornia in the US, while East and West comprise the periphery which often have strong production linkages to the core region.

As a result, the interaction between increasing returns in transportation and industrial location may trigger the industrial agglomeration and determine the structure of the transport network. In fact, it is possible in our model that a region specializing in manufacturing (say, East) spontaneously evolves into a transport hub through which West imports intermediate goods from North.[4]

The plan of the paper is as follows. In Section 2, the setup of the model is introduced. In Section 3, under given transport demand conditions in each region, the possible transport patterns of the products and the associated regional difference in transport advantage are discussed. In Section 4, the specialization pattern, and hence, the transport demand in each region is endogenized, and the equilibrium trade and transport patterns are derived. In Section 5, adjustment dynamics of the economy is introduced, and the stability of equilibria is examined. In Section 6, welfare implications of our model is discussed. Finally, we conclude in Section 7.

2 The model

In this section, we describe the basic setup of the model. In Subsection 2.1, the geography, production technology, and consumer preferences are specified. Then in Subsection 2.2, the transport cost structure, in particular, the working of density economies in transportation, is explained in detail.

2.1 Geography, technology, and preference

In order to model the endogenous transport advantage of a location and its consequence on the industrial location pattern, we need at least three locations in the economy. For this, our economy consists of three regions, called North, East and West. Since it is an unnecessary complication to fully endogenize transport advantages and specialization patterns of all three regions, we consider a simple setup in which those of only two regions, East and West, are simultaneously determined within the model.

To highlight the role of density economies in transportation as a source of regional advantage, we assume symmetry between East and West in both factor endowment and geographical proximity to North. The geography is such that each of East and West is located at one unit of distance away from North, while the distance between East and West is $k > 0$ (refer to Figure 1a). Labor is assumed to be the only primary factor of production in this economy. Each region is endowed with one unit of (a continuum of) immobile workers, where each unit embodies a unit of homogeneous labor.

$$\overline{\text{Figure 1}}$$

There are two types of consumption goods, manufactured and agricultural. All consumers (=workers) have the same preferences, and their utility function is assumed to take the Cobb-Douglas form:

$$U = C_M^\mu C_A^{1-\mu}, \tag{1}$$

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[4]Considering density economies in the transportation of other goods will not change our basic result. Moreover, today, density economies in transportation is most pronounced in the manufacture of final goods which uses a wide variety of fine components obtained from intermediate good suppliers at various locations.

where C_M [resp., C_A] is consumption of the manufactured [resp., agricultural] good, and μ represents the expenditure share of the manufactured good ($0 < \mu < 1$).

The manufactured good is produced by using labor and intermediate goods subject to a constant-returns technology given by

$$M = L^\alpha I^{1-\alpha}, \tag{2}$$

where M is the output, L [resp., I] is the amount of labor [resp., intermediate inputs], and α is a constant ($0 < \alpha < 1$). The asymmetry between North and the other two regions is assumed in that each unit of intermediate good [resp., agricultural good] is produced out of one unit of labor and the production is possible only in North [resp., East and West]. Moreover, the manufactured good can be assembled only in East and West by using their domestic labor. Thus, North exclusively supplies the intermediate good, while East and West produce the manufactured and/or agricultural good (refer to Figure 1b). Finally, all markets are perfectly competitive.

2.2   Transport cost

The transport costs are assumed to be product-specific and subject to Samuelson's iceberg technology (Samuelson, 1952). That is, if the good is transported over a distance, only a fraction of it reaches the destination.[5] For further simplification, there are potential density economies only in the transport of intermediate goods. In this context, the relative transport advantage of East and West is represented by their transport access to North (for the procurement of production inputs), which is endogenous in the model. Any interregional transportation is assumed to take place along the triangle, North-East-West (refer to Figure 1a).

For the transport of intermediate goods on a given link, the transport cost decreases as the transport density on the link increases. Since we do not explicitly consider the timing of shipments here, we interchangeably refer to "transport density" and "(aggregate) transport volume" on a given link. The following functional form is assumed for *the transport rate of intermediate good*, that is, the transport cost (in terms of intermediate goods) for shipping one unit:

$$T(d,Q) = \begin{cases} d & \text{if} \quad Q < \sigma \\ d\sigma/Q & \text{if} \quad Q \geq \sigma \end{cases}, \tag{3}$$

where d and Q are respectively the distance and transport density of intermediate goods on a given transport link (Q is the aggregate quantity on the link which reaches the destination), and σ is a positive constant indicating the degree of density economies (refer to Figure 2 for an illustration). In other words, the transport cost for a unit of intermediate good per distance is one for $Q \leq \sigma$ and σ/Q for $Q > \sigma$. Thus, up to the threshold level σ, density economies are not effective, but beyond σ, the transport rate decreases as the transport density increases. *Economies of density are said to be larger if the transport cost per unit of the product is smaller for a given transport density.* In our formulation, *the smaller the value of σ, the larger the density economies.* Furthermore, it is assumed

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[5]The product-specificity and iceberg technology may not be general properties of transport costs. In particular, an explicit treatment of the transport sector would be an interesting alternative. But, these assumptions dramatically increase the tractability of the spatial general equilibrium models with agglomeration economies.

that *density economies are external to each firm*, and that *transport costs are linear in distance.*[6]

$$\overline{\text{Figure 2}}$$

The transport rate for the manufactured good is given by md, where m is a positive constant and d is the distance. For simplicity, transport of the agricultural good is assumed to be costless.

In this setting, the transport link which happens to attract a larger traffic of intermediate goods can have better transport access, and hence, other things being equal, manufacturing firms are attracted to the region which is a node of the link. This concentration of manufacturing firms in turn enlarges the transport demand there, generating far greater density economies. Hence, even in the absence of agglomerative forces stemming from production technologies or consumer preferences, scale economies in transportation can generate the geographic concentration of industries. The extent to which traffic agglomeration develops depends on the size and spatial distribution of demand for manufactured goods (due to immobile workers in each region), and the size of intermediate good supply (from North). The former has a crucial influence on the structure of the transport network. Namely, if East and West are geographically close, then the traffic for both regions may be pooled along the way from North (i.e., a hub may form) to benefit from density economies. However, a hub formation may not make sense if East and West are far apart, since in this case shippers in at least one of the two regions need to transport over a longer distance than in the case of direct transport from North, and this long-hauling may be very costly. The latter limits the size of transport demand, and hence, limits the scale of density economies attainable.

3   Formation of transport network

In the economy we study, the structure of the transport network and industrial location are determined interdependently. The transport accessibility clearly affects the location of firms, since they want to save shipping costs. On the other hand, the industrial location pattern determines the spatial distribution of transport demand, which in turn influences the shape of the transport network in the presence of density economies. In this section, we focus on the latter effect, and ask "What is the viable structure of the transport network under given transport demand in each region in the presence of economies of traffic density?" Then in the next section, we will discuss the interaction of the two effects, taking the former one into account.

Recall that in the context of our model, the structure of the transport network is relevant only for the transportation of intermediate goods for which density economies are effective. That is, the relevant shippers are manufacturing firms in East and West who import intermediate goods from North. Since the market for intermediate goods is perfectly competitive, the delivered price of an intermediate good is given by $1 + T(d, Q)$ in terms of the transported intermediate good over a link of length d and traffic density Q. Assuming that the transport rate on each link is fully known1 to all firms, we can find viable routes which offer the lowest delivered prices.

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[6] To focus on the influence of density economies, another important scale economies in transportation, *economies of transport distance* or *long-haul economies*, are excluded. See Louveaux, Thisse and Beguin (1981) on this issue.

Now define *the transport network equilibrium* as a state of the transport network such that *given transport density on each transport link, each shipper achieves the lowest transport cost, and has no incentive to change transport routes unilaterally.* Obviously, the transport network equilibrium is a necessary condition for an equilibrium of the economy.

What are the possible structures of transport network? Since the transport rate on each route is perfectly known, the possibility of cross-hauling is excluded in equilibrium. This means that in our economy, there is at most one hub in equilibrium, where by a hub we mean a region (either East or West) through which strictly positive amounts of intermediate goods are transported to both of East and West. Below, the equilibrium conditions for the two key patterns of transportation are derived. Namely, we consider the cases in which all shipments of intermediate goods to one region take the same transport route. The possible route is then either the direct or indirect route: North-East or North-West-East for East-bound shipments, and likewise, North-West or North-East-West for West-bound ones.[7] Denote by I_E [resp., I_W] the demand of intermediate goods in East [resp., West]. In the presence of density economies, the link which happens to attract a larger traffic will benefit from agglomeration economies of transportation. It follows that transport routes which are identical distance-wise may end up being unequal in transport costs once the polarization of traffic takes place. This symmetry-breaking mechanism is explained in Subsections 3.1 and 3.2 for the cases with and without a hub formation, respectively.

3.1 Case without a hub formation

Consider first the case in which only the direct routes are used (i.e., there is no hubbing). The transport rate on each link in this case is shown in Figure 3a. In this case, the traffic density on the North-East [resp., North-West] link is I_E [resp., I_W], while that on the East-West link is zero. It follows that the transport rate on the North-East [resp., North-West] link is given by $T(1, I_E)$ [resp., $T(1, I_W)$], while it is $T(k, 0) = k$ on the East-West link.

$$\overline{\text{Figure 3}}$$

Under what condition is this transport pattern viable? To answer this question, let us see if there is a shipper who wants to use an alternative transport route. In our simple framework, there is only one alternative route for each shipper. Namely, for a manufacturing firm in East [resp., West], intermediate goods may be transported via West [resp., East] indirectly. Note that this round-about transportation may make sense in the presence of density economies if the cost reduction attained by pooling the traffic is sufficiently large. The cost of the round-about transportation under the given transport pattern can be calculated as follows. Recall that density economies are external to each firm, so that the current transport rate on each link is taken as given. Recall also that the transport costs in our model is of the iceberg-type. In this context, for one unit of intermediate good to reach West from East, $1 + k$ units must be shipped from East, where k units melts away while being transported.

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[7]It is possible to have a mixed transport pattern: shippers in the same region are indifferent between direct and indirect routes. But, as will be discussed in Section 4.3, such an equilibrium is unstable under an appropriate adjustment process (refer also to footnote 13). Thus, we omit the discussion of this transport pattern.

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Similarly, for one unit of intermediate good to reach East from North, $1 + T(1, I_E)$ units must be shipped from North. That is, to have one unit in West via the North-East-West route under the present traffic volume on each link, $(1 + k)\{1 + T(1, I_E)\}$ units must be shipped from North, and hence, the transport rate is given by $(1 + k)\{1 + T(1, I_E)\} - 1$. If this amount is not lower than the ongoing transport rate, $T(1, I_W)$, then no shipper in West has an incentive to use the indirect route. In a symmetric manner, we can obtain the condition for all shippers in East to choose the (present) direct route. Accordingly, the no-arbitrage condition for the intermediate good for this case can be written as

$$(1 + k)\{1 + T(1, I_i)\} \geq 1 + T(1, I_j), \quad i \neq j \in \{E, W\}. \tag{4}$$

It is easy to see that we can have different transport rates on the two links, North-East and North-West, in equilibrium, which is due to the presence of density economies and the distance between East and West (represented by k). To understand this, suppose the transport demand in East is larger than that in West, and is large enough for density economies to be effective: $I_E > I_W$ and $I_E > \sigma$. Then, we have $T(1, I_E) < T(1, I_W)$. If East and West are close enough (geographically), the supposed direct transport pattern will not be sustained, since (4) is violated for a sufficiently small k. That is, if the two regions are sufficiently close, then for a shipper in West, the benefit of pooling the traffic overcomes the cost of the round-about transportation via East. However, if the two regions are far from each other, hubbing does not make sense. On the other hand, there should be no incentive for a shipper in East to use a transport route other than the direct North-East one, since it obviously offers the smallest transport rate among all the possible routes.

3.2   Case with a hub formation

Suppose all intermediate goods are transported via either East or West. Without loss of generality, let East be the transport hub. The transport rate on each link is shown in Figure 3b, and is derived as follows. First, since the traffic on the East-West link is I_W, the transport rate on this link is $T(k, I_W)$. It follows that $I_W\{1 + T(k, I_W)\}$ units of intermediate good must be shipped from East to West, which together with the intermediate good demand in East, I_E, make up the traffic density on the North-East link: $I_E + I_W\{1 + T(k, I_W)\}$. Then, the transport rate on the North-East link is given by $T(1, I_E + I_W[1 + T(k, I_W)])$. Consequently, for one unit of intermediate good to arrive in West via East, $\{1 + T(k, I_W)\}\{1 + T(1, I_E + I_W[1 + T(k, I_W)])\}$ units must be shipped from North, and the transport rate on the North-East-West route is this amount minus one.

Now, can this transport pattern be an equilibrium? Let us consider a deviation of a shipper from the present transport pattern. Note that in the above case East is a transport hub so that shippers in East have no incentive to change routes. This can be verified by the fact that the transport rate, $T(1, I_E + I_W[1 + T(k, I_W)])$, on the North-East link is at most $T(1, 0) = 1$ which equals the transport rate on the North-West link. It follows that we only need to see if shippers in West (i.e., the non-hub region) have an incentive to deviate to the direct North-West route. Since there is no traffic there, the transport rate is one. It follows that if the round-about transportation via East costs more than one per unit of intermediate good, then firms will instead use the direct route. The no-arbitrage condition

can then be written as follows:

$$\{1 + T(k, I_W)\}\{1 + T(1, I_E + I_W[1 + T(k, I_W)])\} \leq 2. \tag{5}$$

When is this condition satisfied? Not surprisingly, an increase in the distance between East and West, k, will increase the cost for the round-about transportation for shippers in the non-hub region as we can see from (3) that the LHS of (5) increases as k increases. On the other hand, as long as density economies are at work on the North-East link so that $T(1, I_E + I_W[1 + T(k, I_W)]) < 1$, then the formation of a hub may be an equilibrium for a sufficiently small k. As in the case of the direct transport pattern considered above, the (hub) region which happens to be a node of a frequently used transport link will benefit the most from density economies on the trunk link, and faces a lower delivered price for intermediate good.

When a hub is formed, the benefit from density economies on the trunk link is shared by both East and West, whereas it is localized in the region which is attached to the trunk link if a hub is not formed. It should be noted, however, that the hub formation is not necessarily beneficial to the non-hub region. Since density economies are external to each firm, once a hub is formed, the transport pattern tends to be locked in, and each shipper in the non-hub region may not unilaterally have an incentive to use the direct transport route, even if the direct route may be cheaper if all shippers in the non-hub region collectively choose the direct transport route.

We will see in the following sections that when the demand for intermediate goods in each region is endogenized, density economies in transportation generates agglomeration economies in manufacturing production, and play the central role in driving the specialization patterns and welfare of the regions.

4   Endogenous transport advantage and patterns of trade

In the previous section, we have studied how the difference in transport advantage between the regions can arise for a given size of transport demand in each region. However, the transport demand should also be determined in the model so as to meet the technological and market conditions. Now, we endogenize the specialization patterns of East and West, and find simultaneously the possible equilibrium patterns of production and transportation in the economy. This section is organized as follows. In subsection 4.1, we define the (full) equilibrium of the economy, and discuss possible equilibrium configurations in our model. Though our model potentially generates many different configurations regarding transport and trade patterns, in this subsection we carefully exclude relatively unimportant ones to focus on explaining the basic mechanism of the interdependence between transport density economies and industrial agglomeration. Then in subsections 4.2 and 4.3, we characterize these selected equilibrium configurations.

4.1   Preliminary analysis

First, let us define *an equilibrium* as a state of the economy in which: *(i) consumers maximize utility; (ii) firms maximize profits; (iii) profits are zero; (iv) all workers attain the same utility level in their region; (v) all markets clear; (vi) the transport network equilibrium is attained.*

The analysis of the transport network equilibrium suggested that in the presence of positive externalities in transportation, multiple equilibria may arise under the same set of parameter values, and where each equilibrium corresponds to a different transport and specialization pattern. In our model, the difference in specialization patterns between East and West accrues primarily to the difference in the endogenous transport advantage in each region. Since the transport advantage provides a production advantage in manufacturing, the region which happens to have the transport advantage necessarily exports the manufactured good, while the other region may not. In that sense, it is convenient to classify the possible equilibrium configurations with respect to the trade pattern of the regions. Namely, an equilibrium is called *a convergent equilibrium* if both East and West export manufactured goods, while it is called *a divergent equilibrium* if only one of them does. When dealing with a divergent equilibrium, for convenience, we call the region that exports manufactured goods *the manufacturing region*, and the other *the agricultural region*, based on the region's *relative* specialization. Below, we first derive common conditions for both types of equilibria. Since both convergent and divergent equilibria can have multiple configurations, we exclude relatively unimportant ones for simplicity of presentation. Then, the conditions that are specific to each configuration which we have selected to study are derived in Subsections 4.2 and 4.3. When convenient, we refer to North, East and West by regions N, E and W, respectively, and the equilibrium value of an endogenous variable x is denoted by x^*.

Suppose region $i \in \{E, W\}$ produces manufactured goods. Let p_i^I, w_i and n_i be the delivered price of the intermediate good, the wage rate, and the share of manufacturing workers, respectively, in region i. Then, cost minimization under the production technology given by (2) together with the market clearing of labor and the intermediate good imply that

$$n_i = \frac{\alpha}{1-\alpha} \frac{p_i^I}{w_i} I_i, \qquad i \in \{E, W\}. \tag{6}$$

Given w_i and p_i^I, the unit cost of manufactured good, c, in region i can be calculated by using (2) and (6) as

$$c(w_i, p_i^I) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} w_i^{\alpha}(p_i^I)^{1-\alpha}, \qquad i \in \{E, W\}. \tag{7}$$

Since North produces only intermediate goods, its entire income consists of the sales of this good. Denote by w_N the wage rate (= the total income) in North. The marginal cost pricing implies that the price of the intermediate good in North equals w_N. For a given demand for intermediate goods in region i, I_i, the total sales of intermediate goods in region i is given by $p_i^I I_i$ which includes the transport consumption of the good. It follows that the total income of North, w_N, can be expressed as

$$w_N = p_E^I I_E + p_W^I I_W. \tag{8}$$

Now, what are possible trade flows of manufactured and agricultural goods? Obviously, East or West (or both) should export manufactured goods to North in equilibrium. Since in our model the only source of production advantage in manufacturing is the transport advantage in the procurement of intermediate inputs (due to density economies), the exporter of manufactured goods necessarily

has the transport advantage. Note that in a divergent equilibrium, the manufacturing region may export manufactured goods also to the agricultural region if the transport cost for the good is very low (relatively to that of intermediate inputs). Thus, we have two possible cases: first, either East or West exports manufactured goods to North, while the other region is self-sufficient, and second, either East or West exports manufactured goods to all other regions. However, to explain the basic mechanism which links endogenous transport advantage and concentration of manufacturing production, it is sufficient to consider (an easier) one of these two cases. Here, we limit our analysis to the former one.

The desired situation is guaranteed if the transport rate is higher for the manufactured good than for intermediate inputs, which is often described as *weight/bulk-gaining* manufacturing production. Under the specification of the transport cost for intermediate goods given by (3), it means that transporting a unit of the manufactured good over a unit of distance costs at least one additional unit of the good which is consumed on the way. So, we simply set the transport rate for the manufactured good equal to one, i.e., $m = 1$.[8]

To see why there is no trade of manufactured goods between East and West in the present context, suppose East exports manufactured goods to West in equilibrium. Recall that the agricultural good is free of transport costs, and it can be produced in both East and West under the same constant-returns technology using labor. Then, we must have $w_W^* \leq w_E^*$, where the equality holds if East produces the agricultural good as well. It also follows that we must have $p_E^{I*} < p_W^{I*}$, otherwise, West can produce the manufactured good cheaper than the imports. Since West imports the manufactured good from East, the local price of the manufactured good in West must be equal to the delivered price in West of the manufactured good produced in East, $p_E^{M*}(1 + k)$, where k units of the good is consumed en route. Does it make sense for West to import the manufactured good from East? The answer is "No." Why? Because West can produce manufactured goods cheaper than $p_E^{M*}(1 + k)$, by importing intermediate goods from East. Namely, since $p_W^{I*} \leq p_E^{I*}(1 + k)$ and $w_W^* \leq w_E^*$, we can show that $c(w_W^*, p_W^{I*}) \leq (1 + k)^{1-\alpha} c(w_E^*, p_E^{I*}) < p_E^{M*}(1 + k)$. Hence, if a good is traded at all between East and West, it must be the agricultural good. Though the weight-gaining nature of manufacturing production is not always true in reality, we will see below that it greatly reduces the complexity of the problem without losing the essence of the model.

So far, we have set up the model so that both East and West produce the manufactured good at least for their domestic market. What about agricultural production? Do they both produce the agricultural good? Or, does one of them completely specialize in manufacturing? Not surprisingly, if the demand for the manufactured good in North is not too large, it can be completely satisfied by either East or West. Thus, in this case, there is room for agricultural production in a region even if it is an exporter of the manufactured good. However, if the demand in North is too large, then the supply from only one region may not be enough. In this case, the exporter of manufactured goods will completely specialize in manufacturing, while the other will relatively specialize in agriculture and may export the manufactured good as well. The case of complete specialization may be of interest

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[8] As long as the manufacturing is weight-gaining, our basic results hold. In particular, the specific size of the melting-down factor in transport is not qualitatively important as it depends on the specification of the transport cost for intermediate goods given by (3).

in some other situations. But, for our objective, it does not inspire theoretical interest.[9] Throughout the paper, therefore we restrict the analysis in the context of incomplete specialization in both East and West. One way to guarantee this situation is to assume that consumer preferences are not too inclined towards the manufactured good, that is, μ is not too large (refer to (1)). The calculation below will confirm that the upper bound of the expenditure share of the manufactured good, μ, is given by

$$\widetilde{\mu} \equiv \frac{-1 + \sqrt{1 + 4\alpha(1 - \alpha)}}{2\alpha(1 - \alpha)}. \tag{9}$$

Hence, we maintain the following assumption:[10]

Assumption 1 $\mu < \widetilde{\mu}$.

If region $i \in \{E, W\}$ specializes incompletely, then the equilibrium wage rate in region i, w_i^*, equals the price of the agricultural good. Let the agricultural good be the numeraire. Then, we have

$$w_E^* = w_W^* = 1. \tag{10}$$

Note that share α [resp., $1 - \alpha$] of manufacturing production cost is spent on labor [resp., intermediate good], and that the total sales of intermediate goods should equal to w_N by (8). It follows that the total wage payment in the manufacturing sector in East and West should amount to $w_N^* \alpha/(1 - \alpha)$ in equilibrium, which by (10) is equal to the total labor employment in manufacturing in the two regions. Since each of East and West has in total one unit of labor, the total supply of the agricultural good in the economy is given by $2 - w_N^* \alpha/(1 - \alpha)$. By equilibrating this to the total demand for the agricultural good in the economy, $(1 - \mu)(w_N^* + 2)$, we obtain

$$w_N^* = \frac{2(1 - \alpha)\mu}{1 - (1 - \alpha)\mu}. \tag{11}$$

Next, by a similar argument, the total sales of the manufacturing sector in region i in equilibrium must be equal to $w_i^* n_i^*/\alpha$, while the total expenditure for the consumption of manufactured good in the economy is given by $\mu(w_N^* + w_E^* + w_W^*)$. By equating these two, and using (10) and (11), we can solve for the total worker employment in manufacturing in equilibrium:

$$n_E^* + n_W^* = \frac{2\alpha\mu}{1 - (1 - \alpha)\mu}. \tag{12}$$

The equilibrium manufacturing share in region i, n_i^*, takes its maximum value, $\overline{n}$, when the region's exports completely fulfill the demand in North. In this case, the sales of the manufacturing sector in region i, $w_i^* n_i^*/\alpha$, should match the sum of the expenditure for the manufactured goods in region i and that in North, $\mu(w_i^* + w_N^*)$. On the other hand, n_i^* takes its minimum value, $\underline{n}$, when the manufacturing sector in region i exports nothing. In this case, the equilibrium sales of the manufacturing sector in region i should be equal to the total expenditure for the manufactured good in the region, μw_i^*. Using (10) and (11), we can obtain the values of $\overline{n}$ and $\underline{n}$ as follows:

$$\overline{n} = \alpha\mu \frac{1 + (1 - \alpha)\mu}{1 - (1 - \alpha)\mu}, \tag{13}$$

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[9]The consideration of complete specialization (by East and/or West) complicates the analysis without changing the basic results, just like it is not necessary to derive the basic insight of neoclasical trade models.

[10]$\widetilde{\mu}$ takes the minimum value $2(\sqrt{2} - 1)$ at $\alpha = 1/2$.

$$\underline{n} = \alpha\mu. \tag{14}$$

It can be verified that under Assumption 1, we have $0 < \underline{n} < \overline{n} < 1$. Thus, any equilibrium manufacturing share in region i will satisfy the condition that

$$\underline{n} \le n_i^* \le \overline{n}, \qquad i \in \{E, W\}. \tag{15}$$

Note that if both East and West specialize incompletely, the wage rate of workers is the same for both regions (refer to (10)), and thus only the region with transport advantage (i.e., a lower delivered price for intermediate inputs) will export manufactured goods in equilibrium. This necessarily means that (under Assumption 1) there is a difference in transport advantage in a divergent equilibrium, while there are none in a convergent equilibrium. The following subsections derive the conditions for the two equilibrium configurations.

4.2 Convergent equilibrium

Suppose both East and West export manufactured goods in equilibrium. Then it can be verified that all intermediate inputs used in each region are procured directly from North. It follows that the delivered prices of intermediate goods in these regions are given by

$$p_i^I = w_N\{1 + T(1, I_i)\}, \qquad i \in \{E, W\}. \tag{16}$$

Since the delivered price in North of manufactured goods should be the same no matter where they are produced, the f.o.b. prices (which equal the production costs) in East and West are also necessarily the same: $c(w_E^*, p_E^{I*}) = c(w_W^*, p_W^{I*})$.[11] Since $w_E^* = w_W^* = 1$ by (10), we must have $p_E^{I*} = p_W^{I*}$, which in turn implies $T(1, I_E^*) = T(1, I_W^*)$. There are two possible cases that satisfy this requirement. One is that density economies are effective in an identical manner on both the North-East and North-West links: $I_E^* = I_W^* > \sigma$. The other is that they are not effective on either link: $I_E^*, I_W^* \le \sigma$, and hence, $T(1, I_E^*) = T(1, I_W^*) = 1$. Thus, the degree of density economies, σ, plays a key role. Below, the convergent equilibrium specialization patterns of the regions are derived for each value of σ. The result is also visualized in Figure 4 which plots the equilibrium share of East in the manufacturing labor employment in the economy, $n_E^*/(n_E^* + n_W^*)$, for each value of σ.[12]

<u>Figure 4</u>

By (6), (10), (11) and (16), we can calculate the threshold size of the manufacturing sector in each region to generate a sufficiently large demand for intermediate goods to trigger density economies:

$$I_i^* \lessgtr \sigma \quad \text{if and only if} \quad n_i^* \lessgtr \frac{4\alpha\mu\sigma}{1 - (1 - \alpha)\mu}. \tag{17}$$

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[11] Recall that the transportation of manufactured goods are not subject to density economies.

[12] The plot in Figure 4 is a simulated outcome under the parameter values given in the figure. However, the qualitative nature of the figure does not depend on these specific values of the parameters.

Using (12) and (17), it is straightforward to show that the first case ($I_E^* = I_W^* > \sigma$) arises when density economies are sufficiently large:

$$n_E^* = n_W^* = \frac{\alpha\mu}{1-(1-\alpha)\mu}, \quad \text{if} \quad \sigma < 1/4. \tag{18}$$

On the other hand, the second case arises ($I_E^*, I_W^* \leq \sigma$), if density economies are small ($\sigma \geq 1/4$) so that they are not effective on both transport links when $n_E^* = n_W^*$. In this case, there is a continuum of convergent equilibria in which the specializations of the two regions are not necessarily identical. As long as the manufacturing production (and hence, the traffic demand) is not concentrated too much in one region, it does not trigger density economies, and the transport rate for intermediate goods is given by $T(1, I_E^*) = T(1, I_W^*) = 1$. If density economies are very small ($\sigma > \frac{1+(1-\alpha)\mu}{4}$), it can be shown by using (6) that they will not be effective for any feasible manufacturing shares, n_E^* and n_W^* (i.e., those satisfying (12) and (15)). If density economies are of intermediate extent ($\frac{1}{4} < \sigma \leq \frac{1+(1-\alpha)\mu}{4}$), the values of n_E^* and n_W^* must be sufficiently close so that neither transport link gets traffic large enough to trigger density economies. By using (6) and (12), the range of asymmetry between the regions' specialization patterns, n_E^* and n_W^*, which is consistent with the convergent equilibrium is obtained as $\frac{2\alpha\mu(1-2\sigma)}{1-(1-\alpha)\mu} \leq n_i^* \leq \frac{4\alpha\mu\sigma}{1-(1-\alpha)\mu}$. Since $\underline{n} \leq \frac{2\alpha\mu(1-2\sigma)}{1-(1-\alpha)\mu}$ and $\frac{4\alpha\mu\sigma}{1-(1-\alpha)\mu} \leq \overline{n}$, this range of n_i^* is contained in $[\underline{n}, \overline{n}]$. In sum, for $\sigma \geq 1/4$, at any convergent equilibrium, manufacturing shares, n_E^* and n_W^*, must satisfy the following condition along with the market clearing of the manufactured good, (12) [refer to Figure 4a]:

$$\begin{array}{ll} \frac{2\alpha\mu(1-2\sigma)}{1-(1-\alpha)\mu} < n_i^* < \frac{4\alpha\mu\sigma}{1-(1-\alpha)\mu}, & \text{if} \quad \frac{1}{4} < \sigma \leq \frac{1+(1-\alpha)\mu}{4} \\ \underline{n} < n_i^* < \overline{n}, & \text{if} \quad \sigma > \frac{1+(1-\alpha)\mu}{4} \end{array}, \tag{19}$$

The corresponding demand for intermediate goods (inclusive of transport consumption) in each region can be obtained from (3), (6) and (16) as the following constant:

$$I_i^* \{1 + T(1, I_i^*)\} = \frac{1-(1-\alpha)\mu}{2\alpha\mu} \tag{20}$$

where $T(1, I_i^*) = \sigma/I_i^*$ [resp., $= 1$] if $\sigma < 1/4$ [resp., $\sigma \geq 1/4$]. Thus, the total shipment of the intermediate good from North to either East or West is the same, but the delivered quantity (net of transport consumption) is larger in the region with a transport advantage.

Finally, in order for the assumed (direct) transport pattern to be a transport network equilibrium, the no-arbitrage condition (4) for intermediate goods must be satisfied. Namely, in equilibrium, no manufacturing firm in either region should have an incentive to transport intermediate goods via the other region. If both regions export manufactured goods in equilibrium, we can immediately see that this condition is always satisfied. By summarizing the results obtained, we have the following proposition:

Proposition 1 *There always exists an equilibrium in which both East and West export manufactured goods. Moreover, if $\sigma \leq 1/4$, such an equilibrium exists uniquely, and the specialization patterns of the two regions are identical. If $\sigma > 1/4$, each specialization pattern satisfying (12) and (19) corresponds to a unique convergent equilibrium.*

Note that in the absence of density economies, specialization patterns of the regions are arbitrary as long as (12) and (15) are satisfied as we can see in the case of $\sigma \geq 1/4$. But, in the presence of density economies, possible configurations are limited to the completely symmetric specialization patterns of East and West and the most asymmetric specialization patterns which we study in the next subsection. This shows us the significance of positive externalities in transportation in determining the patterns of trade.

4.3   Divergent equilibrium

In a divergent equilibrium, only either East or West exports the manufactured good (to North). Without loss of generality, East is assumed to be the exporter of the manufactured good. Recall that the manufacturing share of a region takes its maximum value, $\overline{n}$, when the region is the only exporter of the good. Thus, by (12) we have

$$n_E^* = \overline{n} \quad \text{and} \quad n_W^* = \underline{n}. \tag{21}$$

As for transport patterns, there are two key cases. One is that both East and West procure intermediate goods directly from North. The other is that East becomes a transport hub, and all intermediate goods are distributed via East. In this case, East procures intermediate goods directly from North, while West uses the indirect North-East-West route. For convenience, we call an equilibrium associated with the former transport pattern, *the divergent equilibrium without a hub*, and that associated with the latter *the divergent equilibrium with a hub*. There is one more possible equilibrium configuration associated with a mixed transport pattern in which intermediate goods are transported to West both directly via the North-West link and indirectly via the North-East-West route. In this equilibrium, the transport rate on the direct route and that on the indirect route must be exactly the same. But, in the presence of density economies in transportation, such an equilibrium is not stable under appropriate adjustment dynamics.[13] Thus, in the rest of this subsection, we omit the discussion of this configuration, and focus on the two key equilibrium configurations: divergent equilibria with and without a hub.

4.3.1   Case without a hub

We first consider the divergent equilibrium without a hub in which both East and West procures intermediate goods directly from North. In this case, the delivered price of intermediate good in each region is given by (16). By using this, together with (6), (8), (10) and (11), the demand for intermediate goods (inclusive of transport consumption) in each region can be expressed in terms of the region's manufacturing share, n_i^*, which after applying (21) and then (12)-(14), can be completely

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[13]The intuition behind the instability of the mixed-transport pattern is straightforward. Namely, a small perturbation to the transport pattern will increase the traffic on one of the routes, while reducing that on the other. As a result, due to the density economies, the transport access of the more frequently-used route will improve the transport access. It follows that more manufacturing firms will choose this route, which will in turn widen the difference in accessibilities of the two routes. The complete analysis of the divergent equilibrium with a mixed transport pattern can be obtained from the authors upon request.

solved as

$$I_E^*\{1 + T(1, I_E^*)\} = \frac{1 + (1-\alpha)\mu}{2}, \tag{22}$$

$$I_W^*\{1 + T(1, I_W^*)\} = \frac{1 - (1-\alpha)\mu}{2}, \tag{23}$$

where the transport rate on each of the North-East and North-West links is given as follows:

$$T(1, I_E^*) = \begin{cases} \sigma/I_E^* & \text{if} \quad \sigma < \frac{1+(1-\alpha)\mu}{4} \\ 1 & \text{if} \quad \sigma \geq \frac{1+(1-\alpha)\mu}{4} \end{cases}, \tag{24}$$

$$T(1, I_W^*) = \begin{cases} \sigma/I_W^* & \text{if} \quad \sigma < \frac{1-(1-\alpha)\mu}{4} \\ 1 & \text{if} \quad \sigma \geq \frac{1-(1-\alpha)\mu}{4} \end{cases}. \tag{25}$$

Do these configurations satisfy the transport network equilibrium as well? The answer depends on the degree of transport density economies, σ, and the distance between East and West, k. We also need to check the no-arbitrage condition (4). By (24) and (25), we can see that the condition should be examined in three different ranges of σ, where density economies are (i) large: $\sigma < \frac{1-(1-\alpha)\mu}{4}$; (ii) small : $\sigma \geq \frac{1+(1-\alpha)\mu}{4}$; and (iii) intermediate : $\frac{1-(1-\alpha)\mu}{4} \leq \sigma < \frac{1+(1-\alpha)\mu}{4}$. Each case is investigated as follows.

(i) large density economies: $\sigma < \frac{1-(1-\alpha)\mu}{4}$

In this case, the no-arbitrage condition (4) requires $\sigma < \sigma_1(k)$, where $\sigma_1(k)$ is given by

$$\sigma_1(k) \equiv \frac{k}{2} \frac{1 - (1-\alpha)^2\mu^2}{(1+k)\{1 + (1-\alpha)\mu\} - 1 + (1-\alpha)\mu}. \tag{26}$$

It can be verified that

$$\sigma_1(k) \lesseqgtr \frac{1 - (1-\alpha)\mu}{4} \quad \text{if} \quad k \lesseqgtr \frac{2(1-\alpha)\mu}{1 + (1-\alpha)\mu}. \tag{27}$$

Thus, when the two regions are far from each other ($k \geq \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$), the transport network equilibrium is always attained, while when they are close ($k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$), density economies should be sufficiently large ($\sigma \leq \sigma_1$) for this configuration to be an equilibrium. The interpretation is straightforward. Consider an extreme situation in which $\sigma \simeq 0$ so that $T(1, I_i^*) \simeq 0$ ($i \in \{E, W\}$). In this case, the cost of shipping a unit of intermediate good directly from North to East and to West is approximately zero, while the cost of shipping it indirectly via one of the regions to the other is approximately k, since currently there is no traffic between East and West. Once such a transport pattern is established, it is locked in, and no shipper has an incentive to use the round-about route unilaterally, even if it is less costly for all shippers in one region to collectively decide to use the round-about route. Recall (the result in Section 3) that the deviation to the round-about route is more likely when the two regions are geographically close (k is small). This effect can be seen in (26) that $\sigma_1(k)$ is an increasing function of k. That is, the closer the two regions, the greater the degree of density economies required for the direct transport pattern to be established in equilibrium. As (27) shows, the considered transport network is not an equilibrium for $\sigma_1(k) < \sigma$ if $k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$.

(ii) Small density economies: $\sigma \geq \frac{1+(1-\alpha)\mu}{4}$

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In this case, density economies are not effective on either link: $T(1, I_E^*) = T(1, I_W^*) = 1$, which means that the round-about transportation is never cheaper than the present direct one. In this range of σ, the divergence in the specialization patterns of the two regions is not induced by the difference in the production advantage. In fact, any specialization pattern is an equilibrium as long as the aggregate demand and supply of intermediate goods meet (i.e., (12) and (15) are satisfied), as the case for $\sigma > 1/4$ in Proposition 1 indicates. Thus, the distinction between the convergent and divergent configurations is not meaningful in this range of σ.

(iii) Intermediate density economies: $\frac{1-(1-\alpha)\mu}{4} \leq \sigma < \frac{1+(1-\alpha)\mu}{4}$

In this case, the no-arbitrage condition (4) requires $\sigma \geq \sigma_2(k)$, where $\sigma_2(k)$ is given by

$$\sigma_2(k) \equiv \frac{1-k}{4}\{1 + (1-\alpha)\mu\}. \tag{28}$$

It can be verified that

$$\sigma_2(k) \gtreqless \frac{1-(1-\alpha)\mu}{4} \quad \text{if} \quad k \lesseqgtr \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}, \tag{29}$$

which means that the assumed transport network is an equilibrium for the entire range, $\frac{1-(1-\alpha)\mu}{4} \leq \sigma < \frac{1+(1-\alpha)\mu}{4}$, when the two regions are far from each other ($k \geq \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$). Since the round-about transportation makes sense only if the two regions are sufficiently close, this result is not surprising. On the other hand, when the two regions are relatively close ($k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$), the transport network equilibrium requires $\sigma > \sigma_2(k)(> \frac{1-(1-\alpha)\mu}{4})$, that is, density economies must be sufficiently small. It can be understood as follows. Consider an extreme case in which density economies are very small: $\sigma \gg \sigma_2(k)$, so that $T(1, I_E^*) \simeq 1$ and $T(1, I_W^*) \simeq 1$. Thus, the cost of directly transporting a unit of intermediate good to either East or West is approximately one, while that of the round-about transportation is approximately $1 + k$. Hence, neither region would develop a hub. However, as σ decreases (i.e., density economies become larger), the transport rates on both the North-East and North-West tend to decrease. But, due to the manufacturing concentration (and hence, a larger transport demand) in East, the transport rate on the trunk link (which in the present setting is the North-East link) decreases more than that on the other (the North-West link). If density economies are sufficiently large ($\sigma < \sigma_2(k)$), and if the two regions are geographically close ($k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$), then it becomes possible for a shipper in West to decrease the transport cost by using the round-about route via East.

From the comparison between (27) and (29), we can see that when $k \geq \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$, the assumed transport pattern is always consistent with a transport network equilibrium. Since in this case the two regions are very far from each other, the round-about transportation does not make sense even under very large density economies. On the other hand, when $k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$, the direct transport pattern is a transport network equilibrium if and only if density economies are very large or very small, i.e., $\sigma \leq \sigma_1(k)$ or $\sigma_2(k) \leq \sigma$, where $\sigma_1(k) < \frac{1-(1-\alpha)\mu}{4} < \sigma_2(k)$ in this case.

The result obtained above is illustrated in Figure 5 which depicts the equilibrium share of the manufacturing firms in West (the agricultural region) which uses the direct North-West route for each

value of σ under the given set of other parameter values. The curve labeled "equilibria without a hub" shows the range of σ in which the divergent equilibrium without a hub exists.[14],[15] In sum, we have the next proposition:[16]

Proposition 2 *There uniquely exists a (symmetric) pair of divergent equilibria without a hub if and only if one of the following (mutually exclusive) conditions are met: (i) $k \geq \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$; (ii) $k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$ with $\sigma \leq \sigma_1(k)$ or $\sigma_2(k) \leq \sigma$.*

$$\overline{\text{Figure 5}}$$

4.3.2 Case with a hub formation

Next, we consider the divergent equilibrium with a hub. Without loss of generality, East is assumed to be a hub. Then, the equilibrium manufacturing share in each region is given by (21). Note that for a hub to form, density economies must be effective on the North-East link, otherwise shippers in West will want to procure intermediate goods directly from North, that is, the no-arbitrage condition (5) is not satisfied. It follows that the sum of transport cost for all intermediate goods shipped on the North-East link should be σ. Since in total, one unit of intermediate good is produced in North in equilibrium, the amount that reaches East is $1 - \sigma$. Thus, taking into account that $p_N^I = w_N$, the delivered price of intermediate good in East is given by

$$p_E^I = w_N/(1-\sigma). \tag{30}$$

If I_W units are shipped further from East to West, the delivered price of intermediate good in West becomes

$$p_W^I = p_E^I \{1 + T(k, I_W)\}. \tag{31}$$

By using (30) and (31) together with (6), (8), (10) and (11), the demand for the intermediate good in each region can be calculated as

$$I_E^* = \frac{1-\sigma}{2}\{1 + (1-\alpha)\mu\}, \tag{32}$$

$$I_W^*\{1 + T(k, I_W^*)\} = \frac{1-\sigma}{2}\{1 - (1-\alpha)\mu\}. \tag{33}$$

Whether or not density economies are effective on the East-West link depends on the relative values of σ and k. Obviously, density economies work when σ is sufficiently small:

$$I_W^* \lessgtr \sigma \quad \text{if} \quad \sigma \gtrless \sigma_3(k), \tag{34}$$

where

$$\sigma_3(k) \equiv \frac{1 - (1-\alpha)\mu}{2(1+k) + 1 - (1-\alpha)\mu}. \tag{35}$$

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[14] The parameter values used here are the same as those in Figure 4. The same remark in footnote 12 applies.

[15] In the figure, obviously, when the divergent equilibrium without a hub exists, the share of manufacturing firms in West using the direct route is 1.

[16] We have a symmetric pair of equilibria by interchanging the roles of East and West.

Finally, we need to see if the transport pattern under the above configuration is a transport network equilibrium. This time, the relevant no-arbitrage condition is (5). Eq.(34) implies that the condition should be examined in the two cases: $\sigma \leq \sigma_3(k)$ and $\sigma > \sigma_3(k)$. By comparing (33)-(35) and (5), it can be seen that for $\sigma \leq \sigma_3(k)$ [resp., $\sigma > \sigma_3(k)$], the range of σ which satisfies the no-arbitrage condition is limited to $\sigma \leq \frac{1}{2} \frac{1-(1-\alpha)\mu}{1-(1-\alpha)\mu+2k}$ [resp., $\sigma \leq \frac{1-k}{2}$]. It can be verified that $\sigma_3(k) \lessgtr \frac{1}{2} \frac{1-(1-\alpha)\mu}{1-(1-\alpha)\mu+2k}$ [resp., $\sigma_3(k) \lessgtr \frac{1-k}{2}$] if and only if $k \lessgtr \frac{1+(1-\alpha)\mu}{2}$. It basically says that in order for a hub to form in equilibrium, density economies must be sufficiently large (relatively to k) to ensure that the round-about transportation is indeed cost-saving for shippers in West. The threshold value of σ can be expressed as a function of k as follows:

$$\sigma_4(k) \equiv \begin{cases} \frac{1-k}{2} & \text{if} \quad k < \frac{1+(1-\alpha)\mu}{2} \\ \frac{1}{2} \frac{1-(1-\alpha)\mu}{1-(1-\alpha)\mu+2k} & \text{if} \quad k \geq \frac{1+(1-\alpha)\mu}{2} \end{cases} . \tag{36}$$

Thus, the no-arbitrage condition (5) is satisfied if

$$\sigma \leq \sigma_4(k). \tag{37}$$

Note that $\sigma_4(k)$ is a (strictly) decreasing function of k. When the two regions are farther away, a larger degree of density economies are required to sustain the hub formation, since the benefit of traffic bundling must be large enough to overcome the long-haul cost along the round-about route for shippers in West (the non-hub region). The curve labeled "equilibria with a hub" in each diagram in Figure 5 shows the range of σ in which the divergent equilibrium with a hub exists. The next proposition summarizes the result for the case of a hub formation:[17],[18]

Proposition 3 *There uniquely exists a (symmetric) pair of divergent equilibria with a hub if and only if $\sigma \leq \sigma_4(k)$ for each $k > 0$.*

Moreover, it can be shown that

$$\sigma_2(k) < \sigma_4(k). \tag{38}$$

By this, we have the next corollary to Propositions 2 and 3 (see also Figure 5):

Corollary 1 *When no divergent equilibrium without a hub exists, there always exists a divergent equilibrium with a hub.*

Proposition 2 indicated that the divergent configuration without a hub is not an equilibrium if the two regions are geographically close ($k < \frac{2(1-\alpha)\mu}{1+(1-\alpha)\mu}$) and the degree of density economies are intermediate ($\sigma_1(k) < \sigma < \sigma_2(k)$). In this case, the configuration violates the no-arbitrage condition (4): shippers in the agricultural region will want to use the round-about route via the manufacturing region. Corollary 1 assures that in such a case, indeed, there exists a divergent equilibrium with a hub.

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[17] In the figure, when a divergent equilibrium with a hub exists, all intermediate goods imported by West are shipped via East. Hence the share of manufacturing firms using the direct route is 0.

[18] As the diagrams indicate, it can be shown that whenever a divergent equilibrium with a hub and that without a hub coexist, there exists a unique divergent equilibrium with a mixed transport pattern between them.

5 Stability of equilibria

In this section, we study the stability of equilibria derived in the previous section. To this end, we modify our model slightly for the ease of analysis. Namely, we divide the manufacturing sector in each of East and West into two sectors depending on the transport route of intermediate goods. This way, we can examine simultaneously the stability of both the transport and specialization patterns of a given equilibrium. Each individual workers is assumed to belong to one of the two manufacturing sectors or the agricultural sector in each region. Let $n_{iD}(t)$ [resp., $n_{iI}(t)$] be the worker share of manufacturing sector in region $i \in \{E, W\}$ which imports intermediate goods directly [resp., indirectly] from North, and $\mathcal{N}(t) \equiv \{n_{ED}(t), n_{EI}(t), n_{WD}(t), n_{WI}(t)\} \in [0,1]^4$ the distribution of manufacturing shares in the economy at a given point in time $t \geq 0$.[19],[20] We consider a small perturbation to the equilibrium industrial composition and transport pattern of intermediate goods in each region, and examine if the equilibrium is recovered via an appropriate adjustment mechanism. Here, we assume the following dynamics which is often used in the field of economic geography (e.g., Krugman, 1993b):

$$\dot{n}_{ij} = \delta(u_{ij} - \overline{u}_i) n_{ij}, \qquad i \in \{E, W\}, \ j \in \{D, I\}, \tag{39}$$

where $\dot{n}_{ij}$ is a time derivative of n_{ij}, δ a positive constant, u_{ij} the *temporary-equilibrium* utility level of workers in sector j in region i, and $\overline{u}_i \equiv \sum_{j \in \{A,D,I\}} n_{ij}(t) u_{ij}(t)$ the average temporary-equilibrium utility level in region i. The temporary-equilibrium utility levels at each t are determined by using the equilibrium model in Section 4 given the shares of manufacturing workers in each region $n_{ij}(t)$ [refer to Appendix A.1 for the temporary equilibrium conditions]. The adjustment mechanism, (39), assumes that the workers are attracted to the sector associated with a higher utility level, and that the larger the worker share of the sector, the greater the increment of the worker share of the sector. We say that the equilibrium with the distribution of manufacturing shares, $\mathcal{N}^* \equiv \{n_{ED}^*, n_{WI}^*, n_{WD}^*, n_{WI}^*\}$, is (locally and asymptotically) stable, if there exists a small neighborhood of $\mathcal{N}^*$ such that from any point in the neighborhood, the adjustment mechanism (39) leads $\mathcal{N}(t)$ back to the original equilibrium $\mathcal{N}^*$, i.e., $\lim_{t \to \infty} n_{ij}(t) = n_{ij}^*$ for each $i \in \{E, W\}$ and $j \in \{D, I\}$.

Leaving the proof to Appendix A.2, we state the result of the stability analysis as follows:[21]

Proposition 4 *(i) A convergent equilibrium is stable if and only if $\sigma > \alpha/2$ and $\sigma \leq 1/4$; (ii) a divergent equilibrium without a hub is stable if and only if $\sigma < \frac{1+(1-\alpha)\mu}{4}$; (iii) any divergent equilibrium with a hub is stable.*

The stability of an equilibrium depends on the degree of transport density economies, σ, and the size of transport demand. The transport demand is limited by the supply capacity of intermediate goods in North as well as the content of intermediate goods in the manufactured good (represented by $1-\alpha$). An intuition behind the stability condition of the convergent equilibrium can be given as follows. Let us focus on the case in which density economies are effective on the transport links, i.e., $\sigma < 1/4$.

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[19] The density economies are assumed to work only for the shipments moving in the same direction on a given link.

[20] The share, $n_{iA}(t)$, of the agricultural sector in region i at a given time t is given by $1 - \sum_{j \in \{D,I\}} n_{ij}(t)$.

[21] Assumption 1 is irrelevant for (i).

In this context, the specialization patterns of East and West are identical (refer to Proposition 1). Notice that for an equilibrium to be stable, an expansion of the manufacturing employment in any region must result in a decrease in the wage rate of the manufacturing workers in the region (relative to that of the agricultural workers). Consider, at a convergent equilibrium, a small shift of labor from agriculture to manufacture in East. Since the manufacturing production is homothetic, firms would increase their intermediate inputs in proportion to the labor increase, under given factor prices at the equilibrium level. However, this increase in the demand for intermediate goods affects factor prices in the following two ways: (i) it raises the f.o.b. price of the intermediate good by competition for the good between the two regions, and (ii) it reduces the transport margin because of economies of density. Thus, the direction of change in the delivered price, p_E^I, of intermediate good in East is determined by the relative sizes of the two effects.

If σ is very small, the share of transport margin in the delivered price is also small, which makes the second effect negligible. In this case, the intermediate input in East increases less than proportionately to the labor input because of the rise in p_E^I by the first effect. As σ gets larger, the second effect becomes more significant and reduces p_E^I, which induces more demand for the intermediate good. But, obviously the intermediate input in East cannot increase beyond an upper bound which is given by the supply capacity in North. As the demand in East gets closer to the bound, the delivered price will rise unboundedly if the intermediate good is essential for manufacturing (as in the case of the present model). As a result, if $\sigma > \alpha/2$, firms do not increase (rather, they tend to decrease) their intermediate input with a given increase in the labor input. Thus, the delivered price of intermediate good rises, and hence, the wage rate of the manufacturing workers declines in East. In either case, the shifted workers will move back to the agricultural sector, which means that the equilibrium is stable. Since the factor share of the intermediate good is given by $1 - \alpha$, a smaller α implies that the manufacturing production uses the intermediate good more intensively. This in turn means that the given supply capacity in North is relatively small, and accordingly that the upper bound of σ is lower.

For result (ii) in Proposition 4, by (24) and (25), if $\sigma \geq \frac{1+(1-\alpha)\mu}{4}$, density economies are not effective given any feasible specialization pattern. As we have discussed in Sections 4.2 and 4.3.1(ii), the specialization patterns of East and West are arbitrary as long as the aggregate demand and aggregate supply match, that is, as long as (12) and (15) are satisfied. In that sense, no equilibrium is stable in this range of σ. If economies of transport density are sufficiently large ($\sigma < \frac{1+(1-\alpha)\mu}{4}$), then it becomes possible for one transport route to have an absolute cost advantage over the other. This advantage will not vary with a small perturbation of transport pattern, so that the manufacturing localization is stable. When a hub is formed, this advantage in the manufacturing region is even greater, and hence, the equilibrium is stable as stated in result (iii).

6 Welfare

Let us denote by U^c the (common) utility level of East and West in a convergent equilibrium, and by $U_M^{no\text{-}hub}$ and U_M^{hub} [resp., $U_M^{no\text{-}hub}$ and U_A^{hub}] the utility levels of the manufacturing region [resp., agricultural region] respectively in a divergent equilibrium without a hub and that with a hub. The

utility levels can be calculated by using (1), (10), (11) and an appropriate expression for the price of intermediate goods ((16), (30) or (31)). To get the basic insight, let us focus on the values of σ and k under which all of the convergent and divergent equilibria coexist. By Propositions 1, 2, 3, and (38), the desired situation is guaranteed if $\sigma \leq \sigma_4(k)$. In this context, the utility level of workers in each region can be ordered as follows:

$$U_A^{no\text{-}hub} < U^c < U_M^{no\text{-}hub} < U_M^{hub}. \tag{40}$$

The order of U_A^{hub} depends on the distance between East and West, k. The interpretation can be given as follows.

If a hub is not formed, the manufacturing region improves its welfare level (relatively to that of the convergent equilibrium under the same parameter values) at the expense of the agricultural region. It is because the manufacturing region enjoys a lower delivered price of intermediate good, and hence, a lower price of manufactured good, while the agricultural region cannot benefit from density economies on the trunk link between North and the manufacturing region, since the region does not use it (the first two inequalities in (40)). If a hub is formed, the welfare level in the manufacturing region is the highest, since density economies on the trunk link are at their maximum (the last inequality in (40)).

The agricultural region, however, may or may not improve its welfare level by the formation of a hub. If East and West are geographically close, then the benefit of traffic bundling due to density economies outweighs the cost of the round-about transportation via the hub. In fact, if k is very small, we even have $U_M^{no\text{-}hub} < U_A^{hub}$. But, if the two regions are far from each other, then the long-haul costs using the round-about route will be very expensive. In that case, the workers in the agricultural region are better off in the divergent equilibrium without a hub under the same set of the parameter values: $U_A^{hub} < U_A^{no\text{-}hub}$. In general, as k decreases, the ranking of U_A^{hub} goes up from the last to only next to U_M^{hub} in (40).[22]

7 Concluding remarks

In the paper, we presented a general equilibrium model of a spatial economy in which the interregional trade pattern and the structure of the interregional transport network are endogenously determined in the presence of economies of transport density. In particular, we have shown that economies of transport density can be the key source of industrial agglomeration. Agglomeration economies are generated by the circular causation between economies of transport density and industrial localization: a greater concentration of industries in a given region generates a larger transport flow through the region, and lowers the cost of transportation via the region due to density economies, which in turn attracts a larger number of firms to the region. A successful region eventually emerges as an interregional transport hub as well as an industrial center.

Although we primarily focused on the positive aspects, some normative implications of the presence of economies of transport density are also obtained. Namely, the welfare levels of all regions can be enhanced by the formation of a transport hub when the regions are geographically close. Otherwise,

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[22] Details of the welfare analysis can be obtained from the authors upon request.

the hub region improves its welfare at the expense of the non-hub regions. Our stability analysis further indicated that on one hand, noncooperative transport developments by regions might end up with an excess investment on the transport infrastructure. It is because not all of the improved transport routes can attract transport demand, since the transport routes tend to be self-integrated in the presence of economies of transport density. Coordination among regions in financing transport development (especially in developing a hub) may possibly bring about welfare improvement to all regions.

A Appendix

A.1 Temporary equilibrium

We say that *the economy is in temporary equilibrium if given the trade and transport patterns, all markets clear, firms earn zero profits, manufacturing workers employed by firms which use the same transport route achieve the same utility level, and agricultural workers in each region likewise achieve the same utility level.* For convenience, we say that manufacturing firms in region i which use transport route ℓ belong to sector i-ℓ. In this context, the delivered prices of intermediate good in region i can be written as follows:

$$p_{iD}^I = w_N\{1 + T(1, I_i)\}, \tag{41}$$

$$p_{iI}^I = p_{jD}^I\{1 + T(k, I_{iI})\}, \quad i \neq .j \in \{E, W\}, \tag{42}$$

where

$$I_i \equiv I_{iD} + I_{jI}\{1 + T(k, I_{jI})\}, \tag{43}$$

Next, the market clearing of intermediate goods, (8), is rewritten as follows:

$$I_E\{1 + T(1, I_E)\} + I_W\{1 + T(1, I_W)\} = 1. \tag{44}$$

Equation (6) is rewritten as follows:

$$w_{iD} = \frac{\alpha}{1-\alpha}\frac{I_{iD}}{n_{iD}}w_N\{1 + T(1, I_i)\}, \qquad i \in \{E, W\}, \tag{45}$$

$$w_{iI} = \frac{\alpha}{1-\alpha}\frac{I_{iI}}{n_{iI}}w_N\{1 + T(k, I_{iI})\}\{1 + T(1, I_j)\}, \quad i \neq j \in \{E, W\}. \tag{46}$$

The market clearing of the agricultural good assures the following equality:

$$w_N = \frac{1-\alpha}{1/\mu - 1}(2 - n_{ED} - n_{EI} - n_{WD} - n_{WI}). \tag{47}$$

Next, if $n_{iD}, n_{iI} > 0$, the no-arbitrage condition for manufactured good means that $p_i^M \equiv p_{iD}^M = p_{iI}^M$ ($i = E, W$), which in turn implies $w_{iD}/w_{iI} = (p_{iI}^I/p_{iD}^I)^{(1-\alpha)/\alpha}$ by (7), while $w_{iD}/w_{iI} > $ [resp., $<$] $(p_{iI}^I/p_{iD}^I)^{(1-\alpha)/\alpha}$ if $n_{iD} > 0$ and $n_{iI} = 0$ [resp., $n_{iD} = 0$ and $n_{iI} > 0$]. By using (41) and (42), we obtain the following relation:

$$\left(\frac{w_{iD}}{w_{iI}}\right)^{\alpha/(1-\alpha)} - \frac{1 + T(k, I_{iI})}{1 + T(1, I_i)}\{1 + T(1, I_j)\} \begin{array}{c} > \\ = \\ < \end{array} \Leftrightarrow \begin{array}{c} n_{iD} > 0, \ n_{iI} = 0 \\ n_{iD}, n_{iI} > 0 \\ n_{iD} = 0, \ n_{iI} > 0 \end{array} .. \tag{48}$$

This implies that the labor cost differential between the two sectors, i-D and i-I, must be compensated for by the differential of intermediate good prices, if both sectors D and I are active in region i. By using the derivation similar to that for (12), the market clearing of manufactured good can be obtained as follows:

$$\sum_{i \in \{E,W\}} \sum_{j \in \{D,I\}} N_{ij}w_{ij} = \alpha\mu\left(nw_N + \sum_{i \in \{E,W\}} \sum_{j \in \{A,D,I\}} n_{ij}w_{ij}\right), \tag{49}$$

where $w_{iA} = 1$ ($i = E, W$). Finally, if both of East and West are exporting manufactured goods, then we need to add the no-arbitrage condition for the manufactured good:

$$p_E^M = p_W^M. \tag{50}$$

A.2 Proof of Proposition 4

For the equilibria without mixed transport route, i.e., $n_{iD}^* n_{iI}^* = 0$ $(i = E, W)$, by applying the no-arbitrage condition, (4) or (5), the stability analysis can be simplified by reducing the relevant dimension of dynamics (39). Namely, we have the following preliminary result:

Lemma 1 *Suppose that an equilibrium is stable when no perturbation of transport pattern is allowed (i.e., $N_{ij}(t) = 0$ is assumed for any $t > 0$ if $N_{ij}^* = 0$, where $i = E, W$, $j = D, I$). Then, provided that the no-arbitrage condition, (4) or (5), for intermediate goods is satisfied with a strict inequality, the equilibrium is also stable even if an arbitrarily small perturbation of transport pattern is considered.*

Proof: Since proofs are similar for different equilibrium configurations, we only prove the lemma for a divergent equilibrium with a hub in which East is the hub (i.e., $n_{ED}^*, n_{WI}^* > 0$; $n_{EI}^* = n_{WD}^* = 0$). Suppose, the equilibrium $\mathcal{N}^*$ is stable under the adjustment mechanism (39) with a constraint $n_{EI}(t) = n_{WD}(t) = 0$ for all $t > 0$. Next, suppose that in East [resp., West], a fraction of the population $\varepsilon_{EI} > 0$ [resp., $\varepsilon_{WD} > 0$] were made to import intermediate goods indirectly via West [resp., directly from North] and assemble the manufactured good. Let $n_{ED}^* - \varepsilon_{ED} > 0$ [resp., $n_{WI}^* - \varepsilon_{WI} > 0$] be the share of the manufacturing sector in East [resp., West] which imports intermediate goods directly from North [resp., indirectly via East], and let $n_{EA}^* - \varepsilon_{EA} > 0$ [resp., $n_{WA}^* - \varepsilon_{WA} > 0$] be the agricultural share in East [resp., West] after this perturbation. Then, by definition, we have that $\varepsilon_{EI} = \varepsilon_{ED} + \varepsilon_{EA}$ and $\varepsilon_{WD} = \varepsilon_{WI} + \varepsilon_{WA}$. Let us define that $\varepsilon = |\varepsilon_{EI}| + |\varepsilon_{WD}|$. Given this new division of labor, the adjustment process is set off following the dynamics defined by (39), which is to be solved given the initial conditions, $n_{ED}(0) = n_{ED}^* - \varepsilon_{ED}$, $n_{EI}(0) = \varepsilon_{EI}$, $n_{WD}(0) = \varepsilon_{WI}$, and $n_{WI}(0) = n_{WI}^* - \varepsilon_{WI}$. Now, suppose that no-arbitrage condition for intermediate goods, (5), holds with strict inequality under $\mathcal{N}^*$. Let us denote by $\tilde{w}_{EI}(\mathcal{N}^*)$ and $\tilde{w}_{WD}(\mathcal{N}^*)$, the maximum wage rate under $\mathcal{N}^*$ which can be attained in East and West respectively by deviating from the current transport pattern of intermediate goods. Then, it must hold that $\tilde{w}_{EI}(\mathcal{N}^*) < w_{ED}^* \equiv \tilde{w}_{ED}(\mathcal{N}^*)$ and $\tilde{w}_{WD}(\mathcal{N}^*) < w_{WI}^* \equiv \tilde{w}_{WI}(\mathcal{N}^*)$. Therefore, provided that the temporary equilibrium wage rates, w_{ij} $(i = E, W, j = A, D, I)$, are continuous in $\mathcal{N} = (N_{ED}, N_{EI}, N_{WD}, N_{WI})$ in a neighborhood of $\mathcal{N}^* = (N_{ED}^*, 0, 0, N_{WI}^*)$, we can find a sufficiently small ε such that for any initial perturbation of labor division, $\Delta \mathcal{N} \equiv (\varepsilon_{ED}, -\varepsilon_{EI}, -\varepsilon_{WD}, \varepsilon_{WI})$ such that $w_{EI}(0) < \bar{w}_E(0)$ and $w_{WD}(0) < \bar{w}_W(0)$, where $\bar{w}_E(0) \equiv \sum_{i \in \{D,I\}} N_{Ei}(0) w_{Ei}(0) + N_{EA} w_{EA} \approx N_{ED}(0) w_{ED} + N_{EA}(0) w_{EA}(0) \approx w_E^*$ and $\bar{w}_W(0) \equiv \sum_{i \in \{D,I\}} N_{Wi}(0) w_{Wi}(0) + N_{WA} w_{WA} \approx N_{WI}(0) w_{WI} + N_{WA}(0) w_{WA}(0) \approx w_W^*$. Then, again by the continuity of temporary wage functions, this implies that there exists a number $h > 0$ such that $w_{EI}(t) < \bar{w}_E(t)$ and $w_{WD}(t) < \bar{w}_W(t)$ for all $t < h$, which in turn implies that $\dot{n}_{EI}(t), \dot{n}_{WD}(t) < 0$ for all $t < h$. Since this result always holds when we choose ε sufficiently small, and the equilibrium is stable when the transport pattern is fixed, we can conclude that it is also stable even if we allow an arbitrarily small perturbation of transport pattern. Q.E.D.

Since the proofs for (i), (ii) and (iii) are similar, we only prove (i). By Lemma 1, we only need to prove the stability of a symmetric equilibrium when intermediate goods are allowed to be transported

directly to East and West from North. Moreover, since there is no migration of workers across regions, the stability of an equilibrium under the adjustment mechanism (39) is equivalent to that under the modified adjustment mechanism of (39) with u_{ij} and $\overline{u}_i$ replaced by w_{ij} and $\overline{w}_i$ ($\equiv \sum_{j \in \{A,D,I\}} n_{ij} w_{ij}$), respectively. Let n_i be the manufacturing share in region i. Then, the linearization of dynamical system (39) can be written as follows:

$$\dot{\mathbb{N}}(\Delta) = \Phi(\mathcal{N}^*)\mathbb{N}(\Delta), \tag{51}$$

where $\mathbb{N}(\Delta)$ is a 2-by-1 matrix whose first [resp., second] element is $n_E - n_E^*$ [resp., $n_W - n_W^*$]; $\Phi(\mathcal{N}^*)$ is a (E,W)-by-(E,W) matrix whose ij-th element is $\delta(1 - n_i^*)n_i^* w_{ij}^{*\prime}$, where $w_{ij}^{*\prime} \equiv dw_i/dn_j|_{\mathcal{N}=\mathcal{N}^*}$. For a convergent equilibrium, we have

$$w_{ij}^{*\prime} = w_{ji}^{*\prime}, \qquad i,j \in \{E,W\}. \tag{52}$$

It follows that the eigenvalues, λ_1 and λ_2, of $\Phi(\mathcal{N}^*)$ can be written as follows:

$$\lambda_i = w_{EE}^{*\prime} + (-1)^i w_{EW}^{*\prime}, \qquad i \in \{1,2\}. \tag{53}$$

Since intermediate goods are directly transported to East and West, we have

$$I_{iD} = I_i, \qquad i \in \{E,W\}. \tag{54}$$

Suppose $\sigma < 1/4$. Then, economies of density are effective on each transport link in the neighborhood of the equilibrium, i.e.,

$$T(1, I_i) = \sigma/I_i, \qquad i \in \{E,W\}. \tag{55}$$

By substituting (54) and (55) into (41), (44) and (45), we obtain the following:

$$p_i^I = w_N(1 + \sigma/I_i), \qquad i \in \{E,W\}, \tag{56}$$

$$w_i = \frac{\alpha}{1-\alpha}(w_N/n_i)(I_i + \sigma), \qquad i \in \{E,W\}, \tag{57}$$

$$I_E + I_W = 1 - 2\sigma, \tag{58}$$

where w_N is given by (47). Next, by substituting (56) and (57) into the no-arbitrage condition for the manufactured good, (50), and totally differentiating I_E with respect to n_E, we obtain the following:

$$dI_E/dn_E|_{\mathcal{N}=\mathcal{N}^*} = -dI_E/dn_W|_{\mathcal{N}=\mathcal{N}^*} = (1/4)(1/\mu - 1 + \alpha)(1 - 2\sigma)/(\alpha - 2\sigma), \tag{59}$$

where (58) is taken into account. Now, by differentiating (57) with respect to n_E and n_W respectively, and evaluating at the convergent equilibrium, we obtain the following:

$$w_{EE}^{*\prime} = -1/n_E^* + \frac{\alpha}{1-\alpha}\left\{w_{NE}^{*\prime}(I_E^* + \sigma) + w_N^* dI_E/dn_E|_{\mathcal{N}=\mathcal{N}^*}\right\}/n_E^*, \tag{60}$$

$$w_{EW}^{*\prime} = \frac{\alpha}{1-\alpha}\left\{w_{NW}^*(I_E^* + \sigma) - w_N^* dI_E/dn_E|_{\mathcal{N}=\mathcal{N}^*}\right\}/n_E^*. \tag{61}$$

By substituting (47) and (59) into (60) and (61), and using (53), we can solve for the eigenvalues:

$$\lambda_1 = (1/\mu - 1 + \alpha)\{(1 - 2\sigma)/(\alpha - 2\sigma) - 1\} \gtreqqless 0 \Leftrightarrow \sigma \lesseqqgtr \alpha/2. \tag{62}$$

$$\lambda_2 = -\{1 - (1-\alpha)\mu\}^2 / \{\alpha\mu(1-\mu)\} < 0, \tag{63}$$

Recall that for $\sigma > 1/4$, any pair of manufacturing shares, (n_E^*, n_W^*), satisfying (12) and (19), is an equilibrium by Proposition 1. Hence, a symmetric equilibrium is unstable in this range of σ. With this, (62) and (63) indicate that when only the direct transportation of intermediate goods to East and West is allowed, a convergent equilibrium is stable if and only if $\sigma > \alpha/2$ and $\sigma \leq 1/4$. This result combined with Lemma 1 completes the proof of (i). Q.E.D.

Reference

Braeutigam, R.R., Daughety, A.F. and Turnquist, M.A., 1982. The estimation of a hybrid cost function for a railroad firm. Review of Economics and Statistics 64, 394-404.

Braeutigam, R.R., Daughety, A.F. and Turnquist, M.A., 1984. A firm specific analysis of economies of density in the U.S. railroad industry. The Journal of Industrial Economics 33, 3-20.

Brueckner, J.K., Dyer, N.J. and Spiller, P.T., 1992. Fare determination in airline hub-and-spoke networks. The Rand Journal of Economics 23, 309-333.

Brueckner, J.K., and Spiller, P.T., 1991. Competition and mergers in airline networks. International Journal of Industrial Organization 9, 323-342.

Campbell, J.F., 1996. A survey of network hub location. Studies in Locational Analysis, 6. 31-49.

Caves, D.W., Christensen, L.R. and Tretheway, M.W., 1984. Economies of density versus economies of scale: why trunk and local service airline costs differ. The Rand Journal of Economics 15, 471-489.

Fujita, M. and Mori, T., 1996. The role of ports in the making of major cities: self-agglomeration and hub-effect. Journal of Development Economics 49, 93-120.

Hendricks, K., Piccione, M. and Tan, G., 1995. The economics of hubs: the case of monopoly. Review of Economic Studies, 62, 83-99.

Japan Logistic System Association, 1996. Report on logistics costs by the type of industry. Unpublished manuscript. Tokyo: Japan Logistic System Association.

Journal of Commerce, 1997. Trade Hrizons 7(2), A-5-A-6 and A-21-A-22.

Konishi, H., 2000. Hub cities: city formation without increasing returns. Journal of Urban Economics 48, 1-28.

Krugman, P., 1993a, The hub effect: or, threeness in international trade. In W.J. Ethier, E. Helpman and J.P. Neary (eds.), Trade Policy and Dynamics in International Trade, Cambridge: Cambridge University Press, 29-37.

Krugman, P., 1993b. On the number and location of cities. European Economic Review, 37, 293-298.

Mills, E.S., 1972, Studies in the structure of the urban economy, Baltimore: Johns Hopkins University Press, Ch.5.

Samuelson, P.A., 1952. The transfer problem and transport costs: the terms of trade when impediments are absent. Economic Journal 62, 165-186.

Louvex, F., Thisse, J.-F. and Beguin, H., 1981. Location theory and transport costs. Regional Science and Urban Economics 12, 529-545.

Shipping Gazette, 1997. Tokyo: The Japan Press, Ltd., September 22.

Usui, S., 1997. An estimation of cost and freight for ocean-going container transport. Unpublished M.A. thesis. School of Informative Science, Tohoku University.
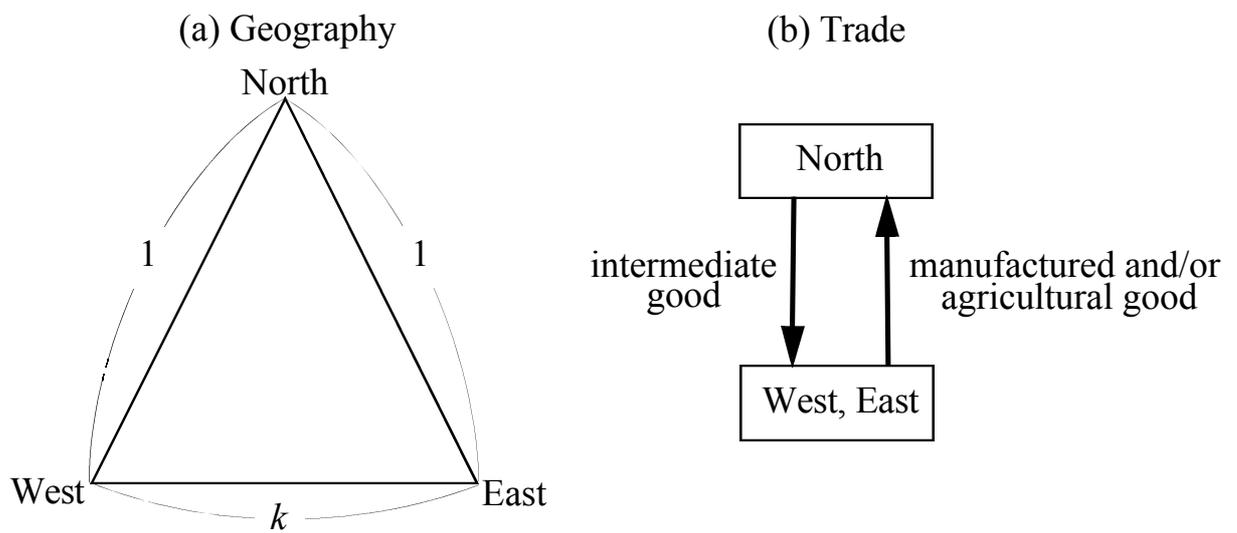
(a) Geography

North

1   1

West   East
          k

(b) Trade

North

intermediate good → West, East

West, East → manufactured and/or agricultural good

Figure 1. Assumed geography and trade pattern in the economy

$T(d,Q)$

d

0 σ Q
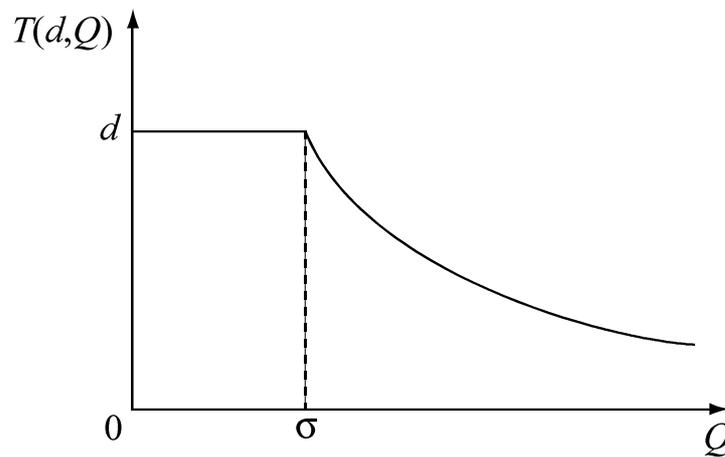
Figure 2. Transport cost with economies of density

(a) Without a hub

North

$T(1,I_W)$ $T(1,I_E)$

West $T(k,0) = k$ East

(b) With a hub

North

$T(1,0) = 1$ $T(1, I_E+I_W[1+T(k,I_W)])$

West $T(k,I_W)$ East
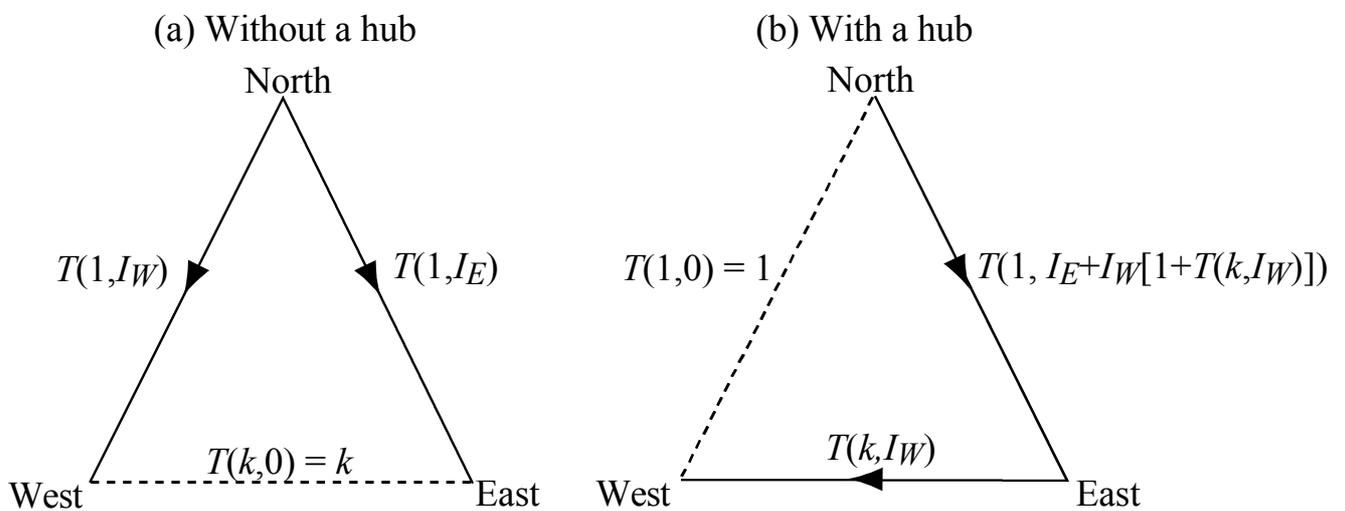
Figure 3. Transport network with and without a hub

$$\frac{n_E^*}{n_E^* + n_W^*}$$

symmetric equilibria

a continuum of
convergent equilibria

σ

$(\alpha = 0.5; \mu = 0.5; \text{large } k = 1.0; \text{small } k = 0.3)$
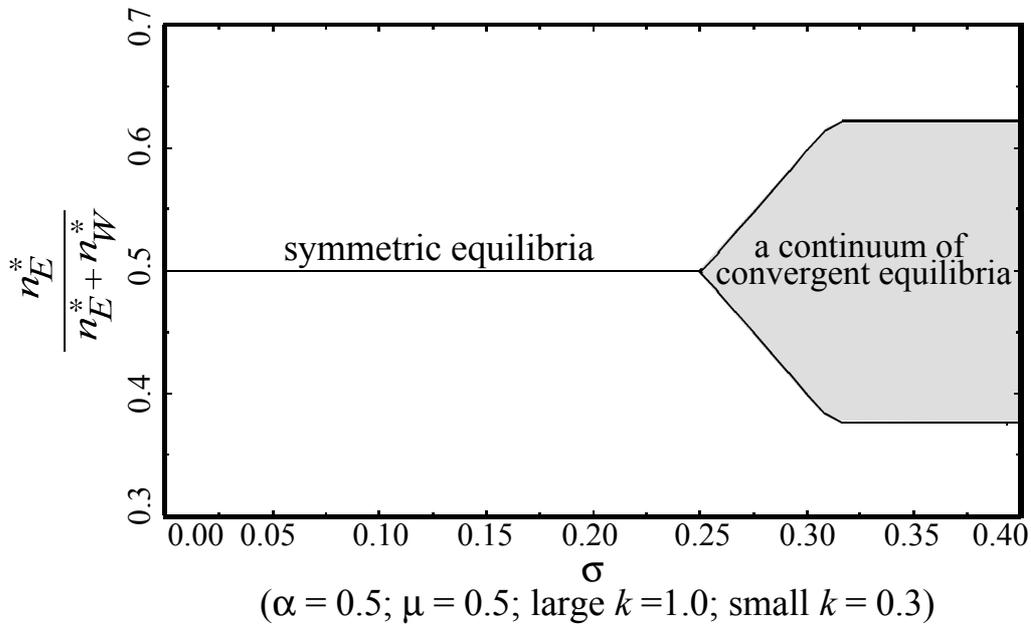
Figure 4. Specialization patterns in convergent equilibria

(a) $k < 2(1-\alpha)\mu / \{1+(1-\alpha)\mu\}$

Share of manufacturing firms using the direct route in the agricultural region

equilibria without a hub σ_1 σ_2 equilibria without a hub

equilibria with a mixed transport route

equilibria with a hub σ_4

σ

(b) $k > 2(1-\alpha)\mu / \{1+(1-\alpha)\mu\}$

Share of manufacturing firms using the direct route in the agricultural region

equilibria without a hub

equilibria with a mixed transport route

equilibria with a hub σ_1
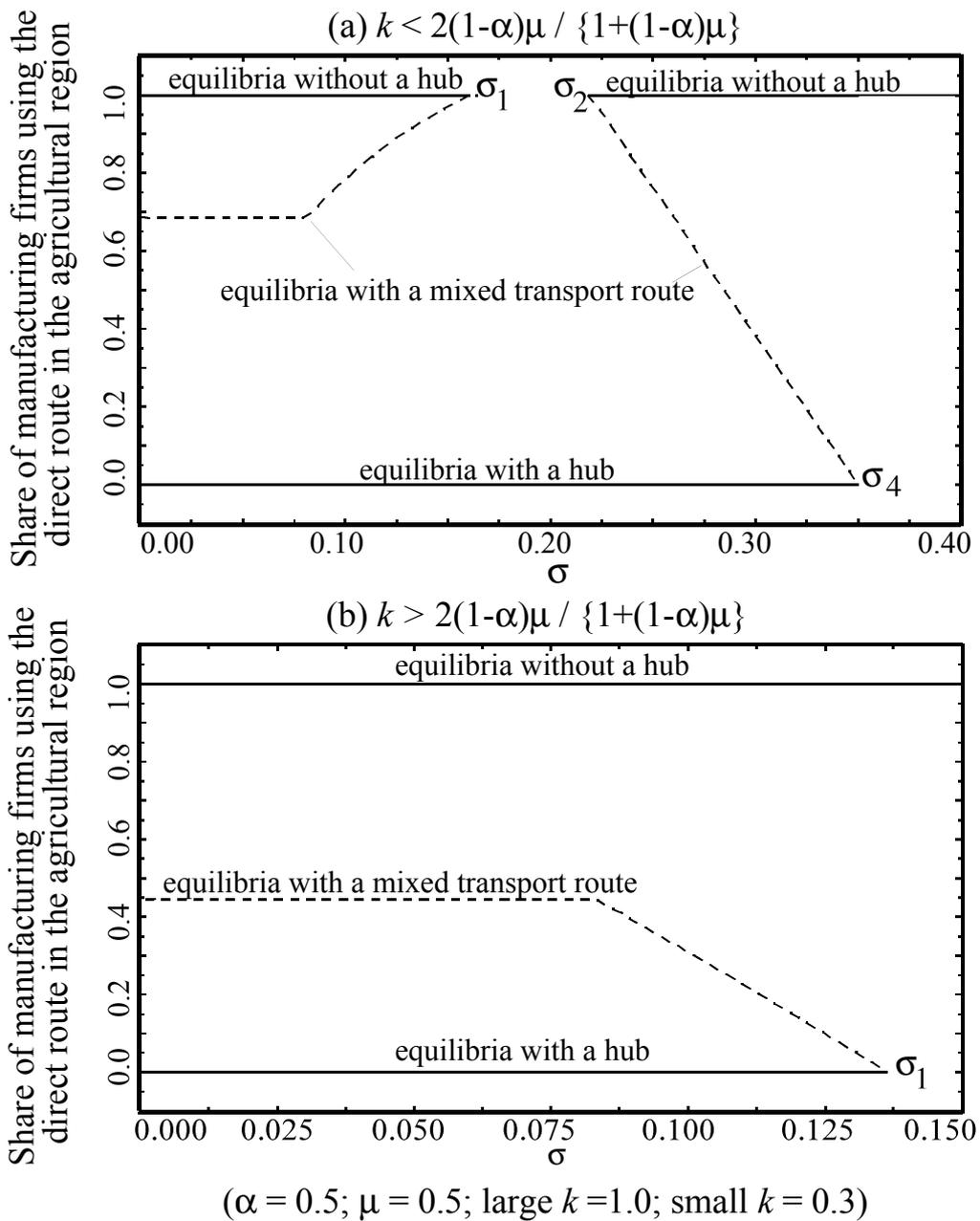
σ

$(\alpha = 0.5; \mu = 0.5; \text{large } k = 1.0; \text{small } k = 0.3)$

Figure 5. Transport patterns in divegent equilibria