CHAPTER 9

Mutual Dependency between Fertility & Socio-economic Factors: Multivariate Time Series Analysis — The Case of Taiwan —

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1. INTRODUCTION

Several numerical techniques or methods of examining causal relationship between fertility and socio-economic factors have been applied to analyze the deterministic factors. When we want to know the time effect on fertility, mutual dependency which is measured by the model based on the stochastic process as the causal relationship is needed. There are many reports where causal relationship between socio-economic factors is analyzed by means of multivariate time series models in the fields of economics. The causal relationship means that the relation which is based on the prediction error from past information to predict the future is defined as well-known Granger's causality. Sims test, Granger test (Granger=Sargent) based on vector autoregressive model (VAR model), relative variance contribution obtained from prediction error of variance decomposition based on vector VAR model, relative power contribution which is a Fouriers transformation relative variance contribution are widely used to test the causality of Granger's sense.

The purpose of this paper is to apply multivariable time series analysis to study the effects of socio-economic factors on total fertility rate and to figure out not only one way direction but also feedback between total fertility rate and socio-economic factors of Taiwan. As application of time series analysis, we measure the mutual dependency between variables by means of relative variance contribution based on VAR model. The idea of the paper is essentially founded on the Noda (1992), and is organized more to the statistical method of multi-variate time series analysis.

The Taiwan economy is characterized by small or medium sizes of enterprises. They are so sensitive to business cycles based on Taiwan's economic fluctuation that a lot of people crowded around the large cities to get jobs at the enterprises in good business and on the contrary return to their native towns in recession. We have a hypothesis that Taiwan's fertility tends to fluctuate along the economic fluctuation though the fertility is declining year by year. The hypothesis is examined in this paper. In Taiwan, total fertility rate started to decline from 1952 and the tendency of declining fertility has been rapid since a start of national family plainning and a structural change happened in about 1975. It is said that main factor of fertility decline is caused by the social and economic background. Therefore it is important to investigate the existence of causal relationship between fertility and socio-economic factors and changing point of the structural change.

Because testing the causal relationship is sensitive to the characteristics of the time period, the choice of the period is an important matter. First we try to figure out the changing point which separate the data into two parts. Then in order to know a structural change of causality between the variables we examine the relative variance contribution each period.

2. FEATURE OF TOTAL FERTILITY RATE AND SOCIO-ECONOMIC FACTORS

Several kinds of time series data were tried for the input data of VAR model. Among the time series data, Noda (1992) selected three variables: Gross Domestic Product at constant price, raitio of higher educational attainment of both sex, and ratio of employed person of both sex as Taiwan socio-ecomomic factors. In this paper, ratio of woman's higher educational attainment is used instead of ratio of higher educational attainment and ratio of employed woman instead of employed person. In particular, export is the key factor determining the extent of ecomonic growth in Taiwan. It is widely said that the rapid expansion of export trade was largely responsible for the ecomonic growth. Moreover export is one of the crucial important matter to strengthen social and economic infrastructural facilities

Table 1 Total Fertility Rate And Socio—Economic Variables in Republic of China

Year	TFR	EXP	EDF(%)	EMPF(%)	GDP	r(%)
1952	6615	12534	3.1194	17.5504	162091	7.7327
1953	6470	15050	3.1125	16.7276	177217	8.4924
1954	6425	11246	3.3423	15.7992	194124	5.7932
1955	6530	14399	3.4738	15.3317	209858	6.8613
1956	6505	16084	3.5042	14.7992	221408	7.2644
1957	6000	18407	3.9557	14.6372	237706	7.7436
1958	6055	18671	4.1549	14.0770	253657	7.3607
1959	5990	22809	4.4598	13.9114	273063	8.3530
1960	5750	25552	4.8016	13.6853	290290	8.8022
1961	5585	32454	5.1780	13.5136	310257	10.4604
1962	5465	33331	5.5560	13.2766	334777	9.9562
1963	5350	43346	6.1434	13.4169	366093	11.8402
1964	5100	55313	6.6037	13.2818	410755	13.4662
1965	4825	68503	6.6888	12.9511	456491	15.0064
1966	4815	81090	7.1787	12.8583	497175	16.3102
1967	4220	93134	9.0464	15.8479	550430	16.9202
1968	4325	117688	9.9811	16.4160	600911	19.5849
1969	4120	146128	10.9092	18.0307	654682	22.3205
1970	4000	187092	11.9138	17.9122	729125	25.6598
1971	3705	249391	13.4577	20.7694	823147	30.2973
1972	3365	332759	14.7338	23.5601	932769	35.6743
1973	3210	413235	16.2216	25.1603	1052467	39.2635
1974	3045	383638	17.4818	25.4373	1064696	36.0326
1975	2830	388742	18.7435	26.0347	1117169	34.7971
1976	3075	526514	20.5428	27.7673	1272017	41.3921
1977	2700	593861	21.9145	28.0721	1401631	42.3693
1978	2710	723353	23.6375	29.6864	1592166	45.4320
1979	2660	766717	24.5196	31.1933	1722309	44.5168
1980	2515	830827	26.3113	30.4820	1848060	44.9567
1981	2455	905744	27.9680	32.0686	1961950	46.1655
1982	2320	924978	29.6575	32.6623	2031623	45.5290
1983	2155	1079089	31.1730	33.7955	2203233	48.9775
1984	2050	1268772	32.7125	34.8869	2436766	52.0679
1985	1885	1299732	34.4118	35.6270	2557447	50.8215
1986	1675	1658744	36.0246	36.1002	2855180	58.0960
1987	1700	1973012	37.5909	37.2363	3207382	61.5147
1988	1850	2084916	38.9695	37.9241	3442826	60.5583
1989	1680	2199711	40.1437	39.9815	3703420	59.3967
1990	1805	2231801	41.4812	39.0648	3892410	57.3373

(Source) Statistical Yearbook of the Republic of China

(Note) TFR: total fertility rate for all women, the rate is per thousand of the estimated mid year population.

EXP: export good and services in expenditure on Gross Domestic Product at 1986 constant prices, the unit is millions of N.T. dollars.

EDF: ratio of woman's higher educational attainment, the ratio$=(uc+s)\times 100/pf$, where uc is university and college, s is senior high school, pf is female population aged 15 years old.

EMPF: ratio of employed women, the ratio$=ef\times 100/pf$, where ef is the employed women.

GDP: Gross Domestic Product in purchaser's price in expenditure on Gross Domestic Product at 1986 constant prices, the unit is millions of N.T. dollars.

r: $r=100\times EXP/GDP$.

and raise the people's standard of living in ten-year plan. According to the Table 1, GDP contains higher ratio of export which is the component of GDP year by year. The ratio is almost 60% by the end of 1980's. Therefore we use export trade as one of the socio-economic factors instead of GDP.

The following notations are used for the variables of fertility and socio-ecomonic annual time series data covered the period from 1952 to 1990 in this paper. TFR: total fertility rate is the sum of the age-specific fertility rate over all age of the childbearing ages. EXP: export of goods and services in expenditure on the gross domestic products at 1986 constant prices. EDF: ratio of woman's higher educational attainment including University, college and senior high school. EMPF: ratio of employed woman. The four variables are shown in Table 1.

As stationary series are needed to apply the VAR model, first of all we must remove trends from all of the variables: the total fertility rate and socio-economic factors which have definitely upward or downward trends. We use two well known methods to detrend variables, one is a polynomial regression model and the other is a differencing. It is known that data detrended by differencing has a property of emphasizing short term variation and residuals from polynomial regression model emphasize long term variation. We try to detrend the variables by using two methods and to figure out the main characteristics of the detrended data..

Firstly, we examine detrended series by the polynomial regression model. The four variables are ploted in Figure 1. It seems that two variables TFR and EMPF have almost linear trends though EMPF has a changing point at 1966, on the contrary EXP and EDF are recognized to have exponential trends. As concerns the variables EXP and EDF we have evidence to use first order polynomial regression model to remove the exponential trend. We assume that x_t is an observation and that x_t^* is a theoretical value which can be written to fit upward exponencial trend at the time t.

$$x_t^* = x_0(1+r)^t$$

where x_0 is an initial value and r is an average growth rate. If the rate is small number, log-transformation of the trend can be approximately represented by first order polynomial of time t.

$$p(t) = \log x_t^* = \log x_0 + \log(1+r)^t$$
$$= a + rt.$$

where a is a constant term $\log x_0$ and $\log(1+r) \sim r$. We assume the following regression model.

$$\log(x_t) = p(t) + y_t.$$

The residuals y_t of EXP and EDF are defined in this form as the detrended series which we want to use for the input data of VAR model. TFR and EMPF are applied to the equation without log transformation. Table 2 shows that the declining rate of TFR is -3.05 and average growth rate of EXP, EDF and EMPF are 15.44, 7.68 and 3.22 respectively in the period: 1952 - 1990. EXP which is one of the motive power for Taiwan's high economic growth rate has the highest average growth rate, EDF has the second highest growth rate among them.

Time plots of the residuals from the first order polynomial regression model are shown in the Figure 2. To show the effect of socio-economic factors on total fertility rate, EXP and TFR, EDF and TFR, EMPF and TFR are ploted together in a graph. TFR is ploted with minus sign to fit the long term fluctuation in the figure. The long term fluctuations of them seems to have almost same patterns.

In order to examine cyclical period of fluctuations of the four variables, we try the spectral analysis.[1] Roughly speaking when spectral density function f(w) has a peak at frequency $w=w_j$, a large amount of variance of x_t comes from the variance σ^2_j. In other words, existence of a peak of f(w) at frequency w_j expresses that variation of x_t can be accounted for a periodic function with frequency w_j. Spectral density function of the four variables detrended by first order polynomial regression model have peaks at low frequencies. The evidence shows that the residuals of the four variables have still long term cyclical variations in them. Removing the long cyclical variation, we postulate three and five order polynomial regression models,

$$p(t) = a_0 + a_1 t + \cdots + a_j t^j \quad j=3,5$$

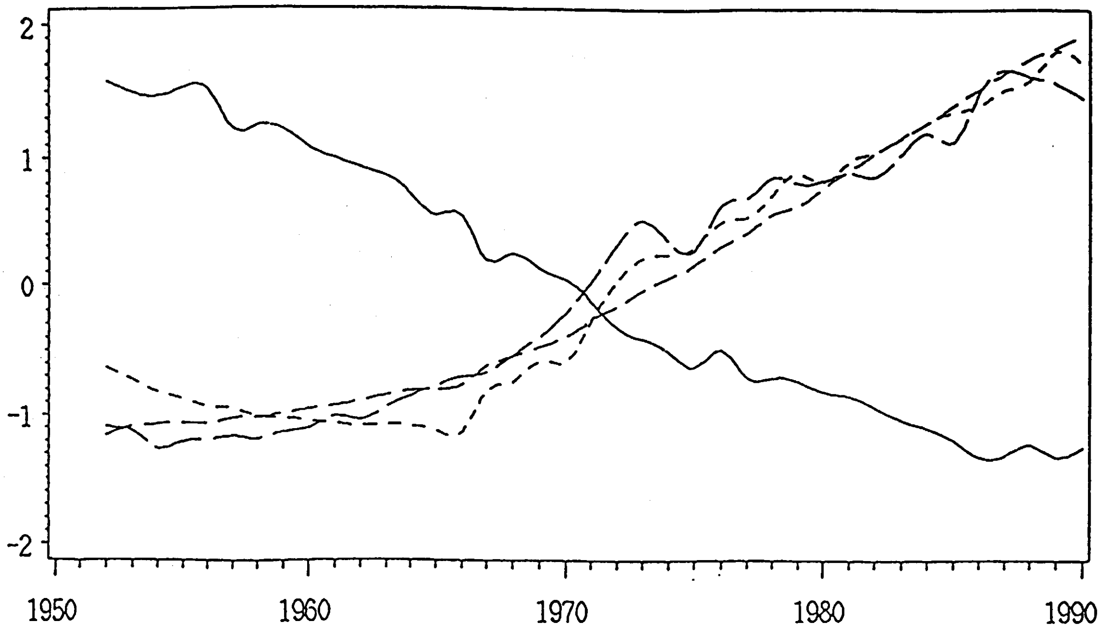
as their trends. The spectral density functions of the four variables detrended by the estimated trends are shown in (1) of Figure 3. For the residuals from the fifth order model, there are no peaks at low frequencies anymore. Wide peak of spectral density of TFR appears at around frequency 0.1, peaks of EDF and EMPF at 0.08 and 0.20. The density of EXP has a peak at 0.13. Since a cyclical period is a reciprocal number of frequency, their peaks mean TFR has 10 years cyclical period, EDF and EMPF have 12 years cycles, and EXP has about 6～7 years cycle.

Secondly, we examine detrended series by the first differencing, that is, y_t is transformed into $\nabla \log y_t$ for EXP and EDF. Log transformation needs to keep the variation of the variables EXP and EDF stable and ∇ denotes differencing operater,

$$\nabla y_t = y_t - y_{t-1}.$$

The variables TFR and EMPF are used in the form ∇y_t without log transformation. Average growth rates of all differencing series are insignificant in Table 3. The spectral density functions of the four variables detrended by the first differencing are shown in (2) of Figure 3. From the figure the spectral density of TFR slightly has peak at around 0.4, EDF at 0.20～0.35, EMPF at 0.20,

Figure 1 Time Plot of Table Fertility Rate and Socio-Economic Variables in Republic of China



(Source) Same as Table 1.
(Note) All variables are standardized in mean 0 and variance 1 to show shapes of them in a graph.
(— :TFR ····· :EXP · · · :EDF -- :EMPF)

Table 2 Average Declining Rate and Growth Rate of Residuals from the First Order Polinomial Regression Model

Period	TFR(a)	EXP	EDF	EMPF
1952−1990	−3.05	15.44**	7.68**	3.22**
1952−1971	−3.09	16.03**	7.82**	−0.04
1975−1990	−3.00	11.24**	5.25**	2.69**

(Source) Prepared by the author.

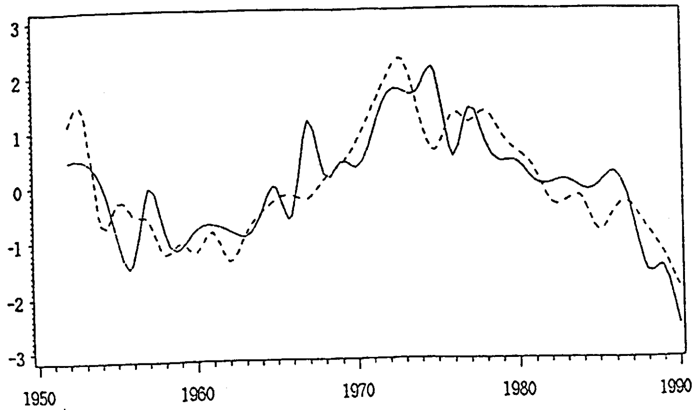
(Notes) ** means significant at 1% level.

* means significant at 5% level.

(a) Average declining rate is defined as the mean value of $(x_t - x_{t-1})/x_t$.

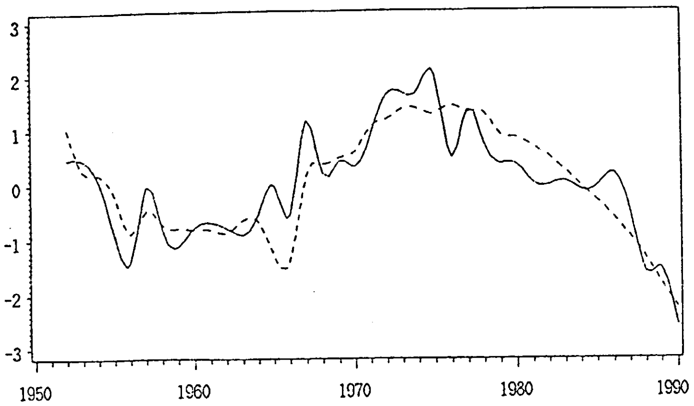
Figure 2 Time Plot of Residual from the First Order Polynomial Model

(1) EXP & TFR



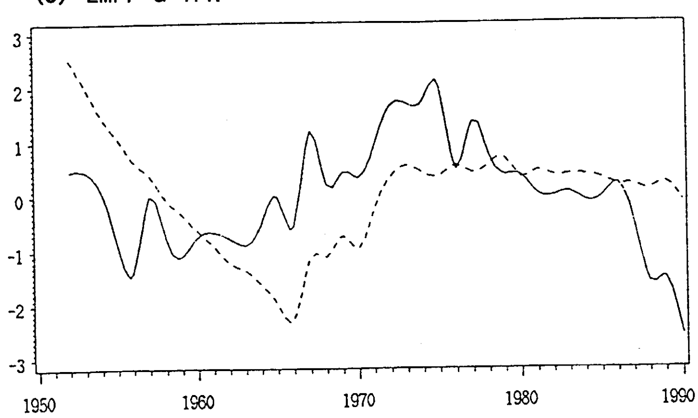
(Source) Prepared by the author.
(Note) TFR has minus sign to fit EXP:
TFR=TFR·(-1).
(—— :TFR ······ ;EXP)

(2) EDF & TFR



(Source) Prepared by the author.
(Note) TFR has minus sign.
(—— :TFR ······ ;EDF)

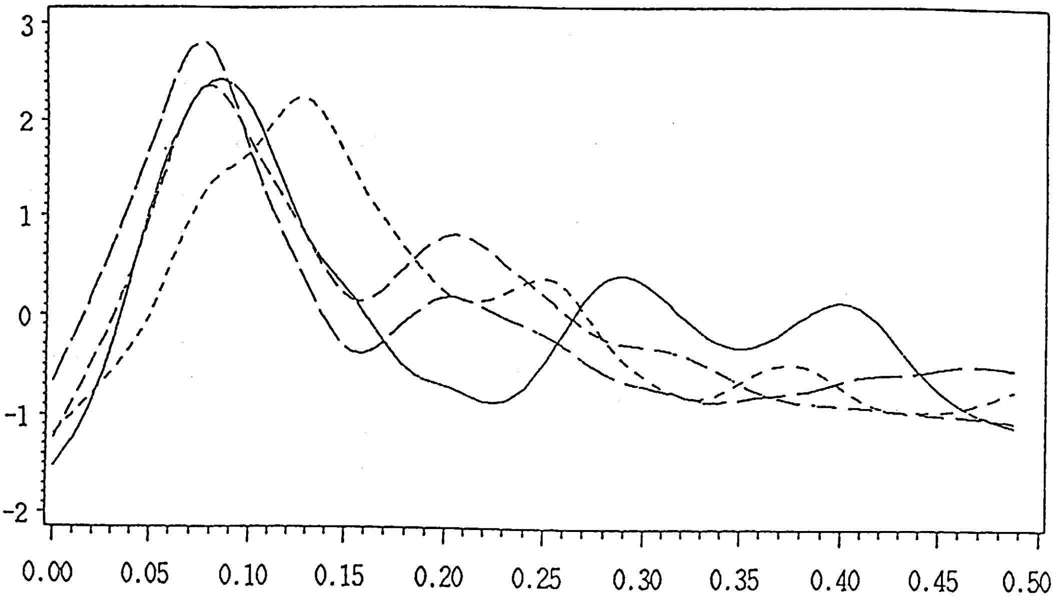
(3) EMPF & TFR



(Source) Prepared by the author.
(Note) TFR has minus sign.
(—— :TFR ······ ;EMPF)

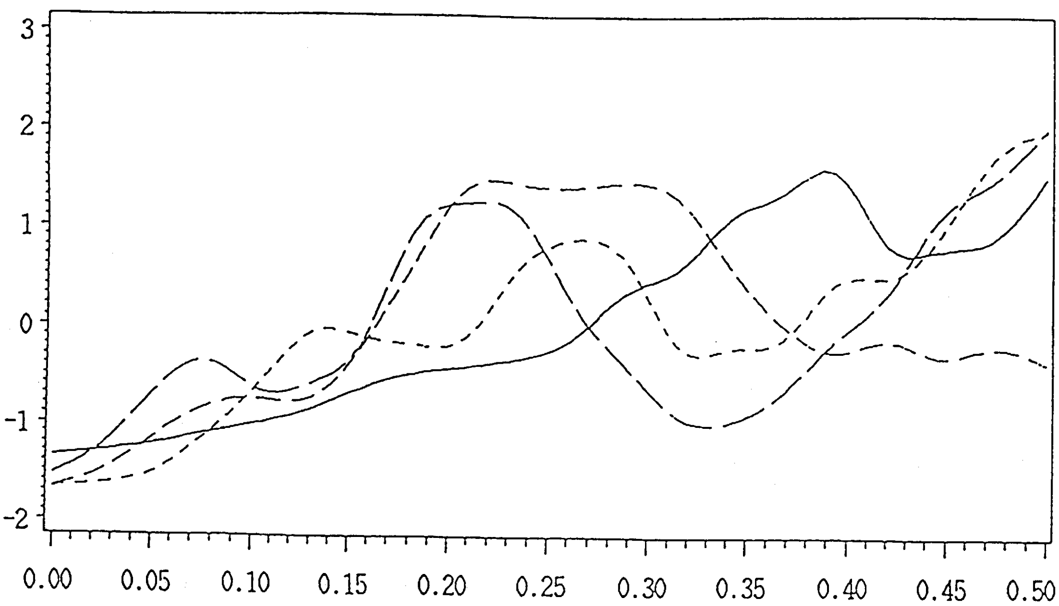
Figure 3 Special Density Function

(1) The residual from the fifth order polynomial regression model



(Source) Prepared by the author.
(Note) All spectral density functions are standardized in mean 0 and variance 1 to show shapes of them in a graph.
(—— :TFR ····· :EXP · · · :EDF – – :EMPF)

(2) The first differencing



(Source) Prepared by the author.
(Note) Same as (1). (——:TFR ····· :EXP · · · :EDF – – :EMPF)

Table 3 AIC(t_p) and Minimum AIC(t_p)

(1) Total fertility rate (TFR)

Year	First	Second	Third
1953	439.653	202.205	.
1954	439.646	195.049m	.
1955	439.127	195.640	355.055
1956	436.890	195.446	353.484
1957	438.525	196.748	355.885
1958	437.510	196.041	355.490
1959	435.117	196.054	353.965
1960	433.538	196.917	353.085
1961	431.676	197.855	351.951
1962	428.490	198.501	349.583
1963	423.143	198.234	344.981
1964	418.781	199.675	341.301
1965	416.898	203.383	340.212
1966	407.820	200.452	331.171
1967	414.225	209.515	338.679
1968	410.049	210.254	335.106
1969	406.277	211.008	331.781
1970	399.382	.	324.887
1971	397.031m	.	322.431m
1972	401.345	.	326.852
1973	404.020	189.042	329.081
1974	405.855	189.043	329.910
1975	407.016	187.636	327.862
1976	406.767	189.269	331.343
1977	406.697	189.550	329.785
1978	407.357	189.536	331.741
1979	408.795	188.861	336.069
1980	410.161	188.124	339.314
1981	411.990	185.791	344.040
1982	413.416	182.567	347.848
1983	414.460	180.235	350.408
1984	415.506	175.874	353.171
1985	416.409	173.837m	.
1986	417.066	177.576	.
1987	419.541	176.966	.
1988	426.820	181.312	.

(Source) Prepared by the author

(Note) Minimum AIC(t_p) is pointed at the symbol "m". The t_p's of AIC(t_p) are calculated from 1953 to 1988 in the first step, and so on.

(2) Export of goods and services (EXP)

Year	First	Second	Third
1953	-112.341	-82.416	.
1954	-111.780	-79.338	.
1955	-111.475	-82.249	.
1956	-111.367	-85.436	.
1957	-111.371	-89.597	.
1958	-112.978	-90.219m	.
1959	-114.229	-89.778	-78.780
1960	-116.710	-88.645	-79.586
1961	-117.986	-85.510	-82.880
1962	-123.684	-85.610	-81.748
1963	-127.834	-81.945	-81.440
1964	-130.322	-76.525	-80.648
1965	-132.118	-71.940	-80.482
1966	-135.147	-69.056	-80.603
1967	-141.322	-67.408	-80.840
1968	-146.131	-65.072	-81.095
1969	-149.990	.	-82.412
1970	-150.368m	.	-84.832m
1971	-145.356	.	-84.664
1972	-138.072	-95.943	-80.029
1973	-133.117	-98.606m	.
1974	-132.966	-93.061	.
1975	-134.110	-95.163	.
1976	-133.990	-94.358	.
1977	-133.997	-94.147	.
1978	-134.261	-94.416	.
1979	-134.141	-95.019	.
1980	-133.304	-95.015	.
1981	-131.664	-94.484	.
1982	-128.166	-92.373	.
1983	-125.442	-92.206	.
1984	-123.351	-92.273	.
1985	-120.301	-95.161	.
1986	-118.887	-96.097	.
1987	-117.784	-94.663	.
1988	-115.801	-94.651	.

Table 3 (Continued)

(3) Ratio of women's higher educational attainment (EDF)

Year	First	Second	Third
1953	-173.513	-102.989	.
1954	-173.547	-106.808	.
1955	-173.539	-110.016	.
1956	-174.112	-110.140	.
1957	-173.969	-111.348	.
1958	-174.438	-112.356	.
1959	-175.168	-113.184	.
1960	-176.196	-113.906	.
1961	-177.600	-114.589	.
1962	-179.767	-115.046	.
1963	-181.519	-116.225	.
1964	-184.309	-118.781	.
1965	-193.314	-118.736	.
1966	-210.326	-126.579m	.
1967	-210.944	-110.665	-99.201
1968	-211.360	-102.659	-98.024
1969	-212.820	.	-97.775
1970	-215.261m	.	-99.582
1971	-213.417	.	-100.583
1972	-211.873	-127.878	-100.641m
1973	-209.769	-133.225	-99.841
1974	-208.918	-139.432	-97.786
1975	-209.097	-148.120	-95.068
1976	-208.587	-153.058	-93.234
1977	-208.668	-157.653	-90.482
1978	-208.655	-158.797	-88.216
1979	-208.939	-158.947m	.
1980	-208.860	-158.661	.
1981	-208.323	-156.900	.
1982	-207.217	-153.864	.
1983	-205.263	-149.406	.
1984	-202.428	-144.407	.
1985	-198.948	-139.774	.
1986	-194.778	-135.100	.
1987	-189.993	-130.391	.
1988	-184.535	-125.350	.

(4) Ratio of employed women (EMPF)

Year	First	Second	Third
1953	-152.321	-117.970	.
1954	-159.659	-124.259	.
1955	-166.865	-130.376	.
1956	-173.199	-131.867m	.
1957	-179.359	-131.390	-69.475
1958	-183.536	-130.068	-72.699
1959	-186.506	-125.143	-76.681
1960	-188.010	-120.402	-81.356
1961	-188.381	-115.790	-87.002
1962	-188.241	-113.552	-92.883
1963	-188.008	-106.519	-101.466
1964	-189.122	-102.274	-109.917
1965	-194.905	.	-110.948
1966	-211.224m	.	-118.000m
1967	-202.802	.	-92.914
1968	-198.128	-128.052	-84.546
1969	-188.713	-133.686	-77.761
1970	-188.300	-157.538	-74.964
1971	-177.764	-173.711m	.
1972	-165.049	-168.828	.
1973	-155.916	-167.906	.
1974	-150.779	-166.941	.
1975	-147.447	-163.530	.
1976	-144.230	-161.037	.
1977	-142.304	-155.770	.
1978	-140.425	-152.672	.
1979	-138.764	-150.977	.
1980	-138.153	-143.740	.
1981	-137.477	-140.013	.
1982	-137.066	-136.112	.
1983	-136.722	-133.169	.
1984	-136.450	-130.760	.
1985	-136.292	-128.262	.
1986	-136.237	-125.451	.
1987	-136.196	-123.267	.
1988	-136.223	-121.243	.

EXP at around 0.28. The density of TFR, EXP and EMPF also have peaks at high frequency. Their peaks mean TFR has 2~3 years cyclical period, EDF 3~5 years cycles, EMPF 3~5 years cycles, and EXP has about 4 years cycle. However we may recognize large parts of their fluctuations depend on irregular variations.

Although the numbers of obserbations for the variables are very small, judging from the results obtained by two types of detrending methods, TFR has combination of 2~3 years short term cycles and around 10 years cycle along the downward trend. Socio-economic factors of Taiwan have also combination of 2 ~5 years short term cycle and around 10 years cycle along the upward trend. The cycles fluctuate in the almost same way as Taiwan's business cycle does.

3. CHANGING POINT IN TFR AND SOCIO-ECONOMIC FACTORS

In order to know a structural change of causality between TFR and socio-economic factors, we need to separate the time period of y_t into two parts at a point where we think of existance of structural change in y_t. We call the point a changing point of the period. In the case of knowing the number of changing points, some methods as Chew test, switching regression model and so on are used to estimate their location. In this section we examine to determine the number and location of the changing points as a structural change in TFR and socio-economic factors by the method of model selection called the Akaike Information Criterion (AIC). Let's think of linear regression.

$$y_t = \begin{cases} a_1 + b_1 t + u_t & t=1 \cdots t_p \\ a_2 + b_2 t + v_t & t=t_p+1 \cdots n \end{cases} \quad \cdots\cdots\cdots (1)$$

We assume that u_t and v_t are normally distributed with mean 0 and variance σ^2 and that t_p moves from 1953 to 1988. As $AIC(t_p)$ is a function of t_p, we define a point t_p at which the AIC of model is the minimum AIC as a changing point. $AIC(t_p)$ of equation (1) is defined as the following.[2]

$$AIC(t_p)=n(\log 2\pi + 1)+n \log \sigma^2+10$$

where n is the number of observation and σ^2 is the maximum likelihood estimater of σ^2. When difference of each AIC is more than 2, the difference is regarded as a level of significance.

Stepwise Chew test is frequently used to figure out the changing point as the test method of structural changes in econometrics. We seldom know the number of changing point in actual data processing. In the case of unknown number of changing point, stepwise Chew test by Ninomiya is treated, however, it is said that the method sometimes makes mistakes to determine the changing point in some typical cases. In this paper, we avoid the kinds of mistakes to estimate the changing point by AIC, the following several steps as in stepwise Chew test are performed. As the first step, we set t_p to separate the given data into two parts as a all posibilities of changing points. Then t_p moves at the point one by one and $AIC(t_p)$ is calculated for the each data which is separated at t_p. The point t_p where the model has the minimun $AIC(t_p)$ is estimated as a changing point. The point t_p separates the data into the first half period and the second half period. The same process is performed in the first half period, after that the process is performed in the second half period. When definit difference appears in $AIC(t_p)$ in a period we don't recognize the existence of changing point in the only period. The process is repeated until we don't recognize the difference in the next period and we can determine the number and the locations of changing point from the results.

Table 3 shows that minimum $AIC(t_p)$ of TFR occurs in 1971 at the first step. That's why t_p is possibly 1970 or 1971. Therefore, the first half period is assumed from 1952 to 1971, and the second period is from 1972 to 1990. At the second step minimum $AIC(t_p)$ occurs in 1954 in the first half period though $AIC(t_p)$ from 1954 to 1963 have almost same values. Minimum $AIC(t_p)$ occurs in 1985 in the second period. As the third step minimum $AIC(t_p)$ occurs in 1971 in the period from 1954 to 1985. Consequently we have an evidence that a changing point t_p of TFR is estimated at 1971 because result of the first step is in agreement with the result of third step. Each of the AIC is standardized and ploted in Figure 4. In a similar way, a changing point of EXP is estimated at 1970, EMPF at 1966. At the beginning of 1966, the total years of the compulsory education were raised from six years to nine years. Concerning EDF, result of the first step is different from the result of third step, however, the $AIC(t_p)$'s from 1970 to 1972 are considered as almost same values. To remove the influence of the first oil shock in Taiwan economy from TFR and socio-economic factors, we may separate the data into two periods, 1952-1971 as the first half period and 1975-1990 as the second half period.

4. VECTOR AUTOREGRESSIVE MODEL AND RELATIVE VARIANCE CONTRIBUTION

Suppose that a vector time series $y_t=[y_1(t) \cdots y_m(t)]'$ with zero mean is a stationary multivariate process and a vector $u_t=[u_1(t) \cdots u_m(t)]'$ with zero mean is a white noise process, then vector auto-regressive model of order p, VAR(p) in short is given by

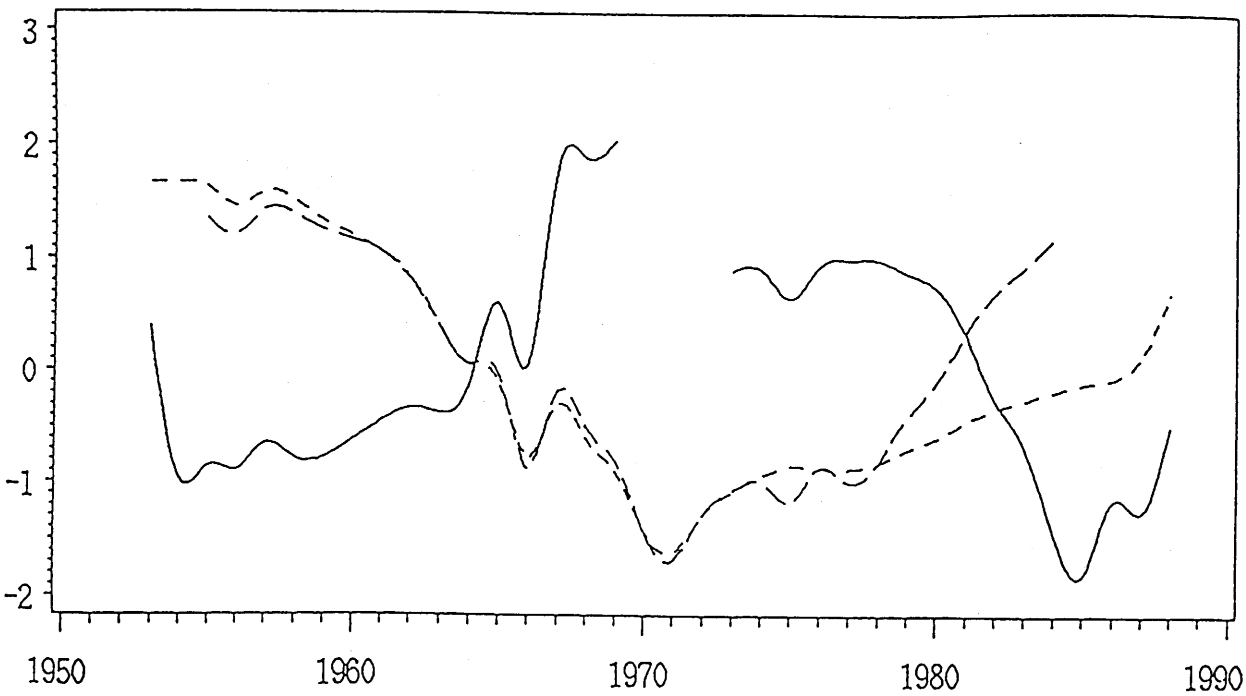
$$y_t=\Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + u_t \quad \cdots\cdots\cdots (2)$$

where Φ_i's are $m \times m$ matrices. The covariance matrix of u_t is represented by

$$G = E\{u_t u_t'\} .$$

Figure 4 Plot of AIC for Changing Point

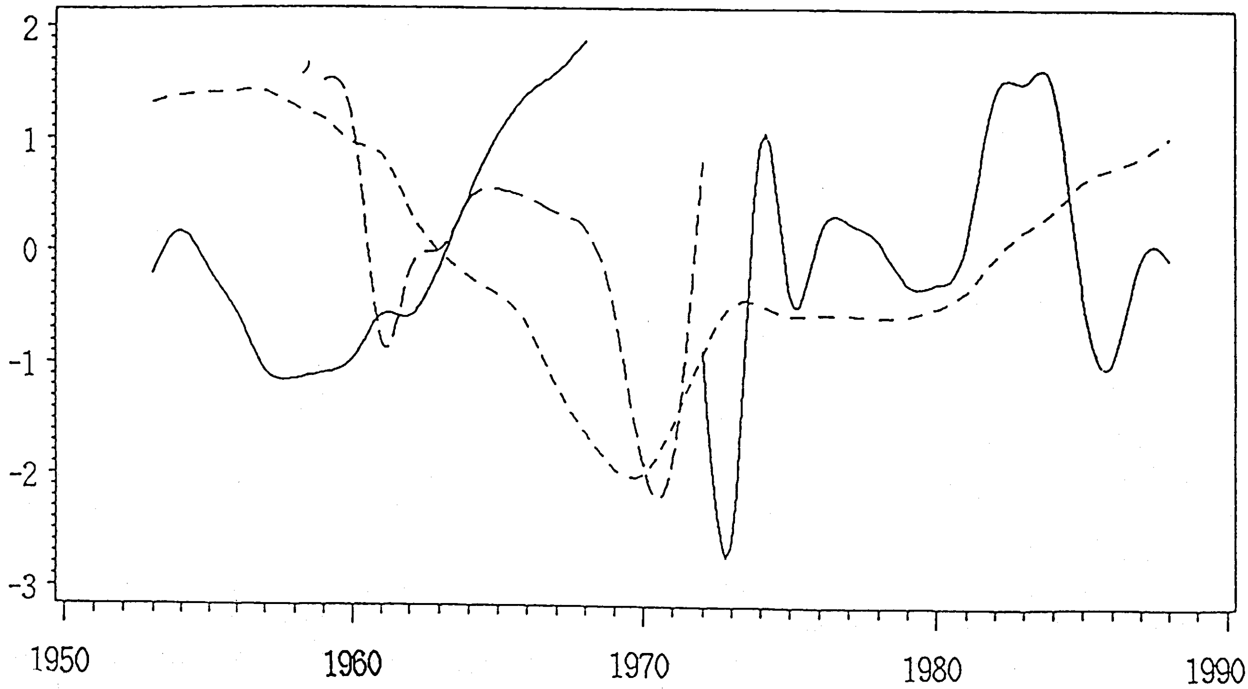
(1) TFR



(Source) Same as Table 3.

(Note) All the plots are standardized in mean 0 and variance 1 to show them in a graph.
(····· :First Step —— :Second Step - - :Third Step)

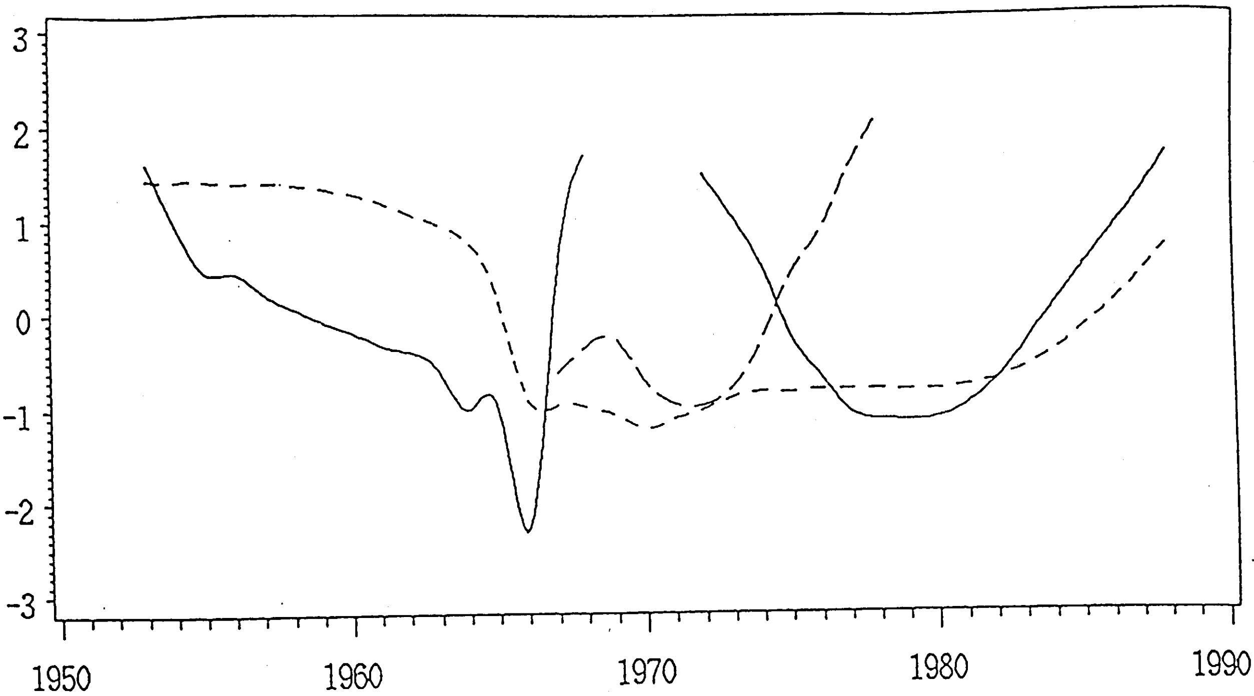
(2) EXP



(Source & Note) Same as (1).

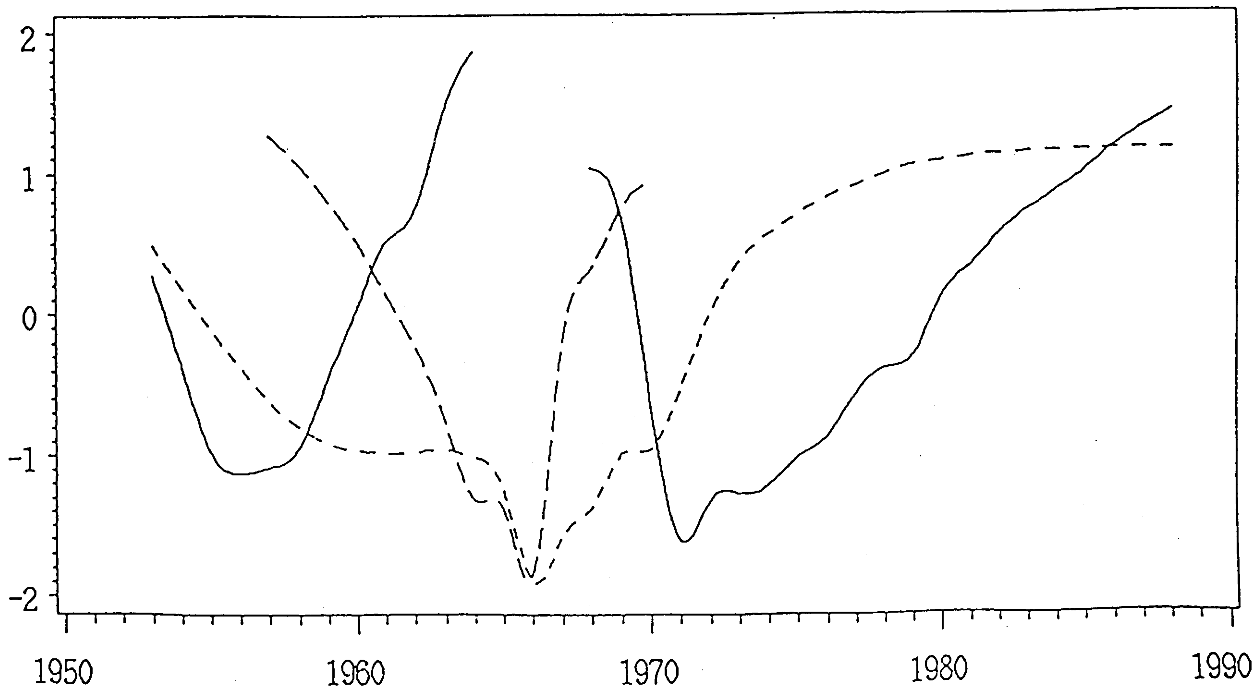
Figure 4 (Continued)

(3) EDF



(Source & Note) Same as (1).

(4) EMPF



(Source & Note) Same as (1).

The matrix G is not necessarily a diagonal one. The order p is determined by the AIC.

$$AIC = n(\log\{2\pi \mid G \mid\} + m) + (2pm^2 + m(m+1))$$

According to the criterion, the model which has a minimum AIC is selected as the best model. If the process $\{y_t\}$ is stationary and invertible, autoregressive process of order p is represented by the moving average process of infinitive order and moving average process vice versa. B denote the backward shift operator, $B^k u_t = u_{t-k}$. The equation (2) can be rewritten.

$$y_t = (I - \Phi_1 B - \cdots - \Phi_p B^p)^{-1} u_t$$
$$= \Psi_0 u_t + \Psi_1 u_{t-1} + \cdots \quad \cdots\cdots\cdots (3)$$

Knowing the values of the Φ_i's, the Ψ_i's can be obtained by the equations as follows.

$$\Psi_1 = \Phi_1$$
$$\Psi_2 = \Phi_1 \Psi_1 + \Phi_2$$
$$\vdots$$
$$\Psi_p = \Phi_1 \Psi_{p-1} + \cdots + \Phi_{p-1} \Psi_1 + \Phi_p$$
$$\Psi_j = \Phi_1 \Psi_{j-1} + \cdots + \Phi_{p-1} \Psi_{j-p+1} + \Phi_p \Psi_{j-p}$$
$$(j > p)$$

Then ij element of impulse response function is a sequence of $\{\phi_{s,ij}\}$ which are coefficient of u_{t-s}, $s=0,1,2 \cdots$. Since elements of u_t are uncorrelated to each other, under the assumption above, the variance of $y_i(t)$ which is the i-th element of y_t is spilt into m variations shown as follows.

$$E\{y_i(t)^2\} = \sum_{j=1}^{m} (\sum_{k=0}^{\infty} \phi_{k,ij2}) \sigma_j^2$$

Multipulication of both sides by $E\{y_i(t)^2\}^{-1}$ yields

$$1 = \sum_{j=1}^{m} [(\sum_{k=0}^{\infty} \phi_{k,ij}^2) \sigma_j^2 / E\{y_i(t)^2\}]$$
$$= \sum_{j=1}^{m} RVC_{j \to i}. \quad \cdots\cdots\cdots\cdots (4)$$

where $RVC_{j \to i}$ is called relative variance contribution (RVC). We consider $RVC_{j \to i}$ as a contribution of variance of $u_j(t)$ toward variance of $y_i(t)$. Since $y_j(t)$ has a proper fluctuation of $u_j(t)$, it is possible to regard $RVC_{j \to i}$ as a contribution of $y_j(t)$ toward $y_i(t)$. In otherwords, relative variance contribution represented by $RVC_{j \to i}$ is a measure of the effect that variable $y_j(t)$ makes on variable $y_i(t)$ in a sense of the Granger's causality. When $RVC_{j \to i}$ is close to 1, the contribution from $y_j(t)$ to $y_i(t)$ is more effective. On the contrary when $RVC_{j \to i}$ is close to 0, $y_j(t)$ does not influence $y_i(t)$.

In order to calculate the relative variance contribution, it is necessary that covariance matrix of residuals from VAR(p) is diagonal. To check an assumption that residuals from the model are uncorrelated to each other, we try to examine the null hypothesis

$$H_0: G = \mathrm{Diag}(g_{11} \cdots g_{pp}). \cdots\cdots\cdots (5)$$

Applying the likelihood ratio test to diagonality of a covariance matrix G, a statistics M below approximately distributed as χ^2 distribution with degrees of freedom $m(m+1)/2$, if the hypothesis is appropriate.

$$M = (n-1)\{\log[\mid \mathrm{Diag}(g_{11} \cdots g_{mm}) \mid] - \log(\mid G \mid)\} \quad \cdots\cdots\cdots (6)$$

where m is the number of variables and n is the sample size. As mentioned before, input data for VAR model are assumed to be stationary, so that we use two types of series, detrended series by the first order polynomial regression model and detrended series by differencing as input data.

Firstly, we examine the detrended series by the first order polynomial regression model. Let y_t be vector of order 4, [TFR EXP EDF EMPF]' as mentioned above. Table 4 shows that the first order is selected as the optimum order of VAR model: VAR(1), when data is obtained from 1952 to 1990. And also the optimum models obtained in the both first period and the second period are VAR(1)'s. Table 4 shows the χ^2(G) which tests the diagonality of the correlation matrix G obtained from the residuals of VAR model. The null hypothesis that all non-diagonal elements of G are zero as shown in (5) is tested by the test statistics M of equation (6). When residual is correlated to each other, that is, the null hypothesis is rejected we call the relation an instantanious causality. Significant level at 5% of χ^2 distribution with degree of freedom 6 is 12.56, therefore, all null hypotheses are rejected from χ^2(G) in the Table 4 of (1). It seems that there are some interrelation between the variables contempolaneously at the time t each period. Under the situation that G is not diagonal, we don't have theoretical foundation to use relative variance contribution, because of the existence of instantanious causality.

Secondly, we examine the detrended series by the first differencing. Judging from Table 4, we determine p=0 for VAR(p), VAR(0) as the optimum model in period: 1952-1990. In the first period: 1952-1971, VAR(2) is assumed as the optimum model, and VAR(3) in the second period: 1975-1990. The results of diagonalities of covariance matrix G obtained from VAR model is shown in Table 4 of (2). The null hypotheses of the diagonalities are rejected at 5% significant level in both the first and the second periods. The same as the result of the first order polynomial regression model, we don't have theoretical foundation to use relative variance contribution RVC** because of the existence of instantanious causality.

The differencing is not only unstable to make a model because the order of VAR(p) is higher for the small sample, but also has no evidence of theoretical foundation, so we only use the first order polynomial regression model to detrend.

Table 4 Order of VAR(p) and Test Statistics of Diagonality

(1) The first order polinomial regression model

Period	1952－1990	1952－1971	1975－1990
sample size	39	20	16
p of VAR(p)	1	1	1
χ^2(G)	59.548**	78.894**	21.714**
χ^2(D)	3.200	5.617	5.430

(2) The first differencing

Period	1952－1990	1952－1971	1975－1990
sample size	38	19	16
p of VAR(p)	0	2	3
χ^2(G)	.	33.754**	47.220**
χ^2(D)	.	17.938**	16.895**

(Source) Prerared by the author
(Note) ** means significant at 1% level.
* means significant at 5% level.
. means there are no correlation matricies G or D in the model because order of VAR(p) is 0.

Table 5 Relative Variance Contribution RVC* by Choleski Decomposition

	TFR	EXP	EDF	EMPF	Total
1952－1990					
TFR	4.311**	0.400	1.468	0.959	100.0
EXP	－5.320	9.520**	1.846*	－1.014	100.0
EDF	－1.709	－1.626	8.834**	－3.543	100.0
EMPF	－11.481	－0.793	－0.567	10.111**	100.0
1952－1971					
TFR	2.615**	5.067**	－0.090	－1.726	100.0
EXP	－0.958	5.658**	0.294	－0.664	100.0
EDF	0.216	－1.187	2.110*	1.030	100.0
EMPF	－0.451	－0.616	1.447	1.955*	100.0
1975－1990					
TFR	17.870**	－7.536	3.311**	－12.382	100.0
EXP	－3.057	8.354**	－0.919	－2.841	100.0
EDF	－8.873	－1.183	17.979**	－8.429	100.0
EMPF	－20.550	－3.461	0.702	28.286**	100.0

(Source) Prerared by the author.
(Note) ** means significant at 1% level.
* means significant at 5% level.

5. MODIFIED RELATIVE VARIANCE CONTRIBUTION

When covariance matrix G is not diagonal, we can not perform the calculation of the ordinal RVC directly. We need to diagonalize the matrix G. In this section we try to diagonalize G and to modify the ordinal RVC by two methods, well-known Choleski decomposition and a single factor model. All variables used for the following analyses are detrended by the first order polynomial regression model.

1) Choleski decomposition

Since a covariance matrix is symmetric and positive definite, the matrix can be decomposed by Choleski's method as G=LL', where L is lower matrix. If lower matrix W^{-1} has unit diagonal elements, L is a product of W^{-1} and $D^{1/2}$,

$$L = W^{-1}D^{1/2}.$$

We get the following result.

$$G = LL' = W^{-1}DW'^{-1}$$

$$WGW' = D$$

Where D is a diagonal matrix. To diagonalize the covariance matrix of u_t, we have transform matrix obtained by Choleski's method.

$$u_t^* = Wu_t$$

The covariance matrix u_t^* is

$$\begin{aligned} E\{u_t^* u_t^{*\prime}\} &= WE\{u_t u_t'\}W' \\ &= WGW' \\ &= \mathrm{Diag}\{\sigma^{*2}_1 \cdots \sigma^{*2}_m\} . \end{aligned}$$

It shows that $u_1(t)^* \cdots u_m(t)^*$ are uncorrelated to each other. Therofore we can use the method shown above to get the impulse response function and RVC. By substituting u_t^* for u_t, we get

$$\begin{aligned} y_t &= (I - \Phi_1 B - \cdots - \Phi_p B^p)^{-1} W^{-1} u^*_t \\ &= \Psi_0 W^{-1} u^*_t + \Psi_1 W^{-1} u^*_{t-1} + \cdots \\ &= \Psi_0^* u^*_t + \Psi_1^* u^*_{t-1} + \cdots \end{aligned}$$

where $\Psi_s^* = \Psi_s W^{-1}$ $s=1,2 \cdots$. Then ij element of impulse response function is a sequence of $\{\psi^*_{s,ij}\}$. The following RVC* also can be obtained by substituting ψ^* and σ^* for ψ and σ in the equation (4).

$$RVC^*_{j\to i} = \sum_{k=0}^{\infty} \psi^{*2}_{k,ij} \sigma^{*2}_j / E\{y_i(t)^2\}$$

Choleski decomposition is a powerful method to diagonal matrix G, but the method includes a severe problem that ordering of elements in vector y_t makes changes of values of the transform matrix W. Therefore average RVC* is considered as a magunitude of contribution. There are 4!=24 times for arangement of vector y_t=[TFR EXP EDF EMPF]', so we caluculate RVC* every cases. Table 5 shows the result of the t test for a null hypothesis H_0: $RVC^*_{j\to i}=20$, aga nst an alternative hypothesis H_1: $RVC^*_{j\to i}>20$. The contribution from EXP to TFR is significant at 1% level in the first half period: 1952−1971 and the contribution from EDF to TFR is significant in the second half period: 1975 - 1990. However there don't exist any causality to TFR in the period of all data : 1952-1990. As concerns EXP, EXP is caused by EDF at 5% significant level, at the same time EXP is also caused by EXP oneself at 1% level.

2) A single factor model

Next, to diagonalize a covariance matrix G of the residuals from VAR model, we apply the factor analysis to the residuals. In this paper we use a single factor model.

$$u_t = af_t + e_t$$

where vector $a'=[a_1 \cdots a_m]$ is factor loading, f_t common factor with $E\{f_t\}=0$ and $E\{f_t^2\}=1$, $e_t'=[e_1(t) \cdots e_m(t)]$ unique factor. The covariance matrix of et is diagonal matrix D. These relationship can be expressed as

$$G = aa' + D$$

The purpose of factor analysis is the determination of the elements a and D. For the solution of the maximum-likelihood loading equation, we will adopt a cyclic method for a single factor model. Knowing a and D, y_t is rewritten

$$\begin{aligned} y_t &= \Psi_0(af_t + e_t) + \Psi_1(af_{t-1} + e_{t-1}) + \cdots \\ &= (\Psi_0 e_t + \Psi_1 e_{t-1} + \cdots) \\ &\qquad + (\Psi_0 af_t + \Psi_1 af_{t-1} + \cdots) \end{aligned}$$

Since the elements of e_t are independently distrib-uted and common factor f_t is also independently distributed, the variance of $y_i(t)$ is split into m variations from the unique factors and a variation from the common factor shown as follows:

$$E\{y_i(t)^2\} = \sum_{j=1}^{m} (\sum_{k=0}^{\infty} \psi_{k,ij}^2) v_j^2 + \sum_{k=0}^{\infty} (\sum_{j=1}^{m} \psi_{k,ij} a_j)^2$$

where v_j^2 is j-th diagonal element of covariance matrix D. Multiplying E $\{y_i(t)^2\}^{-1}$ both sides yields

$$\begin{aligned} 1 &= \sum_{j=1}^{m} [(\sum_{k=0}^{\infty} \psi_{k,ij}^2) v_j / E\{y_i(t)^2\}] \\ &\quad + \sum_{k=0}^{\infty} (\sum_{j=1}^{m} \psi_{k,ij} a_j)^2 / E\{y_i(t)^2\} \end{aligned}$$

$$1 = \sum_{j=1}^{m} RVC^{**}_{j\to i} + \mu_i$$

RVC** is similar form as the equation (4).

Table 4 of (1) shows the χ^2(D) which tests the diagonality of the correlation matrix D obtained from the residuals of VAR model with a single factor model. The null hypothesis that all diagonal elements of D are zero is tested by the test statistics M of (6). We don't have any evidences to reject the hypotheses from χ^2(D) in the Table 4 of (1). Therefore we have theoretical foundation to use relative variance contribution of RVC** adequately.

From the Table 6 of (1952-1971), the contribution from TFR to TFR is 14.500%, contribution from EXP to TFR 30.233%, contribution from common variation to

Table 6 Relative Variance Contribution RVC And Correlation Matrix D by a Sigle Factor Model**

	TFR	EXP	EDF	EMPF	common	total
1952−1990						
RVC**						
TFR	15.132	13.967	2.557	37.356	33.677	100.0
EXP	0.542	28.818	3.130	33.745	33.765	100.0
EDF	1.018	1.796	6.845	32.243	56.306	100.0
EMPF	0.481	4.815	1.130	43.549	50.025	100.0
a	−0.418	0.339	0.484	0.283	−	−
D						
TFR	1.000	−0.041	−0.027	0.187	−	−
EXP	.	1.000	−0.050	0.193	−	−
EDF	.	.	1.000	0.014	−	−
EMPF	.	.	.	1.000	−	−
1952−1971						
RVC**						
TFR	14.500	30.233	0.974	6.672	48.082	100.0
EXP	0.509	36.611	1.645	4.770	56.465	100.0
EDF	0.054	2.485	1.411	0.312	95.739	100.0
EMPF	0.098	2.788	0.529	6.187	90.399	100.0
a	−0.636	0.497	0.782	0.658	−	−
D						
TFR	1.000	−0.214	−0.059	0.239	−	−
EXP	.	1.000	−0.151	0.300	−	−
EDF	.	.	1.000	0.015	−	−
EMPF	.	.	.	1.000	−	−
1975−1990						
RVC**						
TFR	69.628	4.727	18.762	0.413	6.470	100.0
EXP	8.854	24.765	0.576	6.437	59.369	100.0
EDF	6.691	4.082	39.249	1.847	48.131	100.0
EMPF	2.674	1.118	8.507	56.896	30.806	100.0
a	0.012	0.736	0.617	0.515	−	−
D						
TFR	1.000	−0.346	0.188	0.383	−	−
EXP	.	1.000	0.002	0.003	−	−
EDF	.	.	1.000	−0.008	−	−
EMPF	.	.	.	1.000	−	−

(Source) Prerared by the author

(Note) ** means significant at 1% level.

* means significant at 5% level.

. means the value of ij element is same as ji element, $r_{ij}=r_{ji}$ because correlation matrix is synmetric.

− means filler.

TFR 48.082%, and total is 100%. From the same table, the contribution from EXP to EXP is 36.611% and common variation to EXP is 56.465%. The table shows that TFR causes TFR with 14.5%, EXP causes TFR with 30.233% in the first half period: 1952-1971, and that TFR causes TFR with 69.628%, EDF causes TFR with 18.762% in the second half period: 1975-1990, and that TFR causes TFR with 15.132%, EXP causes TFR with 13.967%, EMPF causes TFR with 37.356%, common variation with 33.677% in the full period: 1952−1990. In the first period EXP has higher contribution to TFR, on the contrary in the second period EXP doesn't influence TFR anymore and EDF begins to contribute more.

In the second half period, ratio of common variation decreases compared with the common variation in the first half period, on the contrary fertility increases the contribution for oneself. It is very difficult to determine the other influences owing to variation of data.

6. CONCLUSION

Taiwan's total fertility rate definitely began to decline from 1952 and at the same time the TFR is recognized to be not only fluctuated with the combination of 2∼5 year short term cycle and around 10 year cycles along the downward trend. The rate also seemed to exist the structural change at the point of 1971. Roughly speaking we recognized that a cycle of TFR fluctuates in almost the same way as Taiwan's business cycle does.

Moreover the TFR is said to be caused by the socio-economic factors. Therefore the hypothesis that there exists the causal relationship between TFR and socio-economic factors in Taiwan was tested by relative variance contribution. As the socio-economic factors for the TFR, three variables: export, ratio of women's higher education and ratio of employed women were selected in this paper. From the model selection by using AIC, we have evidence that a changing point t_p of TFR is estimated at 1971, EXP at 1970, EMPF at 1966 and EDF at the beginning of 1970s. Removing the infulence on the first oil shock in Taiwan economy, we may separate the data of TFR into two periods, 1952-1971 as the first half period and 1975-1990 as the second half period.

It is widely known that most economic data are nonstationary. The first order polynomial regression model and the first differencing were tried to remove the nonstationarities. The differencing is unstable to specify a model because the order of VAR(p) is higher for the small sample, and has no evidence of theoretical foundation to use relative variance contribution, so we only use the first order polynomial regression model to detrend.

Consequently we support the hypothesis from the consistent results by two types of modified relative variance contributions, RVC* and RVC**. In the first period export has higher contribution to total fertility rate, on the contrary in the second period export doesn't influence TFR anymore and ratio of higher educated woman begins to cause. Contribution of common variation in the second half period decreases compared with one of the first half period, on the contrary fertility increases the contribution for oneself.

NOTES

1. Let x_t denote complex stationary process with $E\{x_t\}=0$, a_j ($j=1 \cdots m$) a set of independent complex random variables with $E\{a_j\}=0$ and $E\{a_j a_k\}=\sigma^2_j \delta_{jk}$ and i an imaginary number. A generating process

$$x_t = \sum_{j=1}^{m} a_j \exp\{i(2\pi)^{-1} w_j t\}$$

is a sum of periodic functions with period

$2\pi w_j^{-1}$, w_j $j=1 \cdots m$ ($| w_j | < 1/2$).

The autocorrelation function of x_t is represented by

$$c_k = E\{x_t x_{t-k}\} / E\{x_t x_t\}$$

$$= \sum_{j=1}^{m} \sigma_j^2 \exp\{ i(2\pi)^{-1} w_j m \} \Big/ \sum_{j=1}^{m} \sigma_j^2$$

$$= \int_{-0.5}^{0.5} \exp\{ i(2\pi)^{-1} wm \}\ dF(w)$$

Assuming $dF(w)=f(w)dw$ under certain conditions and continuity of $f(w)$, $f(w)$ is called spectral density function of x_t.

2. Let y be n dimensional vector of the observation, $e=(u_t\ v_t)'$ normally distributed with mean zero and covariance matrix $\sigma^2 I$, X $n\times4$ matrix, a n dimensional vector. In matrix notation, the model of (1) is represented as the model with four explanatory variables.

$$y = Xa + e \quad \cdots\cdots\cdots (2)$$

where

$$y = (y_1 \cdots y_n)' \qquad e = (e_1 \cdots e_n)'$$

$$X' = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 & \cdots & 0 \\ 1 & 2 & \cdots & t_p & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ 0 & \cdots & 0 & 1 & 2 & \cdots & n-t_p \end{bmatrix} \qquad a = \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{bmatrix}$$

log likelihood of the model (2) is,

$$L(a, \sigma^2) = -(n/2)\log 2\pi\sigma^2 - (2\sigma^2)^{-1}(y - Xa)'(y - Xa).$$

maximum likelihood estimators of a and σ^2,

$$\hat{a} = (X'X)^{-1}X'y$$

$$\hat{\sigma}^2 = n^{-1}(y - X\hat{a})'(y - X\hat{a}).$$

and maximum log likelihood is

$$L(\hat{a}, \hat{\sigma}^2) = -(n/2)(\log 2\pi\hat{\sigma}^2 + 1)$$

As the number of parameters in the model is 5 because the model has 4 unknown parameters and σ^2. AIC of the model is represented as the function of t_p

$$AIC(t_p) = -2(\text{maximum log likelihood}) + 2(\text{number of free parameters})$$

$$=n(\log 2\pi + 1) + n \log \hat{\sigma}^2 + 10$$

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