Discussion Papers are preliminary materials circulated to stimulate discussions and critical comments

DISCUSSION PAPER No. 204

Trade Coefficients and the Role of Elasticity in a Spatial CGE Model Based on the Armington Assumption

Asao ANDO^{\dagger}, Bo MENG^{\ddagger} and Chao QU*

Abstract

The Armington Assumption in the context of multi-regional CGE models is commonly interpreted as follows: Same commodities with different origins are imperfect substitutes for each other. In this paper, a static spatial CGE model that is compatible with this assumption and explicitly considers the transport sector and regional price differentials is formulated. Trade coefficients, which are derived endogenously from the optimization behaviors of firms and households, are shown to take the form of a potential function. To investigate how the elasticity of substitutions affects equilibrium solutions, a simpler version of the model that incorporates three regions and two sectors (besides the transport sector) is introduced. Results indicate: (1) if commodities produced in different regions are perfect substitutes, regional economies will be either autarkic or completely symmetric and (2) if they are imperfect substitutes, the impact of elasticity on the price equilibrium system as well as trade coefficients will be nonlinear and sometimes very sensitive.

Keywords: Armington Assumption, spatial CGE, elasticity of substitution, trade coefficient **JEL classification:** C68, R13, R15

[†]Professor, Graduate School of Information Science, Tohoku University. (ando@m.tains.tohoku.ac.jp)

[‡]Research Fellow, Institute of Developing Economies, JETRO. (mengbo @ ide.go.jp)

^{*} Graduate School of Information Science, Tohoku University. (qu@se.is.tohoku.ac.jp)

The Institute of Developing Economies (IDE) is a semigovernmental, nonpartisan, nonprofit research institute, founded in 1958. The Institute merged with the Japan External Trade Organization (JETRO) on July 1, 1998. The Institute conducts basic and comprehensive studies on economic and related affairs in all developing countries and regions, including Asia, the Middle East, Africa, Latin America, Oceania, and Eastern Europe.

The views expressed in this publication are those of the author(s). Publication does not imply endorsement by the Institute of Developing Economies of any of the views expressed within.

INSTITUTE OF DEVELOPING ECONOMIES (IDE), JETRO 3-2-2, Wakaba, Mihama-ku, Chiba-shi Chiba 261-8545, JAPAN

©2009 by Institute of Developing Economies, JETRO

Trade Coefficients and the Role of Elasticity in a Spatial CGE Model Based on the Armington Assumption

ANDO Asao, MENG Bo^{\dagger} and QU Chao[‡]

July 8, 2009

Abstract

The Armington Assumption in the context of multi-regional CGE models is commonly interpreted as follows: Same commodities with different origins are imperfect substitutes for each other. In this paper, a static spatial CGE model that is compatible with this assumption and explicitly considers the transport sector and regional price differentials is formulated. Trade coefficients, which are derived endogenously from the optimization behaviors of firms and households, are shown to take the form of a potential function. To investigate how the elasticity of substitutions affects equilibrium solutions, a simpler version of the model that incorporates three regions and two sectors (besides the transport sector) is introduced. Results indicate: (1) if commodities produced in different regions are perfect substitutes, regional economies will be either autarkic or completely symmetric and (2) if they are imperfect substitutes, the impact of elasticity on the price equilibrium system as well as trade coefficients will be nonlinear and sometimes very sensitive.

Keyword: Armington Assumption, spatial CGE, elasticity of substitution

^{*}Graduate School of Information Sciences, Tohoku University. (E-mail: ando@m.tains.tohoku.ac.jp)

[†]Institute of Developing Economies (IDE-JETRO) and the OECD. (E-mail: mengbo@ide.go.jp)

[‡]Graduate School of Information Sciences, Tohoku University. (E-mail: qu@se.is.tohoku.ac.jp)

1 Introduction

According to traditional trade theory (e.g. Samuelson (1953)), the phenomenon known as "cross-hauling" or "two-way trade" may not appear under perfect competition. On the contrary, it is quite common for a pair of countries to trade the same commodities with each other. Brander (1981) explains the existence of cross-hauling by introducing "imperfect competition" (strategic interaction among firms) into traditional trade theory. In addition to theoretical explanations, cross-hauling can also be interpreted from the following statistical viewpoints: (1) every practical classification of a commodity involves great diversity in quality, (2) a country often represents a highly aggregated area, and (3) trade statistics capture transactions in a finite period during which a country may seek supplies of a commodity from various countries due to seasonality and other factors. It must be noted that the first point explains the "intra-industry trade" of half-products that belongs to the same category as final products.

In many multi-regional models, potential type interregional trade coefficients are formulated to accommodate the observation of cross-hauling. For example, the most popular formulation assumes that the quantities of interregional trade are positively related to production (supply) capacities and negatively related to CIF prices. This kind of formulation can be derived from Wilson's entropy model (see Wilson (1970)). However, the problem of such formulations is that they are based on analogies in physics or on statistical principles; they do not provide a theoretical explanation from the view of rational and deterministic decision making of firms or individuals. Therefore, when such a formulation is used, there may be inconsistencies in economic models.¹ On the other hand, in CGE literature, the most commonly used method to justify the phenomenon of cross-hauling under a perfect competition market is to employ the Armington Assumption. This assumes that same commodities produced in different origins are imperfect substitutes for each other. The assumption of perfect competition, to some extent, is out of touch with economic reality. Thus, an imperfect competition approach appears to be more preferable in the CGE model to justify the existence of cross-hauling. However, this requires additional information on industry agglomeration (number of firms) as well as scale economies (data about fixed cost) for model calibration, and this is often extremely difficult to obtain. This is particularly true when developing economies or relatively small regions are studied. In addition, the "cross-hauling" caused by the statistical reasons described above is difficult to explain with imperfect competition. Therefore, in many spatial CGE (SCGE) models, perfect competition and the Armington Assumption are still the most popular and standard assumptions used by CGE modelers.

The Armington assumption is easy to incorporate and can also be used to justify the existence of cross-hauling under a perfect competition market. However, relationships among the Armington elasticity, trade coefficient, spatial price equilibrium (SPE), and model solutions have not been carefully clarified in existing SCGE literature. One reason is that existing studies tend to regard the transport sector as an ordinary service sector or as an imaginary transport agency that requires no resource for producing transport services (see Miyagi and Hongbu (1993) and

¹Meng and Ando (2005) shows that very similar potential type interregional trade coefficients can be logically derived from the economic principle of deterministic decision making of firm or individual under the framework of multi-regional input-output framework rather than from the vague and irrelevant concepts of social physics.

GTAP ² (1997)). The problem is that transport conditions, fares in particular, are a source of regional price differentials and should be consistent with the SPE system. Without an explicit consideration of the unique characteristics of the transport sector, it is difficult to explain how transport conditions affect trade patterns and the SPE system under given Armington elasticity. It is also difficult to show how Armington elasticity affects trade patterns and the SPE system under given transport conditions. To justify considering the behavior of transport firms explicitly in SCGE model, refer to Harker (1987)³, Haddad and Hewings (2001), and Macann (2005).

The application of the Armington Assumption also requires information about elasticity of substitution between goods from different regions, and this is normally difficult to estimate when the number of regions and sectors is large. In many existing SCGE models, such information is based on existing literature or given by the modelers without adequate verification of its accuracy. Without significant information on the elasticity of substitution, use of the Armington assumption may lead to arbitrary model simulation results. This is another reason why a detailed estimation of the property of the Armington Assumption in the SCGE model is important.⁴

Based on the Armington Assumption and the assumption of a perfect competitive market, a 3-region, 2-sector (besides the transport sector) SCGE model is formulated in Section 2 of this paper. The main feature of this model is that behaviors of the transport sector and transportation networks are explicitly considered. In Section 3, the computation algorithm of the SCGE model is discussed. This is followed by a detailed evaluation of the Armington Assumption property summarized relative to simulation results based on three different benchmark calibrations. Section 4 provides conclusions.

2 The SCGE Model

In this section, basic assumptions of the model are introduced and followed by detailed descriptions of the behavior of individual economic agents (general industries, households, and the transport sector). It is then shown: (1) trade coefficients can be endogenously derived from the deterministic decision making of firms or households under the Armington Assumption and (2) that the spatial price equilibrium condition can be obtained from the cost-minimization behavior of transport firms. Finally, general equilibrium conditions of the entire system are summarized. Definitions of all notations used in the formulations are given in Appendix A.

2.1 Basic Assumptions

(1) Numbers of regions and sectors: Three regions and two industrial sectors (non-transport sector) are assumed.

 $^{^2\}mathrm{Developed}$ by the World Trade Analysis Center in 1992. See http://www.gtap.age
con.purdue.edu/ for details.

³Harker (1987) introduced transport firms and networks into the framework of Takayama and Judge (1971). This made the SPE model a specific antecedent to development of the SCGE model.

 $^{^{4}}$ Due to the similar reason, Lofgren and Robinson (2002), Florenz (2005) and Ando and Meng (2009) use perfect substitution assumption to avoid using the Armington assumption in their CGE models.

- (2) Two factors of production: Two production factors of labor and physical capital are considered; both of these are immobile across regions and sectors⁵.
- (3) Three types of economic agents: General industrial sectors (non-transport firms), transport firms and households are assumed.
- (4) Transport demand: The demand for transport services is assumed to consist solely of derived demand that accompanies purchases of other commodities⁶. Transport services are supplied by the region of origin, and all transport costs are paid at origin.
- (5) Final demand: Final demand is only from household expenditure, household disposable income is equal to household consumption expenditure.
- (6) Imperfect substitutes: Commodities produced in different regions are imperfect substitutes for each other (Armington Assumption).

2.2 Behavior of Economic Agents

2.2.1 General Industries (Non-transport Firms)

The (aggregate) production function of sector j in region s combines the two factor inputs of labor L_j^s and capital stock K_j^s of sector j in region s, with the intermediate inputs x_{ij}^{rs} of commodity i produced in region r as follows:

$$X_{j}^{s} = A_{j}^{s} \prod_{i \neq 3} (\sum_{r} (x_{ij}^{rs})^{-\rho_{ij}^{s}})^{\frac{\alpha_{ij}^{s}}{-\rho_{ij}^{s}}} (L_{j}^{s})^{\alpha_{L_{j}}^{s}} (K_{j}^{s})^{\alpha_{K_{j}}^{s}}.$$
 (1)

The upper level of the production function uses a Cobb-Douglas type technology, and the lower level for intermediate inputs from different regions employs a CES type technology. X_j^s denotes the amount of output produced by industry j in region s, ρ_{ij}^s the substitution parameter⁷, and A_j^s the scale parameter. The notation "3" represents the transport sector. The following is assumed for the parameters α_{ij}^s , α_{Kj}^s and α_{Lj}^s :

Assumption 1 The production function is linearly homogeneous for each region: $\sum_{i \neq 3} \alpha_{ij}^s + \alpha_{Lj}^s + \alpha_{Kj}^s = 1^8.$

As a whole, non-transport firms face the problem of choosing a combination of $\{x_{ij}^{rs}, K_j^s, L_j^s\}$ to maximize their profits. This is described as follows:

$$\pi_j^s = p_j^s X_j^s - \sum_{i \neq 3} \sum_r (p_i^r + c_i^{rs}) x_{ij}^{rs} - \omega_j^s L_j^s - \gamma_j^s K_j^s.$$
(2)

⁷The elasticity of substitution can be written as follows: $\sigma_{ij}^s = \frac{1}{1 + \rho_{ij}^s}$, where $\rho_{ij}^s \ge -1$.

⁵This assumption can be easily modified to facilitate mobile capital and (or) labor.

⁶For simplicity, transport services are considered as freight transport. Passenger transport is combined with the "other services" sector.

⁸According to Basic Assumption (4), transport services (i = 3) are not considered as an intermediate input.

where p_j^s is the producer's (FOB) price of commodity j in region s, c_i^{rs} the transport cost for shipping a unit commodity i from region r to s, and $p_i^r + c_i^{rs}$ the purchasing (CIF) price of region s for the intermediate commodity i produced in region r. γ_j^s and ω_j^s are respectively the capital rent and wage rate.

One of the first-order conditions of equation (2) can be written as:

$$\frac{\partial \pi_j^s}{\partial x_{ij}^{rs}} = \frac{p_j^s \alpha_{ij}^s X_j^s (x_{ij}^{rs})^{-\rho_{ij}^s}}{x_{ij}^{rs} \sum_r (x_{ij}^{rs})^{-\rho_{ij}^s}} - (p_i^r + c_i^{rs}) = 0.$$
(3)

According to the Chenery-Moses assumption, the intermediate input in physical terms may be written as follows:

$$x_{ij}^{rs} = a_{ij}^{rs} X_j^s = t_i^{rs} a_{ij}^s X_j^s, (4)$$

where a_{ij}^{rs} is the interregional input coefficient in physical terms. t_i^{rs} and a_{ij}^{s} are respectively the regional trade coefficient and the regional input coefficient. Based on the above equation, equation (3) can be then simplified to the following:

$$\alpha_{ij}^{s} = \frac{\sum_{r} (t_i^{rs})^{-\rho_{ij}^{s}}}{p_j^{s}} \cdot \frac{t_i^{rs} (p_i^{r} + c_i^{rs})}{(t_i^{rs})^{-\rho_{ij}^{s}}} \cdot a_{ij}^{s}.$$
(5)

The above solution for x_{ij}^{rs} is available for any region r'. Thus, a similar result for $x_{ij}^{r's}$ is obtained:

$$\alpha_{ij}^{s} = \frac{\sum_{r} (t_i^{rs})^{-\rho_{ij}^{s}}}{p_j^{s}} \cdot \frac{t_i^{r's} (p_i^{r'} + c_i^{r's})}{(t_i^{r's})^{-\rho_{ij}^{s}}} \cdot a_{ij}^{s}.$$
(6)

Dividing (5) by (6) yields the following equation:

$$\frac{t_i^{rs}}{t_i^{r's}} = \left(\frac{p_i^r + c_i^{rs}}{p_i^{r'} + c_i^{r's}}\right)^{\frac{-1}{1+\rho_{i_j}^s}}.$$
(7)

Summing both sides with r' and using the condition $\sum_{r'} t_i^{r's} = 1$, trade coefficients can be derived as follows:

$$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{\frac{-1}{1+\rho_{ij}^s}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{-1}{1+\rho_{ij}^s}}}.$$
(8)

This form implies that trade coefficients depend on producer prices p_i^r and transport costs c_i^{rs} . This is very similar in form to a potential type function.

The total of intermediate inputs in production function (1) can also be considered as a composite good D_{ij}^s , specifically

$$D_{ij}^{s} = \left(\sum_{r} (x_{ij}^{rs})^{-\rho_{ij}^{s}}\right)^{-\frac{1}{\rho_{ij}^{s}}}.$$
(9)

The profit function (2) may be rewritten as follows:

$$\pi_{j}^{s} = p_{j}^{s} X_{j}^{s} - \sum_{i \neq 3} q_{i}^{s} D_{ij}^{s} - \gamma_{j}^{s} K_{j}^{s} - \omega_{j}^{s} L_{j}^{s},$$
(10)

where q_i^s is considered to be the purchasing price index of composite good D_{ij}^s in region s.

First-order conditions to the profit-maximization problem of equation (10) may be written as follows:

$$\alpha_{ij}^{s} = \frac{q_{i}^{s} D_{ij}^{s}}{p_{j}^{s} X_{j}^{s}} , \quad \alpha_{Lj}^{s} = \frac{\omega_{j}^{s} L_{j}^{s}}{p_{j}^{s} X_{j}^{s}} , \quad and \quad \alpha_{Kj}^{s} = \frac{\gamma_{j}^{s} K_{j}^{s}}{p_{j}^{s} X_{j}^{s}}.$$
(11)

The above parameters are simply the regional input coefficients measured in monetary terms. Since the regional input coefficients in physical terms can be given as

$$a_{ij}^{s} = \frac{D_{ij}^{s}}{X_{j}^{s}}$$
, $a_{Lj}^{s} = \frac{L_{j}^{s}}{X_{j}^{s}}$, and $a_{Kj}^{s} = \frac{K_{j}^{s}}{X_{j}^{s}}$, (12)

the relationship between monetary and physical regional input coefficients can be written as follows:

$$a_{ij}^{s} = \frac{p_{j}^{s}}{q_{i}^{s}} \alpha_{ij}^{s} , \quad a_{Lj}^{s} = \frac{p_{j}^{s}}{\omega_{j}^{s}} \alpha_{Lj}^{s} , \quad and , \quad a_{Kj}^{s} = \frac{p_{j}^{s}}{\gamma_{j}^{s}} \alpha_{Kj}^{s}.$$
(13)

Alternatively, according to Shephard's duality, the composite price can be written in the following form:

$$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs}.$$
 (14)

This equation indicates that the composite price (market price) q_i^s can be regarded as an average value of CIF prices for a commodity *i* supplied from various regions weighted by a trade coefficient (see Appendix B).

2.2.2 Households

The source of income for households is the gross regional domestic product V^s comprising rent and wage payments:

$$V^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s, \tag{15}$$

where regions are assumed to be closed in terms of factor income. For simplicity, firms and their capital are considered to be owned by the households of the region where they are located. Further, since tax and income transfer are ignored, household disposable income W^s should equal V^s in the model.

The aggregate utility function of households in region s is considered to depend only on y_i^{rs} , the amount of commodity *i* produced in region *r* consumed in region *s*. The problem of households is thus to choose $\{y_i^{rs}\}$ such that their utility is maximized:

$$\underset{y_i^{rs}}{\operatorname{Max}} \qquad U^s = \prod_{i \neq 3} \left(\sum_r (y_i^{rs})^{-\delta_i^s} \right)^{\frac{\beta_i}{-\delta_i^s}},\tag{16}$$

under the budget constraint

$$s \, . \, t. \qquad \sum_{i \neq 3} \sum_{r} (p_i^r + c_i^{rs}) y_i^{rs} = W^s, \tag{17}$$

where W^s is the disposable income of households, $\delta_i^s \ge -1$ the substitution parameter, and β_i^s the final demand parameter.

Parallel to the production function, linear homogeneity of the utility function is assumed:

Assumption 2 The utility function is linearly homogeneous, $\sum_{i\neq 3} \beta_i^s = 1$.

From the above first-order condition, the final demand parameter for y_i^{rs} may be derived as follows:

$$\beta_i^s = \frac{\lambda^s (p_i^r + c_i^{rs}) \sum_r (y_i^{rs})^{-\delta_i^s}}{U^s (y_i^{rs})^{-\delta_i^s - 1}}.$$
(18)

The same parameter for $y_i^{r's}$ may be given as:

$$\beta_i^s = \frac{\lambda^s (p_i^{r'} + c_i^{r's}) \sum_r (y_i^{rs})^{-\delta_i^s}}{U^s (y_i^{r's})^{-\delta_i^s - 1}}.$$
(19)

Dividing (18) by (19), the trade coefficient of final demand goods $(t_{i(h)}^{rs})$ can be obtained:

$$t_{i(h)}^{rs} = \frac{(p_i^r + c_i^{rs})^{\frac{-1}{1+\delta_i^s}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{-1}{1+\delta_i^s}}}.$$
(20)

The form of this trade coefficient is very similar to that in equation (8). For simplicity, the following assumption is used in the model:

Assumption 3 Substitution parameters of general industries and households are dependent only on their destinations and commodities. Both are equal to each other: $\rho_{ij}^s = \rho_i^s$ and $\delta_i^s = \rho_i^s$.

Under the above assumption, a general trade coefficient which includes both intermediate inputs and final demands emerges:

$$t_i^{rs} \equiv \frac{T_i^{rs}}{\sum_r T_i^{rs}} = \frac{\sum_j x_{ij}^{rs} + y_i^{rs}}{\sum_r (\sum_j x_{ij}^{rs} + y_i^{rs})} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}.$$
(21)

From this equation and the first-order condition, the composite consumption of commodity i by households in region $s (y_i^s = (\sum_r (y_i^{rs})^{-\delta_i^s})^{-\frac{1}{\delta_i^s}})$ can be written as follows:

$$y_i^s = \frac{\beta_i^s W^s}{q_i^s}.$$
(22)

2.2.3 Transport Sector

Under Basic Assumption (4), all demands of this sector are derived from purchases of other commodities. Non-transport firms can determine output levels to maximize their profits, but transport firms are required to provide transport services needed to fulfill demands of other commodities and services. Thus, they seek to minimize costs given the level of services.

For convenience, the following assumption concerning transport cost payments is introduced:

Assumption 4 Transport costs are paid at origin. This applies to purchases by the transport sector itself. However, they do not recognize the imputed costs that accompany their own purchases from the regions in which they are located⁹.

⁹Transport costs that accompany intra-regional purchases of transport sectors are paid to the transport sectors themselves. Thus, they can be deducted from the total cost of producing the transport services required.

Total transport demands originating in region s, in monetary terms, would be given by the LHS of the following formula:

$$\sum_{i \neq 3} \sum_{r} c_i^{sr} (\sum_j x_{ij}^{sr} + y_i^{sr}) = p_3^s X_3^s.$$
(23)

Under Assumption 4, these demands would be fulfilled by transport firms in region s, and their monetary output $p_3^s X_3^s$ must exceed these demands. The cost to provide services required may then be written as follows:

$$C_3^s = \sum_{i \neq 3} \sum_{r \neq s} (p_i^r + c_i^{rs}) x_{i3}^{rs} + \sum_{i \neq 3} p_i^s x_{i3}^{ss} + \omega_3^s L_3^s + \gamma_3^s K_3^s.$$
(24)

The production function of transport firms is also given by equation (1). The problem is to choose $\{x_{i3}^{rs}, K_3^s, L_3^s\}$ so that the total cost (24) is minimized while satisfying transport demands (23).

The first-order condition of intermediate inputs may be written with the Lagrange multiplier μ^s associated with (23) as follows:

$$a_{i3}^{s} = \frac{\mu^{s} p_{3}^{s}}{q_{i}^{s}} \alpha_{i3}^{s} = \frac{\mu^{s} p_{3}^{s}}{p_{i}^{s} + \mu^{s} c_{i}^{ss}} \alpha_{i3}^{s}.$$
(25)

The first expression represents purchases from other regions, x_{i3}^{rs} $(r \neq s)$. The second expression is for intra-regional purchases. From the above equation, the relation between FOB and CIF prices may be expressed as:

$$q_i^s = p_i^s + \mu^s c_i^{ss}.$$
 (26)

Finally, conditions for factor inputs can be written as follows:

$$a_{K3}^{s} = \frac{\mu^{s} p_{3}^{s}}{\gamma_{3}^{s}} \alpha_{K3}^{s} \quad and \quad a_{L3}^{s} = \frac{\mu^{s} p_{3}^{s}}{\omega_{3}^{s}} \alpha_{L3}^{s}.$$

$$(27)$$

2.3 Equilibrium Conditions

In this section, equilibrium conditions are summarized. Many are obtained by incorporating first-order conditions of individual agents into the price and output equations of the interregional input-output system.

2.3.1 Price Equations

Price equations correspond to column sums of the input-output table. Three different patterns of equations must be prepared for non-transport and transport sectors as well as for final demands. The equation for non-transport sectors may be written as follows:

$$p_{j}^{s}X_{j}^{s} = \sum_{i \neq 3} \sum_{r} p_{i}^{r}t_{i}^{rs}a_{ij}^{s}X_{j}^{s} + \sum_{i \neq 3} \sum_{r} c_{i}^{rs}t_{i}^{rs}a_{ij}^{s}X_{j}^{s} + \omega_{j}^{s}a_{Lj}^{s}X_{j}^{s} + \gamma_{j}^{s}a_{Kj}^{s}X_{j}^{s}.$$
(28)

Using (13) to eliminate a_{ij}^s , and dividing both sides by $p_j^s X_j^s$,

$$1 = \sum_{i \neq 3} \frac{\alpha_{ij}^s}{q_i^s} \sum_r (p_i^r + c_i^{rs}) t_i^{rs} + \alpha_{Lj}^s + \alpha_{Kj}^s.$$
(29)

According to the definition of market price q_i^s (equation (14)), it is easy to see that the above equation is simply Assumption 1 of linear homogeneity in general sectors.

A similar argument can be applied to final demand:

$$W^{s} = \sum_{i \neq 3} \frac{\beta_{i}^{s} W^{s}}{q_{i}^{s}} \sum_{r} (p_{i}^{r} + c_{i}^{rs}) t_{i}^{rs}.$$
(30)

This equation is consistent with Assumption 2, specifically $\sum_{i\neq 3} \beta_i = 1$. Similarly the price equation of transport sectors may be written as follows:

$$\frac{1}{\mu^s} = \sum_{i \neq 3} \sum_r \frac{(p_i^r + c_i^{rs}) t_i^{rs}}{q_i^s} \cdot \alpha_{i3}^s + \alpha_{L3}^s + \alpha_{K3}^s, \tag{31}$$

where costs accompanying intra-regional purchases of its own are taken into account. $\mu^s = 1$ must hold in order to comply with Assumption 1. Equation (26) can then be rewritten as follows:

$$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} = p_i^s + c_i^{ss}.$$
(32)

This is the only meaningful condition derived from the price equations.

2.3.2 Output Equations

Output equations correspond to row sums of the input-output table. Output levels for nontransport sectors can be measured in physical units. Hence,

$$X_{i}^{r} = \sum_{s} \frac{t_{i}^{rs}}{p_{i}^{s} + c_{i}^{ss}} (\sum_{j} \alpha_{ij}^{s} p_{j}^{s} X_{j}^{s} + \beta_{i}^{s} W^{s}).$$
(33)

For the special property of transport sectors defined in *Basic Assumption* 4, output level in the transport sector can only be written in monetary terms:

$$p_3^r X_3^r = \sum_{i \neq 3} \sum_s \frac{c_i^{rs} t_i^{rs}}{p_i^s + c_i^{ss}} (\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s).$$
(34)

2.3.3 Factor Market and Final Demand

According to Basic Assumption 2, capital rent and wage rate are determined as follows:

$$\omega_j^s = \alpha_{Lj}^s p_j^s \frac{X_j^s}{L_j^s},\tag{35}$$

and

$$\gamma_j^s = \alpha_{Kj}^s p_j^s \frac{X_j^s}{K_j^s}.$$
(36)

The formula for the expenditure item can be summarized as follows:

$$W^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s.$$
(37)

3 Simulation and Analysis

In this section, equilibrium conditions and variables used in the model are summarized. The computational procedure applied for calculation of benchmark equilibriums is then explained. Finally, the benchmark equilibrium solutions are used to evaluate in detail the relationships among Armington elasticity, transport conditions and endogenous solutions of the model.

3.1 Equations, Variables, and Calculations

Equations describing equilibrium are summarized in Table 1. Since prices in transport sectors cannot be distinguished from their quantities, their product $(p_3^r X_3^r)$ is considered to be an independent variable. Variables and parameters of the system are summarized in Table 2. There are 54 endogenous variables, and this is equal to the number of equilibrium conditions.

The model is composed of a system of nonlinear simultaneous equations. However, each equation is not uniformly interconnected with other equations. Several blocks of equations can be identified that are relatively independent from other blocks. Considering this structural features of the equation system, the entire system may be divided into three blocks (see Table 1). These include a trade coefficient block (T), a price block (P) and the block (X,W, ω , γ) for other endogenous variables. Each block takes a nonlinear programming form to minimize the sum of squared errors from relevant equilibrium conditions. The system solution constitutes a series of convergence calculations using an iterative procedure based on a quasi-Newton algorithm.

It should be noted that interregional transport costs are considered exogenous to the model. Actual transport costs c_i^{rs} , which are different among sectors, can be assumed to be proportional to interregional time-distances d^{rs} : $c_i^{rs} = \xi_i d^{rs}$. d^{rs} can be based on the shortest time paths between pairs of regional geographical centers or capitals.

3.2 Results of Simulation

Three different benchmark equilibriums were calculated to test the impact of Armington elasticity on trade coefficients, price system and other endogenous solutions in detail under given transport conditions. Benchmark 1 represents an economic system in which the distribution pattern of interregional transport costs is completely uniform. Benchmark 2 provides an economy in which the distribution pattern of transport costs is completely symmetric with relatively low intra-regional transport costs. Benchmark 3 shows a non-symmetric economic system in which transport cost between two selected regions are lower than other regions. These three benchmark situations are compared under the following two scenarios: (1) Armington elasticities are perfect substitutes ($\sigma_i^s = \infty$) and (2) Armington elasticities are imperfect substitutes.

3.2.1 Benchmark 1

The parameters and exogenous variables used in Benchmark 1 are shown below: $\alpha_{ij}^s = 0.25, \quad \forall \ i, j, s$ $\alpha_{Lj}^s = \alpha_{Kj}^s = 0.25 \quad \forall j, s$ $\beta_i^s = 0.50 \quad \forall \ i, s$ $L_j = K_j = 100.00 \quad \text{for } j = 1, 2, \ L_3 = K_3 = 40$ $c_i^{rs} = 0.20 \quad \forall \ r, s, i \text{ (see Figure 1)}$.

	1	
Equations	Numbers	Blocks
General Sectors:	3×2	
$X_i^r = \sum_s \frac{t_i^{rs}}{p_i^s + c_i^{ss}} \left(\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s \right)$	eq.(33)	
Transport Sectors:	3	
$p_3^r X_3^r = \sum_{i \neq 3} \sum_s \frac{c_i^{rs} t_i^{rs}}{p_i^s + c_i^{ss}} (\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s)$	eq.(34)	Х
Wage Rate:	$3 \times 2 + 3$	W
$\omega_j^s = \alpha_{Lj}^s p_j^s X_j^s / L_j^s$	eq.(35)	ω
Capital Rent:	$3 \times 2 + 3$	γ
$\gamma_j^s = \alpha_{Kj}^s p_j^s X_j^s / K_j^s$	eq.(36)	
Households:	3	
$W^s = \sum_j \omega^s_j L^s_j + \sum_j \gamma^s_j K^s_j$	eq.(37)	
Price System:	3×2	
$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs} = p_i^s + c_i^{ss}$	eq.(32)	Р
Trade Coefficient:	$3 \times 2 \times 3$	
$\underline{t_i^{rs}} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}$	eq.(21)	Т
	Subtotal: 54	

Table 1: Equilibrium Conditions

Table 2	2: Variables and Parameters
	$X_i^r (3 \times 2) , p_3^s X_3^s (3) ,$
Endogenous Variables	$p_i^r (3 \times 2)$, $\omega_i^s (3 \times 2 + 3)$, $\gamma_i^s (3 \times 2 + 3)$,
and Subtotal	$W^s(3), t_i^{rs}(3 \times 2 \times 3)$ Subtotal: 54.
Exogenous Variables	$K_{j}^{s}, L_{j}^{s}, c_{i}^{rs}(c_{i}^{rs} = \xi_{i}d^{rs}).$
Parameters	α_{ij}^s , α_{Kj}^s , α_{Lj}^s , β_i^s , σ_i^s , ξ_i .
	$\sum_{i \neq 3} \alpha_{ij}^{s} + \alpha_{Lj}^{s} + \alpha_{Kj}^{s} = 1 , \sum_{i \neq 3} \beta_i^{s} = 1.$

11

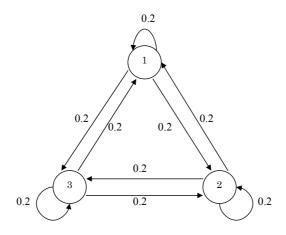


Figure 1: c_i^{rs} in Benchmark 1

The endogenous variables for calculation of convergence are initialized with the following values:

 $\begin{array}{l} p_i^s = 1.00, \quad \forall i,s \\ X_j^s = 100.00, \quad \forall j,s \end{array}$

From these conditions, it is easy to see that the given economy is a completely uniform system. Here, the system is solved under the following two scenarios:

Scenario 1: $\sigma_i^s = 10, \quad \forall i, s$

Scenario 2: $\sigma_i^s = \infty$, $\forall i, s$, where $\sigma_i^s = 1/(1 + \rho_i^s)$

The calculation results for Benchmark 1 are summarized in Table 3. Obviously, endogenous solutions are also completely uniform under both scenarios. This means that the relationship between Armington elasticity and model solutions is very robust when the distribution pattern of interregional transport costs is completely uniform.

	10010 0	· Solut		, Dila		, taria		Bonor	iiiiai ii	$-, (\circ_i$	10	0, 00)	·
	p p	p_i^r	X	$\sum_{i=1}^{r}$	$p_3^r X_3^r$	ω_i^r			W^r		t_i^{rs}		U^r
	i= 1	2	1	2		1	2	3		s=1	2	3	
r = 1	1.00	1.00	100	100	40	0.25	0.25	0.25	120	1/3	1/3	1/3	2500
r = 2	1.00	1.00	100	100	40	0.25	0.25	0.25	120	1/3	1/3	1/3	2500
r = 3	1.00	1.00	100	100	40	0.25	0.25	0.25	120	1/3	1/3	1/3	2500

Table 3: Solutions of Endogenous Variables in Benchmark 1, $(\sigma_i^s = 10 \text{ or } \infty)$

3.2.2 Benchmark 2

Benchmark 2 presents a relatively real situation in which the interregional transport costs are changed as follows:

 $c_i^{rs} = 0.20 \quad \forall i, \text{ when } r \neq s,$

 $c_i^{rr} = 0.10 \quad \forall r, i \text{ (see Figure 2)}$.

Other initialization conditions are the same as those in Benchmark 1.

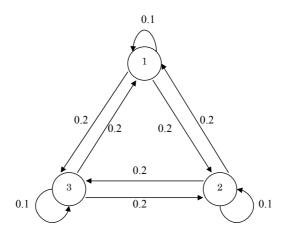


Figure 2: c_i^{rs} in Benchmark 2

The calculation results for Benchmark 2 under Scenario 1 and 2 are summarized in Table 4 and 5 respectively. Both give completely symmetric solutions. Specifically, comparing with those for Benchmark 1 under Scenario 1, the elasticities of substitution are 10; the outputs of non-transport sectors, regional utilities, and factor prices in non-transport sectors for each region go up. This means that the reduction of intraregional transport costs has a positive effect on outputs of non-transport sectors, regional utilities, and factor prices. However, this directly decreases the revenue of the transport sector, thus, output and labor wages in transport sector seem to be damaged. Conversely, relatively lower intraregional transport costs have a positive impact on intraregional trade coefficients. This is easily understood since the reduction of intraregional transport costs will lower production costs inside the region. Firms and households can then enjoy relatively low CIF prices inside the region. As an extreme case, when Armington elasticity approaches to an infinite value, the economic system develops an autarkic pattern for each region. This means that when intermediate inputs can be perfectly substituted with each other among regions, every region will simply import goods and services from its own region since CIF prices inside the region are the lowest.

Table 4: Solutions of Endogenous Variables in Benchmark 2, $(\sigma_i^s = 10)$

	p_i^r	X_i^r	$p_3^r X_3^r$	ω	r_i	W^r		t_i^{rs}		U^r
	i=1, 2	1, 2		1, 2	3		s=1	2	3	
r = 1	1.00	101.57 \uparrow	27.37 ↓	$0.254\uparrow$	$0.17\downarrow$	115.25 ↓	$0.54\uparrow$	0.23↓	0.23↓	$2744\uparrow$
r=2	1.00	101.57 \uparrow	27.37 ↓	$0.254\uparrow$	$0.17\downarrow$	$115.25\downarrow$	$0.23\downarrow$	$0.54\uparrow$	0.23↓	$2744\uparrow$
r = 3	1.00	$101.57\uparrow$	27.37 ↓	$0.254\uparrow$	$0.17\downarrow$	$115.25\downarrow$	$0.23\downarrow$	0.23↓	$0.54\uparrow$	$2744\uparrow$

3.2.3 Benchmark 3

Compared with Benchmarks 1 and 2, Benchmark 3 has initial economic conditions that are nonuniform and non-symmetric. The transport costs of Commodity 1 from Region 1 to Region 2

	p_i^r	X_i^r	$p_3^r X_3^r$	ú	ω_i^r	W^r		t_i^{rs}		U^r
	i=1,2	1, 2		1,2	3		s=1	2	3	
r = 1	1.00	$101.95\uparrow$	20.47 ↓	$0.26\uparrow$	0.13 ↓	112.18 ↓	$1.00\uparrow$	0 ↓	0↓	2600↑
r=2	1.00	$101.95\uparrow$	20.47 ↓	$0.26\uparrow$	$0.13\downarrow$	112.18 ↓	0↓	$1.00\uparrow$	0↓	$2600\uparrow$
r = 3	1.00	$101.95\uparrow$	20.47 ↓	$0.26\uparrow$	0.13 ↓	112.18 ↓	0↓	0 ↓	$1.00\uparrow$	$2600\uparrow$

Table 5: Solutions of Endogenous Variables in Benchmark 2, $(\sigma_i^s = \infty)$

decrease from 0.2 to 0.15. This not only provides a check of the impact of Armington elasticity on model solutions but also helps to simulate the impact of transport cost reduction for a selected pair of regions on the whole economy. Interregional transport costs for Benchmark 3 are as follows:

 $\begin{array}{l} c_i^{rs} = 0.20 \ \, \forall \ \, i, \, {\rm when} \ \, r \neq s, \\ c_i^{rr} = 0.10 \ \, \forall \ \, r, i \\ c_1^{12} = 0.15 \ \, ({\rm see \ Figure \ } 3). \end{array}$

Other initialization conditions are the same as those in Benchmark 2.

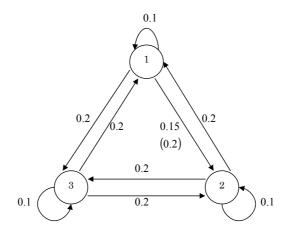


Figure 3: c_i^{rs} in Benchmark 3

The calculation results of Benchmark 3 under Scenarios 1 and 2 are shown in Tables 6 and 7 respectively. Obviously, under the non-symmetric distribution pattern of interregional transport costs, equilibrium solutions yield a non-symmetric image. Under Scenario 1, the prices of Commodity 1 for Regions 1 and 2 are reduced because the interregional transport cost for Commodity 1 from Region 1 to Region 2 lowers CIF prices in both Region 1 and Region 2. Comparing decreasing CIF prices in these two regions, the CIF price in Region 3 inevitably becomes relatively high. This high price will have a negative impact on the output of Commodity 1 in Region 3, since high prices result in low demand. It should be noted that the output of Commodity 2 in Region 1 and 2 also produces negative effects. This is because the relatively lower CIF price of Commodity 1 in Regions 1 and 2 will boost the demand for Commodity 1 produced in these regions to satisfy the increased demand. Non-transport firms in these regions must transfer resources originally used for the production of Commodity 2. As a result, outputs of Commodity 2 in both regions decline. The pattern of trade coefficients also becomes non-symmetric. Compared with Benchmark 2, every region tends to import relatively more of Commodity 1 from Regions 1 and 2. This also results from the reduction of transport costs between Region 1 and Region 2. However, when Armington elasticity approaches infinity, the economic system becomes autarkic for each region as in the solution of Benchmark 2. This means that when intermediate inputs are perfectly substituted for each other among regions, every region will simply import goods and services from their own region, even if the initial condition is non-symmetric.

	p	p_i^r	X	r_i	$p_3^r X_3^r$		ω_i^r		W^r		t_1^{rs}		U^r
	i=1	2	1	2		1	2	3		s=1	2	3	
r = 1	0.99	1.00	112.13	97.91	23.26	0.28	0.26	0.15	116.15	0.56	0.29	0.27	2811
r=2	0.97	1.00	115.00	96.36	24.18	0.28	0.24	0.15	115.85	0.29	0.58	0.33	2850
r = 3	1.04	1.00	71.27	103.72	22.61	0.17	0.26	0.14	100.31	0.15	0.13	0.41	2006

Table 6: Solutions of Endogenous Variables in Benchmark 3, $(\sigma_i^s = 10)$

Table 7: Solutions of Endogenous Variables in Benchmark 3, $(\sigma_i^s = \infty)$

		p_i^r	X	r_i	$p_3^r X_3^r$		ω_i^r		W^r		t_1^{rs}		U^r
	i=1	2	1	2		1	2	3		s=1	2	3	
r = 1	1.00	1.00	101.95	101.95	20.46	0.26	0.26	0.13	112.18	1.00	0	0	2600
r=2	1.00	1.00	101.95	101.95	20.46	0.26	0.26	0.13	112.18	0	1.00	0	2600
r = 3	1.00	1.00	101.95	101.95	20.46	0.26	0.26	0.13	112.18	0	0	1.00	2600

Summarizing, if elasticity of substitution is set at an infinite value, only two solution patterns can be obtained: (1) a completely symmetric regional economic structure and (2) complete autarkic regional economies. For details of other calculation results, refer to Appendix C.

3.2.4 Sensitivity Check of Armington Elasticity

Using the parameters of Benchmark 2, a sensitivity test was conducted to show how the change of Armington elasticity may affect the price equilibrium system and trade coefficients under given transport conditions. Armington elasticity σ_2^1 is the target parameter tested. Other elasticities were set as follows: $\sigma_i^s = 2.0, \forall s \neq 1, i \neq 2$.

Figures 4 and 5 show results of the sensitivity check of Armington elasticity on price system and trade coefficients respectively. Obviously, a change in σ_2^1 does not affect the price and trade coefficient of Commodity 1 since the elasticity between Commodity 1 and Commodity 2 equals 1 (see production function (1)). However, the price and trade coefficients of Commodity 2 show a changing pattern that is very sensitive and non-linear. This implies that Armington elasticity should be given carefully since it is not robust enough to be given arbitrarily.

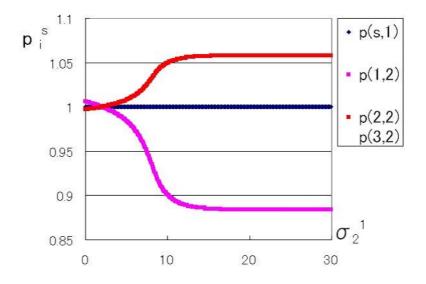


Figure 4: Sensitivity Check of Armington Elasticity on the Price Equilibrium System

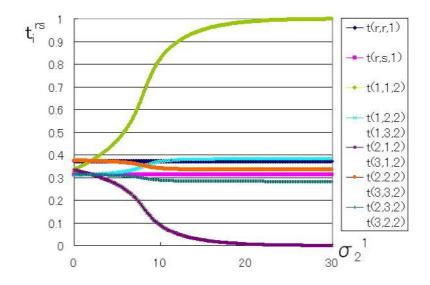


Figure 5: Sensitivity Check of Armington Elasticity on Trade Coefficients

4 Conclusion

The Armington Assumption takes products with the same name and coming from different countries of origin to be imperfect substitutes for each other. This assumption has been widely used in existing CGE models. To investigate in detail the impact of this assumption on a spatial price equilibrium system, trade coefficients and other model solutions, a 3-region, 2commodity simple spatial CGE model was formulated with explicit consideration of transport sector and regional price differentials. With the Armington Assumption, the model shows that trade coefficients can be endogenously derived from rational and deterministic decision making of firms or households. Using this trade coefficient, model solutions show that: (1) if commodities produced in different regions are perfect substitutes, the regional economies will become autarkic or develop a complete symmetric pattern and (2) if they are imperfect substitutes, the impact of elasticity on the price equilibrium system and trade coefficients is nonlinear and sometimes very sensitive.

Appendix A: Symbols in the Paper

Symbols	Definitions
X_j^s	production of industry j in egion s
	interregional input of good i from region r to region s , used in industry j
D_{ij}^{s}	composite intermediate input goods i of industry j in region s
L_{i}^{s}	labor input of industry j in region s
$\check{K_{i}^{s}}$	capital input of industry j in region s
π_{i}^{s}	profit of industry j in region s
p_j^s	FOB price of good j produced in region s
$\begin{array}{c} x_{ij}^{rs} \\ D_{ij}^{s} \\ L_{j}^{s} \\ K_{j}^{s} \\ \pi_{j}^{s} \\ p_{j}^{s} \\ q_{i}^{s} \\ c_{i}^{rs} \\ U^{s} \end{array}$	CIF price of good i used in region s
c_i^{rs}	transport cost for shipping goods i from region r to region s
	utility of households in region s
y_i^{rs}	households' consumption of region s for good i produced in region r
y_i^s	composite final consumption of good i in region s
W^s	income of households in region s
A_j^s	scale parameter in production function
$ ho_{ij}^s$	substitution parameter used in production function
$lpha_{ij}^s$	regional input coefficient of intermediate goods, measured in monetary terms
α^s_{Lj}	regional input coefficient of labor, measured in monetary terms
α^s_{Kj}	regional input coefficient of capital, measured in monetary terms
ω_j^s	wage rate of industry j in region s
γ_j^s	capital rent of industry j in region s
a_{ij}^s	regional input coefficient of intermediate goods, measured in physical terms
t_i^{rs}	trade coefficient (physical term)
δ^s_i	substitution parameter used in utility function s
$\rho_{ij}^{s} \\ \alpha_{ij}^{s} \\ \alpha_{Lj}^{s} \\ \omega_{j}^{s} \\ \alpha_{ij}^{s} \\ \lambda_{j}^{s} \\ \alpha_{ij}^{s} \\ \tau_{i}^{s} \\ \delta_{i}^{s} \\ \beta_{i}^{s} $	final demand parameter (monetary term)
σ_i^s	parameter representing the elasticity of substitution s

Table A.1: Symbols in the Paper

Appendix B: Alternate Derivation of Trade Coefficients

In modern microeconomics, it is well known that a duality exists between production function and cost function. According to "Shephard's duality", the unit cost function can be represented as follows:

$$p_j^s = \prod_i \left[\frac{1}{\alpha_{ij}} \left(\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}} \right)^{\frac{1+\rho_i}{\rho_i}} \right]^{\alpha_{ij}} \left[\frac{\gamma_j^s}{\alpha_{Kj}^s} \right]^{\alpha_{Kj}^s} \left[\frac{\omega_j^s}{\alpha_{Lj}^s} \right]^{\alpha_{Lj}^s}.$$
(B.1)

Shephard's lemma was used in this paper to produce the unit demand function for input x_{ij}^{rs} shown below. This theoretically equals the interregional input-output coefficient.

$$\frac{\partial p_j^s}{\partial (p_i^r + c_i^{rs})} = \frac{\alpha_{ij} p_j^s}{p_i^r + c_i^{rs}} \frac{(p_i^r + c_i^{rs})^{\frac{\mu_i}{1 + \rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1 + \rho_i}}} = a_{ij}^{rs}$$
(B.2)

Using equations (4) and (13) and arranging the above equation,

$$t_i^{rs} = \frac{q_i^s}{p_i^r + c_i^{rs}} \frac{(p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}.$$
(B.3)

Moving the term $p_i^r + c_i^{rs}$ to the left side and computing \sum_r for both sides,

$$q_i^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs}.$$
 (B.4)

This implies that the purchasing price index can be considered as an average value of purchasing prices weighted by trade coefficients. Since $\sum_{r} t_i^{rs} = 1$, summarizing both sides of (B.3) by r, a different expression of purchasing prices indices is obtained as follows:

$$q_i^s = \frac{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_i}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}.$$
(B.5)

Substituting the above equation into equation (B.3) to calculate t_i^{rs} results in the followings

$$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}{\sum_r (p_i^r + c_i^{rs})^{-\frac{1}{1+\rho_i}}}.$$

This is the same as earlier equation (8).

Appendix C: Results of Simulation

$\overline{x_{ij}^{rs}}$			1			2			3	
		1	2	3	1	2	3	1	2	3
	1	12.56	12.56	3.36	5.26	5.26	1.42	5.26	5.26	1.42
1	2	12.56	12.56	3.36	5.26	5.26	1.42	5.26	5.26	1.42
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
2	2	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56	3.86
3	2	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56	3.86
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table C.1: x_{ij}^{rs} in Physical Terms for Benchmark 2, $(\sigma_i^s = 10)$

Table C.2: y_i^{rs} in Physical Terms for Benchmark 2, $(\sigma_i^s = 10)$

y_i^{rs}		1	2	3
	1	28.51	11.94	11.94
1	2	28.51	11.94	11.94
	3	0.00	0.00	0.00
	1	11.94	28.51	11.94
2	2	11.94	28.51	11.94
	3	0.00	0.00	0.00
	1	11.94	11.94	28.51
3	2	11.94	11.94	28.51
	3	0.00	0.00	0.00

$\overline{x_{ij}^{rs}}$			1			2			3	
		1	2	3	1	2	3	1	2	3
	1	12.56	12.56	3.39	5.26	5.26	1.42	5.26	5.26	1.42
1	2	12.56	12.56	3.39	5.26	5.26	1.42	5.26	5.26	1.42
	3	5.29	5.29	1.79	4.88	4.88	1.68	4.88	4.88	1.68
	1	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
2	2	5.26	5.26	1.42	12.56	12.56	3.39	5.26	5.26	1.42
	3	4.88	4.88	1.69	5.29	5.29	1.79	4.88	4.88	1.69
	1	5.26	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56
3	2	5.26	5.26	5.26	1.42	5.26	5.26	1.42	12.56	12.56
	3	4.88	4.88	1.69	4.88	4.88	1.68	5.29	5.29	1.79

Table C.3: x_{ij}^{rs} in FOB Prices for Benchmark $2, (\sigma_i^s = 10)$

Table C.4: y_i^{rs} in FOB Prices for Benchmark $2, (\sigma_i^s = 10)$

	1	2	3
1	28.51	11.94	11.94
2	28.51	11.94	11.94
3	12.39	11.44	11.44
1	11.94	28.51	11.94
2	11.94	28.51	11.94
3	11.44	12.39	11.44
1	11.94	11.94	28.51
2	11.94	11.94	28.51
3	11.44	11.44	12.39
		2 28.51 3 12.39 1 11.94 2 11.94 3 11.44 1 11.94 2 11.94	228.5111.94312.3911.44111.9428.51211.9428.51311.4412.39111.9411.94211.9411.94

Table C.5: x_{ij}^{rs} in Physical Terms for Benchmark $2, (\sigma_i^s = \infty)$

$\overline{x_{ij}^{rs}}$		1				2		3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

y_i^{rs}		1	2	3
	1	50.99	0.00	0.00
1	2	50.99	0.00	0.00
	3	0.00	0.00	0.00
	1	0.00	50.99	0.00
2	2	0.00	50.99	0.00
	3	0.00	0.00	0.00
	1	0.00	0.00	50.99
3	2	0.00	0.00	50.99
	3	0.00	0.00	0.00

Table C.6: y_i^{rs} in Physical Terms for Benchmark $2, (\sigma_i^s = \infty)$

Table C.7: x_{ij}^{rs} in FOB Prices for Benchmark $2, (\sigma_i^s = \infty)$

			ij							
x_{ij}^{rs}	$\begin{bmatrix} s \\ ij \end{bmatrix}$ 1			2			3			
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	9.27	9.27	1.86	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
	3	0.00	0.00	0.00	9.27	9.27	9.27	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	9.27	9.27	1.86

0	/				
-	y_i^{rs}		1	2	3
		1	50.99	0.00	0.00
	1	2	50.99	0.00	0.00
		3	20.40	0.00	0.00
		1	0.00	50.99	0.00
	2	2	0.00	50.99	0.00
		3	0.00	20.40	0.00
-		1	0.00	0.00	50.99
	3	2	0.00	0.00	50.99
-		3	0.00	0.00	20.40

Table C.8: y_i^{rs} in FOB Prices for Benchmark $2, (\sigma_i^s = \infty)$

$\overline{x_{ij}^{rs}}$			1			2			3		
		1	2	3	1	2	3	1	2	3	
	1	14.42	12.71	3.02	7.85	6.80	1.71	4.40	6.14	1.34	
1	2	13.74	12.11	2.88	5.76	4.99	1.25	3.85	5.37	1.71	
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	1	7.14	6.29	1.50	14.89	12.90	3.24	5.24	7.32	1.60	
2	2	5.76	5.07	1.21	13.75	11.92	3.00	3.85	5.37	1.17	
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	1	3.90	3.44	0.82	3.32	2.88	0.72	6.62	9.25	2.02	
3	2	5.76	5.07	1.21	5.76	5.00	1.25	9.19	12.83	2.80	
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

Table C.9: x_{ij}^{rs} in Physical Terms for Benchmark $3, (\sigma_i^s = 10)$

Table C.10: y_i^{rs} in Physical Terms for Benchmark 3,($\sigma_i^s = 10$)

y_i^{rs}		1	2	3
	1	30.15	16.36	11.87
1	2	28.73	12.00	10.39
	3	0.00	0.00	0.00
	1	14.93	31.03	14.15
2	2	12.03	28.65	10.40
	3	0.00	0.00	0.00
	1	8.15	6.93	17.89
3	2	12.03	12.00	24.81
	3	0.00	0.00	0.00

Table C.11: x_{ij}^{rs} in FOB Prices for Benchmark $3, (\sigma_i^s = 10)$

$\overline{x_{ij}^{rs}}$		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	14.30	12.60	2.99	7.78	6.74	1.70	4.36	6.08	1.33
1	2	13.74	12.11	2.88	5.76	4.99	1.25	3.85	5.37	1.17
	3	5.59	5.26	1.70	5.12	4.80	1.62	4.43	5.08	1.61
	1	6.90	6.08	1.44	14.39	12.47	3.13	5.06	7.07	1.54
2	2	5.76	5.07	1.21	13.75	11.92	2.99	3.85	5.37	1.71
	3	5.36	5.05	1.65	5.64	5.26	1.73	4.60	5.32	1.66
	1	4.06	3.58	0.85	3.46	3.00	0.75	6.90	9.64	2.10
3	2	5.76	5.07	1.21	5.76	4.99	1.25	9.17	12.83	2.80
	3	4.71	4.48	1.52	4.59	4.35	1.51	4.36	4.99	1.59

y_i^{rs}		1	2	3
	1	29.88	16.21	11.77
1	2	28.73	12.00	10.39
	3	12.55	11.52	11.12
	1	14.43	29.99	13.68
2	2	12.03	28.65	10.39
	3	12.06	12.64	11.58
	1	8.49	7.22	18.64
3	2	12.03	12.00	24.81
	3	10.70	10.45	10.94

Table C.12: $\underline{y_i^{rs}}$ in FOB Prices for Benchmark $3, (\sigma_i^s = 10)$

Table C.13: x_{ij}^{rs} in Physical Terms for Benchmark $3, (\sigma_i^s = \infty)$

			*		-					
x_{ij}^{rs}		1			2			3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	4.65	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

		v			
-	y_i^{rs}		1	2	3
		1	50.99	0.00	0.00
	1	2	50.99	0.00	0.00
		3	0.00	0.00	0.00
		1	0.00	50.99	0.00
	2	2	0.00	50.99	0.00
		3	0.00	0.00	0.00
		1	0.00	0.00	50.99
	3	2	0.00	0.00	50.99
		3	0.00	0.00	0.00

Table C.14: y_i^{rs} in Physical Terms for Benchmark $3, (\sigma_i^s = \infty)$

x_{ij}^{rs}			1			2		3		
		1	2	3	1	2	3	1	2	3
	1	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
1	2	23.17	23.17	4.65	0.00	0.00	0.00	0.00	0.00	0.00
	3	9.27	9.27	1.86	0.00	0.00	0.00	0.00	0.00	0.00
	1	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
2	2	0.00	0.00	0.00	23.17	23.17	23.17	0.00	0.00	0.00
	3	0.00	0.00	0.00	9.27	9.27	9.27	0.00	0.00	0.00
	1	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
3	2	0.00	0.00	0.00	0.00	0.00	0.00	23.17	23.17	4.65
	3	0.00	0.00	0.00	0.00	0.00	0.00	9.27	9.27	1.86

Table C.15: x_{ij}^{rs} in FOB Prices for Benchmark $3, (\sigma_i^s = \infty)$

Table C.16: y_i^{rs} in FOB Prices for Benchmark $3, (\sigma_i^s = \infty)$

y_i^{rs}		1	2	3
	1	50.99	0.00	0.00
1	2	50.99	0.00	0.00
	3	20.40	0.00	0.00
	1	0.00	50.99	0.00
2	2	0.00	50.99	0.00
	3	0.00	20.40	0.00
	1	0.00	0.00	50.99
3	2	0.00	0.00	50.99
	3	0.00	0.00	20.40

References

- Ando A. and B. Meng, 2009. The transport sector and regional price differentials: a spatial CGE model for Chinese provinces. *Economic Systems Research*, 21(2), 89-113.
- [2] Brander, James A., 1981. Intra-industry trade in identical commodities. *Journal of International Economics*, 11, 1-14.
- [3] Eduardo A. Haddad and Geoffrey J.D. Hewings, 2001. Transportation Costs and Regional Development: An Interregional CGE Analysis, In P. Friedrich and S. Jutila, eds. *Policies of Regional Competition*, Baden-Baden, Nomos Verlag, 83-101.
- [4] Harker, P.T., 1987. Predicting Intercity Freight Flows, VNU Science Press.
- [5] Lofgren, H. and Robinson, S., 2002. Spatial-network, general-equilibrium model with a stylized application. *Regional Science and Urban Economics*, 32, 651-671.
- [6] Meng, B. and A. Ando, 2005. An economic derivation on trade coefficients under the framework of multi-regional I-O analysis, *IDE Discussion Papers*, 29.
- [7] Miyagi, T. and K. Honbu, 1993. Estimation of interregional trade flows based on the SCGE model. Proc. of Infrastructure Planning, 16, 879-886. (in Japanese)
- [8] Florenz, P., 2005. The advantage of avoiding the Armington assumption in multi-region models. *Regional Science and Urban Economics*, 35, 777-794.
- [9] McCann, P., 2005. Transport cost and new economic geography. Journal of Economic Geography, 5, 305-318.
- [10] Samuelson, P., 1953. Prices of factors and goods in general equilibrium. Review of Economic Studies, 21, 1-20.
- [11] Takayama, I. and G.G. Judge, 1971. Spatial and Temporal Price and Allocation Models, North-Holland.
- [12] Wilson, A.G., 1970. Entropy in Urban and Regional modeling, Pion.