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IDE DISCUSSION PAPER No. 121

A Mathematical Representation of “Excitement” in Games from the Viewpoint of a Neutral Audience

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October 2007

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Researchers have long believed the concept of “excitement” in games to be subjective and difficult to measure. This paper presents the development of a mathematically computable index that measures this concept from the viewpoint of an audience. One of the key aspects of the index is the differential of the probability of “winning” before and after one specific “play” in a given game. If the probability of winning becomes very positive or negative by that play, then the audience will feel the game to be “exciting.” The index makes a large contribution to the study of games and enables researchers to compare and analyze the “excitement” of various games. It may be applied to many fields especially the area of welfare economics, ranging from allocative efficiency to axioms of justice and equity.

Keywords: game, game system, excitement
JEL classification: C69, D63

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Researchers have long believed the concept of "excitement" in games to be subjective and difficult to measure. This paper presents the development of a mathematically computable index that measures this concept from the viewpoint of an audience. One of the key aspects of the index is the differential of the probability of "winning" before and after one specific "play" in a given game. If the probability of winning becomes very positive or negative by that play, then the audience will feel the game to be "exciting." The index makes a large contribution to the study of games and enables researchers to compare and analyze the "excitement" of various games. It may be applied to many fields especially the area of welfare economics, ranging from allocative efficiency to axioms of justice and equity.

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1. Introduction

Researchers have long believed the concept of “excitement” in games to be subjective and difficult to measure. It is true that “excitement” may differ significantly from person to person even though all are playing or watching the same game. The concept of “excitement” involves various factors such as having fun seeing or doing something fantastic or promoting a favorite sports team.

In this paper, “excitement” in games is refined through the development of a mathematically computable index that measures this concept from the viewpoint of the audience. Significantly, and perhaps more essentially, it contributes to further research on the concept “excitement” of games from the viewpoint of the player.

The key concept of the mathematical index is that the core element of “excitement” is the differential of the probability of “winning” before and after one specific “play” in a match[1]. If the probability of winning becomes very positive or negative by that play, the audience feels the game to be “exciting.” However, this may not be the only measure of “excitement”, and the key concept may have problems. These problems and an alternative concept are explored in a later part of this paper.

This paper is constructed as follows: In Section 2, the concept of “excitement” is refined for mathematical formulation. Section 3 includes the first mathematical representation of “excitement” in games based on the above mentioned key concept. In Section 4, problems with the key concept are discussed, and an alternative concept of “excitement” is proposed. Conclusions are presented in the final section.

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[1] You can find the very similar concept in use at http://live.protrade.com/, but it is not clear who is the first person hit on this idea (I reached the idea in 1997). This concept of “excitement” in games might be just like the “folk theorem” of game theory.

2. Refining the Concept of "Excitement"

2.1 Two Factors of Excitement

Although there are various sources of "excitement" in games, such sources may be categorized into two groups:

The first group of sources includes doing something fun such as running, chasing after something, jumping, or shooting for example. This group contains both physical and virtual forms. For example, the "excitement" of shooting can be both physical (using real guns) and virtual (as in a computer game). Henceforth, this group of "excitement" will be termed the "fun factor".

The second group of sources includes seeking success or winning the game. This group of "excitement" arises from a formal system of rules (termed a "game system"). The "excitement" of chess comes primarily from the game system of chess, not from physically moving a knight or a pawn. This source of "excitement" will be termed the "game system factor".

In constructing an objective index of "excitement" in games, it is crucial to consider only the "game system factor"[2]. The "fun factor" is simply too subjective to be analyzed numerically.

[2] Play as fun was analyzed in some literature, notably Huizinga(1955) and Caillois(1961). However, there was little research on this factor prior to that of Salen and Zimmerman(2004).

2.2 Two Viewpoints of Excitement

It is also useful to separate two viewpoints of “excitement” in games: the viewpoint of the “player” and that of the “audience.” The “excitement” of games from the viewpoint of the player is much more complex than that of the audience. Consider the following points:

First, players of a game put much effort into playing the game. Thus, the “excitement” of games from the viewpoint of the player must be analyzed using a “cost-benefit” type of analysis. Conversely, audience “excitement” can be analyzed without considering audience “efforts” in watching the game.

Second, player “excitement” tends to contain more of the “fun factor” and may be more objective than audience “excitement”. For example, football can be exciting without any rules because kicking and chasing a ball is in itself “fun” and “exciting”.

Third, player “excitement” tends to be strongly influenced by the result of the game, that is, by who wins and who loses. An “exciting match” is “not exciting” at all for the loser. On the other hand, audience “excitement” tends to be more subjective and “process oriented.” However, “excitement” of a member of the audience who is a fanatic fan of a player or team can be quite different from that of others. Therefore, for purposes of this study, a hypothetical “neutral” audience that does not care about who wins (evaluates the process of the match as neutral) will be used as sole evaluator of “excitement” in games.

3. "Excitement" of Games from the Viewpoint of a Neutral Audience

Figures 1 and 2 show the movement of the probability of winning for hypothetical baseball game scores seen in Tables 1 and 2. The probability of winning before the game is 0.5 because it is unknown which team will win, and both hypothetically have an equal chance of winning. After the game, the probability of winning for a team converges to 1 or 0 (the team has either won or lost). The probability of winning during the game moves with the score.

Table 1: Baseball Game I

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Total*
N	9	0	0	6	0	0	0	0	0	15
B	0	0	0	0	0	0	0	0	0	0

Table 2: Baseball Game II

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Total*
N	0	0	0	0	0	0	0	0	1	1
B	0	0	0	0	0	0	0	0	2x	2

Figure 1: Movement of the Probability of Winning (Game I)

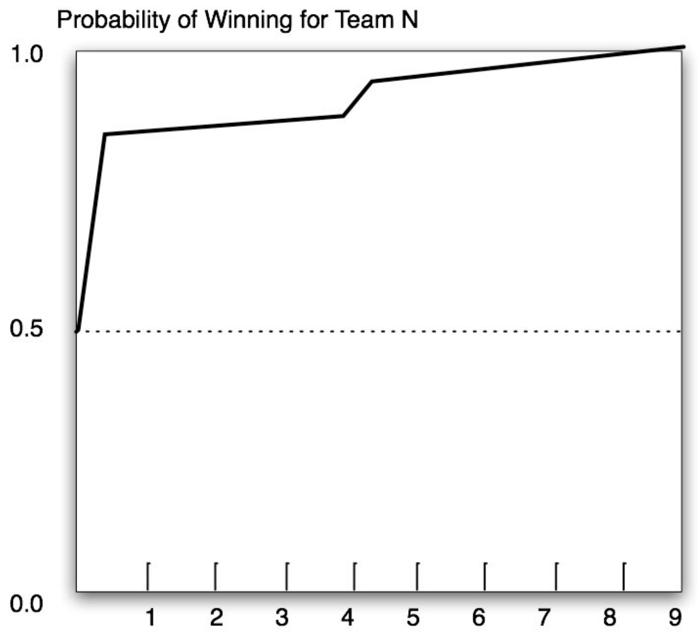
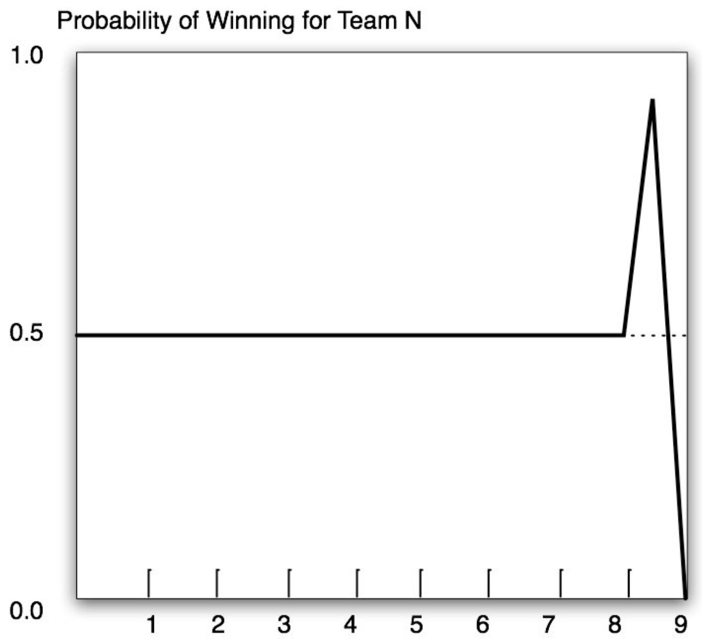


Figure 2: Movement of the Probability of Winning (Game II)



A possible mathematical formulation of “excitement” from the viewpoint of the neutral audience for one specific “play” or “move” at time *t* in a match, X_t, is as follows:

$$X_t = | p_t - p_{t-1} | \quad \text{... (1)}$$

where p_t is the probability of winning for a team at time *t*.

Thus, the index of the “excitement” of the game, *G*, is constructed by summing X_t from the beginning of the game to the end of the game:

$$G = \sum_{t=1}^{t=T} | p_t - p_{t-1} | \quad \text{... (2)}$$

where p_t is the probability of winning for a team at time *t*, and *T* is the time for the end of the game.

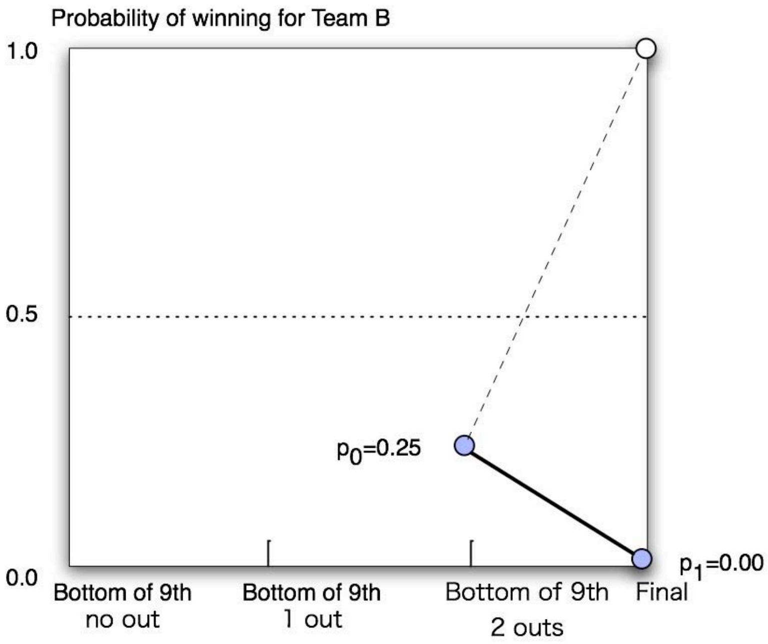
4. Problems of an *Ex Post* Index and Advantages of an *Ex Ante* Index

According to index *G*, Game I takes the value of 0.5 and Game II takes the value of approximately 1.5. Therefore, index G indicates that Game II should be more exciting than Game I. This quantitative result would seem to make intuitive sense in this case, but such is not always the case. Consider the following situation that could arise in a baseball game:

- It is the bottom of the 9th inning.
- The score is N 1 – 0 B, and team B is at bat.
- Bases are loaded with two outs.

Assume the batting average of the hitter at bat to be .250. Further, for simplicity, assume that there is no base on balls, and that a single is enough for 2 runs (team B wins). Assume the hitter is then struck out and that the game is then over. Figure 3 shows the movement of the probability of winning for team B in this situation.

Figure 3 Movement of the Probability of Winning for Team B



In this situation, the probability of winning for Team B before the play (p_0) is 0.25 since the batting average of the hitter at bat equals the probability of winning for Team B. Obviously, the probability of winning for Team B after the play (p_1) is 0. Thus, the change in the probability of winning is $X = |0.25 - 0.00| = 0.25$.

Actually, the outcome of the game is difficult to know precisely. Thus, the value of X=0.25 seems to be reasonable post game (*ex post)*. This is one of the most exciting situations in baseball. X might be 0.75 if the hitter had at least hit a single or more.

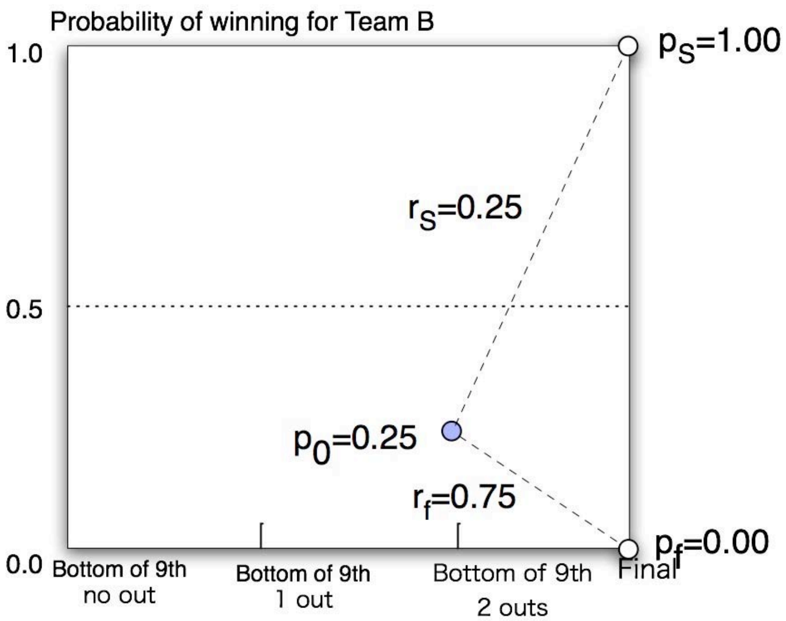
This analysis shows that the index of "excitement" (G) proposed in the previous section is inherently determined post game (*ex post)*. The result is evaluated, not the situation that might lead to higher “excitement”. Index G is thus reasonable for measuring the “excitement” of games after they have ended. This has a drawback. If a baseball game is like the one above, but with no run at all until the last inning, then the value of G for that game becomes quite small relative to a game with many runs. In this case, Index G does not seem to make intuitive sense relative to a “feeling” of “excitement”.

To solve this issue, another index of “excitement” is proposed as follows:

$$X' = E(X) = r_s \mid p_s - p_0 \mid + r_f \mid p_f - p_0 \mid \quad .... (4)$$

where r_s and r_f are the probabilities of successful play and failed play respectively, p_s and p_f are the probabilities of winning after the successful/failed play respectively, and p_0 is the probability of winning before the play (Figure 4).

Figure 4 Expected Movement of the Probability of Winning for Team B



This index incorporates the expected “excitement” of the situation. By allowing multiple outcomes and summing *X’*, another index of “excitement” for games may be constructed as follows:

$$G' = \sum_{t=1}^{T} X'_t$$

where $$X'_t = E_t(X_t) = \sum_{i=1}^{I} r_t^i \,|\, p_t^i - p_{t-1} \,|$$

and $$p_{t-1} = \sum_{i=1}^{I} r_i^t p_t^i .$$

I is the number of possible outcomes, r_t^i is the probability of the realization of play *i* at time *t*, and p_t^i is the probability of winning after play *i* at time *t*.

Call *G’* an *ex ante* index of “excitement” and *G* an *ex post* index of “excitement”. *G’* captures “excitement” before the play while *G* captures “excitement” after the play.

5. Conclusion

While it might appear to have relevance only to games, this research may well have a large impact on several fields of social science, especially the area of welfare economics. Traditionally, welfare economics has only considered the utility of goods distributed through some interaction of rational individuals. This research indicates that the process of interaction among people may have utility when considered by itself, and it may be possible to analyze this utility mathematically.

A zero-sum game may not be zero-sum when “excitement” of the game is considered. Therefore, the welfare of ALL people even in a zero-sum restriction can be improved by improving the game system or “institution.” The process of allocation may sometimes be more important than the allocation itself, and the formulations developed in this paper may provide the base for exploring such matters. Applications are possible in many fields ranging from allocative efficiency to axioms of justice and equity.

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