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International Real Business Cycles: A Re-Visit

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December 2010

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It is well known that several quantitative properties of international real business cycle models with are at odds with the data. First, the cross-country correlations are much higher for consumption than for output, while in the data the opposite is true (the BKK puzzle). Second, cross-country correlations of employment and investment are negative, while in the data they are positive. This paper quantitatively shows that preferences with a zero income effect on labor supply help generate a correct cross-country correlation in employment even without any restrictions on financial markets. In a bond economy, a zero income effect in labor supply, combined with time-to-build investment, can generate a positive cross-country correlation in investment, and the BKK puzzle is also resolved when the inter-temporal elasticity of substitution in labor supply is low.

Keywords: International Real Business Cycles, Income Effects, GHH Preferences
JEL classification: E21, E32, F41

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1 Introduction

It is well known that several quantitative properties of international business cycle models are at odds with empirical data. First, the cross-country correlations are much higher for consumption than for output, while in the data the opposite is true (the BKK puzzle). Second, cross-country correlations of employment and investment are negative, while in the data they are positive. Baxter (1995) wrote in her paper that: "It has proved particularly difficult to write down plausibly-parameterized models which can generate positive comovement of labor and investment across countries...Thus a major challenge to the theory is to develop a model which can explain international comovement in labor input and investment."

The main reason for the negative comovements is that in the model of one-good with the internationally mobile capital, there is a strong tendency to move capital to the most productive location in response to persistent productivity shocks. The movement of capital to the more productive country leads to a rise in labor returns there accompanied by a fall in labor returns in the other country, hence inducing the negative comovement in labor and output in the model via the substitution effect. As a result, labor input in different country is negatively correlated unless the cross-country correlation of the innovations to the country-specific shocks is very high. Baxter (1995) argues that although the innovations to Solow residuals are positively correlated across countries, this correlations is not strong enough to overcome the natural mechanisms leading to negative comovement.

However, there is another effect, which is not often emphasized, that causes the cross country negative comovement of employment in these models: the income/wealth effect. Intuitively, when a positive productivity shock hits the foreign country, consumers in the home country expect "spillover" to the home productivity and an increase in wealth because of risk sharing via financial markets, hence increasing consumption goods and *leisure*. In other words, the positive wealth effect, combined with the substitution effect that already raise leisure with declining home wage rates, magnifies the decrease of the home country's labor supply in response to a foreign country's positive productivity shock. As a result, labor inputs are negatively correlated across country despite positive correlations in productivity innovations. In this paper, I quantitatively shows that without the income effect on labor supply, relatively small positive correlations in cross-country productivity innovations, as suggested by empirical studies, are sufficient to generate significant positive comovement in employment even without any restrictions on financial markets.[1]

The paper then shows that when there are delays in investment (time-to-build), restricted financial markets of state non-contingent bonds, combined with a zero income effect on labor supply, can account for the positive comovement in investment. The intuition goes as follows: it takes time for firms to implement many planned projects each time so by the time firm can implement new projects, productivity differences between countries have become relatively less significant, which limits the cross-country capital movement, especially under a restricted financial market.

Finally, the paper shows that under an economy of restricted financial markets and time-to-build investment, the BKK puzzle is resolved when the intertemporal elasticity of substitution in labor supply is low.[2] Intuitively, restricted financial market structures limit risk-sharing capacity of consumers, hence lower correlation in consumption across countries. Moreover, when the elasticity is low, labor supply is less responsive to a country-specific productivity shocks and because of non-separability, the country-specific effect on consumption is lower. Consequently, the cross-country correlation in consumption becomes significantly weaker.

Devereux et al (1992) is the first to introduce non-separable preferences without income effects into two-country one good models. They quantitatively show that there is a realistic cross-country correlation of consumption in a bond economy model with independently random productivity shocks. However, they did not compare this value of correlation in consumption to that of output and did not examine the possibility of the positive comovement in employment, output and investment. My work also differs from their paper in another critical aspect. I consider a different productivity shock structure: there is persistence in productivity shocks and positive correlations in cross-country innovations, as suggested by empirical studies. Moreover, I show that under this productivity shock structure, positive comovement in employment and output and a

[1] It means that the positive comovement result holds both in the complete markets where where people in both countries can trade a full-set of contingent claims.

[2] The value is still in the range suggested by empirical studies.

realistic cross-country correlation in consumption can be generated even under complete financial markets.

There has been also a growing literature focusing on the role of credit market imperfections in explaining the positive comovements. Backus et al. (1992), Baxter and Crucini (1995) and Heathcote and Perri (2002) assume financial autarky, i.e. countries cannot trade financial claims. These papers find that extreme restrictions in the trade of financial assets, by largely reducing international capital mobility, can generate the positive cross-country correlation in output. Kehoe and Perri (2002) analyze a model where the borrowing constraints of a country are endogenous and change over the cycle. In particular, they consider the case in which the borrowing capacity of a country depends on the value that the country attributes to future access to international financial markets. The borrowing constraint requires that, in each period, allocations have a higher discounted utility than would prevail if the country were excluded from all further intertemporal and international trade. When the foreign country is hit by a positive shock, its output cannot increase too much otherwise the value of defaulting would become higher than the penalty of being excluded from international financial markets in the future. Therefore, the flow of capital from the home country to the foreign country is limited, which help to account for the positive cross-country correlations under relative small correlations in productivity innovations.

My work takes a different approach from the above literature in examining the anomalies between the data and theoretical models. It shows that many properties of international business cycle models can be reconciled with the data by simply assuming a zero wealth effect in labor supply via non-separable GHH preferences, hence attempting to provide insights from different point of view.

The structure of this paper is as follows. Section 2 introduces models' setting and calibration. Section 3 discusses quantitative results. Conclusions follow in section 4.

2 Model

The world consists of two countries: the home country and the foreign country. Consumers in each country value leisure and consumption of the single tradable produced good while labor is internationally immobile. Firms in each country produce the single good by identical Cobb-Douglass production functions and are subject to exogenous shocks to total factor productivity.

In this model, the foreign country is distinguished from the home country by means of a star attached to all foreign-country variables. When there are no stars, the variable, parameter, or function is assumed to be identical across countries. All variables are in per capita terms.

Preferences. The representative household in each country maximizes its expected lifetime utility defined over random sequences of consumption goods (c_t) and labor disutility (l_t):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, l_t\big), \quad \text{Home country;} \tag{2.1}$$

$$U^* = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t^*, l_t^*\big), \quad \text{Foreign country} \tag{2.2}$$

I consider two types of preferences in this paper. The first one is "standard" separable Cobb-Douglas preferences, which have been used in the international business cycle literature of one good two country models.[3]

$$U(c_t, l_t) = \frac{\big(c_t^{\gamma}(1-l_t)^{1-\gamma}\big)^{1-\sigma} - 1}{1-\sigma} \tag{2.3}$$

Parameter γ will determine the value of hours at the steady state.

The second one is non-separable GHH preferences:

$$U(c_t, l_t) = \frac{(c_t - \kappa \frac{l_t^{\omega}}{\omega})^{1-\sigma} - 1}{1-\sigma} \tag{2.4}$$

Similar to γ, parameter κ will determine the value of hours at the steady state and parameter ω determine the elasticity of labor supply. It is well known that GHH preferences imply zero elasticity of leisure to income.

Technology. Production functions are in Cobb-Douglass forms, hence exhibiting constant returns to scale; production of the single final good requires the input of both labor and capital. Capital used in production in a specific country is not necessarily owned

[3]The type of preferences was used by King, Plosser, and Rebelo, 1988

by residents of that country; thus, k_t represents capital in place in the home country, not necessarily capital owned by residents of the home country. Labor is internationally immobile.

$$y_t = A_t(k_t)^{\alpha}(l_t)^{1-\alpha} \quad \text{Home country;} \tag{2.5}$$

$$y_t^* = A_t^*(k_t^*)^{\alpha}(l_t^*)^{1-\alpha} \quad \text{Foreign country.} \tag{2.6}$$

where A_t represents the stochastic level of productivity home country and k_t is the capital stock installed home country at time t.

Productivity evolves according to the bivariate autoregressive process:

$$\begin{bmatrix} log(A_{t+1}) \\ log(A_{t+1}^*) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} log(A_t) \\ log(A_t^*) \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1}^* \end{bmatrix} \tag{2.7}$$

where a_1 measures the persistence in productivity shocks and a_2 measures the degree of international spillovers. The variance in the innovations is denoted by σ_ϵ^2 and the correlation between ϵ_t and ϵ_t^* is σ_{12}.

I follow Kydland and Prescotts (1982) to assume (four-period) time-to-build investment technology. That is, in the time-to-build models:

$$i_t = \omega_1 s_{1t} + \omega_2 s_{2t} + \omega_3 s_{3t} + \omega_4 s_{4t} \tag{2.8}$$

where i_t is the investment at time t and s_{jt} is the volume of projects j periods away from completion at the beginning of period t and ω_j is the resource cost associated with work on a project j periods away from completion, for j = 1, 2, 3, 4. Investment projects progress according to $s_{j,t+1} = s_{j+1,t}$ for j = 1, 2, 3; and starts during period t are represented by s_{4t}. The capital stock thus evolves according to:

$$k_{t+1} = (1-\delta)k_t + s_{1t} - \frac{\phi}{2}k_t\left[\frac{s_{1t}}{k_t} - \delta\right]^2 \tag{2.9}$$

This law of motion includes investment adjustment costs governed by ϕ and is such that there are no adjustment costs in steady state. Notice that when $\omega_1 = 1, \omega_2 = \omega_3 = \omega_4 = 0$, we have the regular one-time-to-build investment.

Market Structure. I assume that there is frictionless international trade in output, so that there is an unified world resource constraint for the single produced good:

$$(y_t - c_t - i_t) + (y_t^* - c_t^* - i_t^*) = 0 \tag{2.10}$$

Regarding the financial structure, I consider both complete-markets and bond economies. The difference of the two structures lies in the number of assets available to the agents. When markets are complete, the representative agents in both countries can trade a full set of contingent claims. Accordingly, the budget constraint of the home country's representative household can be expressed as:

$$c_t + i_t + \sum_{s_{t+1}} p(s_{t+1}, s_t) b(s_{t+1}) = y_t + b(s_t) \tag{2.11}$$

where s_t indicates the state in period t and $b(s_{t+1})$ denotes the quantity of contingent claims purchased in period t and paying off one unit of consumption the following period, conditional on the state of the world being s_{t+1} next period. $p(s_{t+1}, s_t)$ denotes the price of these contingent assets.

By contrast, in a bond economy, there is only one-period real discount bonds. Let b_{t+1} denote the per capita quantity of these discount bonds purchased by the home economy, which mature in period $t+1$, and p_t^b is its price at time t.

The flow budget constraints for the bond economy are: [4]

$$c_t + i_t + p_t^b b_{t+1} + \frac{\pi_b}{2}(b_{t+1})^2 = y_t + b_t; \quad \text{home country} \tag{2.12}$$

$$c_t^* + i_t^* + p_t^b b_{t+1}^* + \frac{\pi_b}{2}(b_{t+1}^*)^2 = y_t^* + b_t^*; \quad \text{foreign country} \tag{2.13}$$

The world market clearing condition for bonds is:

$$b(s_{t+1}) + b^*(s_{t+1}) = 0; \quad \text{complete markets} \tag{2.14}$$

$$b_{t+1} + b_{t+1}^* = 0 \quad \text{bond economy} \tag{2.15}$$

Calibration. This paper follows closely calibration from Baxter and Crucini (1995), Kollmann (1996), and Kehoe and Perri (2002). See Table 1 for details.

In particular, parameter ω, which determines the inter-temporal elasticity of substitution in labor supply, [5] is set to 2 as a benchmark. The unit benchmark elasticity is equal to the value implied by standard preferences as in form (2.3). For sensitivity

[4]Following Boileau et al (2008) and others, I impose quadratic portfolio adjustment costs to induce stationarity in incomplete markets. See Boileau et al (2008) for more details about other methods.

[5]The inter-temporal elasticity of substitution in labor supply is approximately $\frac{1}{\omega-1}$.

analysis, ω is set from 1.58 [6] to 6, which then implies the intertemporal elasticity of substitution varies from 1.7 to 0.2 accordingly. This is a range suggested by empirical studies.

Parameters, κ, γ in GHH preferences and Cobb-Douglass preferences, respectively are chosen such that the hours of working in the steady state are 0.25.

Portfolio adjustment costs parameter, π_b is set to 0.0005 such that the implied volatility of the ratio of net exports to output in bond economy models is the same in corresponding financial complete market models.

In regular one-time-to-build investment environment, investment adjustment cost parameter, ϕ is set such that the ratio of investment volatility to that of output match the data, which is equal to 3.24. In the four-time-to-build investment environment, ϕ is set to zero.

Table 1: Parameter values

	Parameters
Preferences	$\beta = 0.99$, $\sigma = 2$
	hours at s.s $l = 0.25$
	$\omega = 2$ as benchmark
Technology	$\alpha = 0.3$, $\delta = 0.03$
	$\omega_1 = \omega_2 = \omega_3 = \omega_4 = 0.25$
Productivity shocks	$a_1 = 0.95$, $a_2 = 0$
	var(ϵ_1)=var(ϵ_2)=0.07^2, corr(ϵ_1,ϵ_2)=0.25
Adjustment cost	$\pi_b = 0.0005$

3 Quantitative Results

There are four types of models with different financial market structures and different investment technologies: models with complete financial markets and models with restricted one period state non-contingent bonds, models with regular one-time-to-build

[6] 1.58 is the value used by Devereux et all (1992) in their two-country model; the value was first used by Greenwood et al (1988) in a closed-economy model.

investment and models with four-time-to-build investment. I solve and simulate these models by the perturbation method [7] and Figures 1-4 present impulse responses of these models with respect to one unit of positive shock in the foreign countries. For comparison, I combine impulse responses of models with Cobb-Douglas separable preferences and those with non-separable GHH preferences in one figure. [8] Table 2 and Table 3 provide business cycle statistics from data and those implied by these models.

Figures 1-4 show differences between the impulse responses of models with GHH preferences and those with Cobb-Douglas preferences, particularly in consumption and labor input. In response to a positive shock in the foreign country, consumptions in both countries exhibit a relatively smooth and similar dynamic pattern for models with Cobb-Douglas preferences while for GHH preferences, the impulses are more responsive and move in opposite directions. This explains why the cross-country correlation in consumption is higher in Cobb-Douglas preferences. The impulse responses also present evidence of more risk-sharing in the complete markets, where consumption in the home country increases relatively more than it does in a bond economy.

By contrast, employment (hours) in home country decreases significantly less in models with GHH preferences. The reason is straightforward. With a zero wealth effect in labor supply, consumers in the home country consume less leisure in response to a positive productivity shock in the foreign country, hence reducing labor supply by a relatively smaller amount compared to a positive wealth effect case. Tables 2 and 3 show that the cross-country correlations in labor and output are positive in the models with GHH preferences while it is negative in the models with Cobb-Douglas preferences.

Figures 3- 4 show impulse responses of models with time-to-build investment. As pointed out in Kydland et al (1982), models with time-to-build display a more persistence in all time series variables. The above arguments for consumption and labor input dynamics hold for models with time-to-build investment. However, there is a significant difference between models with complete markets and models with restricted financial markets. Restricted financial markets, combined with time-to-build investment, can largely reduce the international capital mobility. The last column of Table 2 shows that

[7]For more details, see Schmitt-Grohe and Uribe (2004)

[8]In each figure, ω is chosen such that the implied standard deviations of output for the two types of preferences are the same.

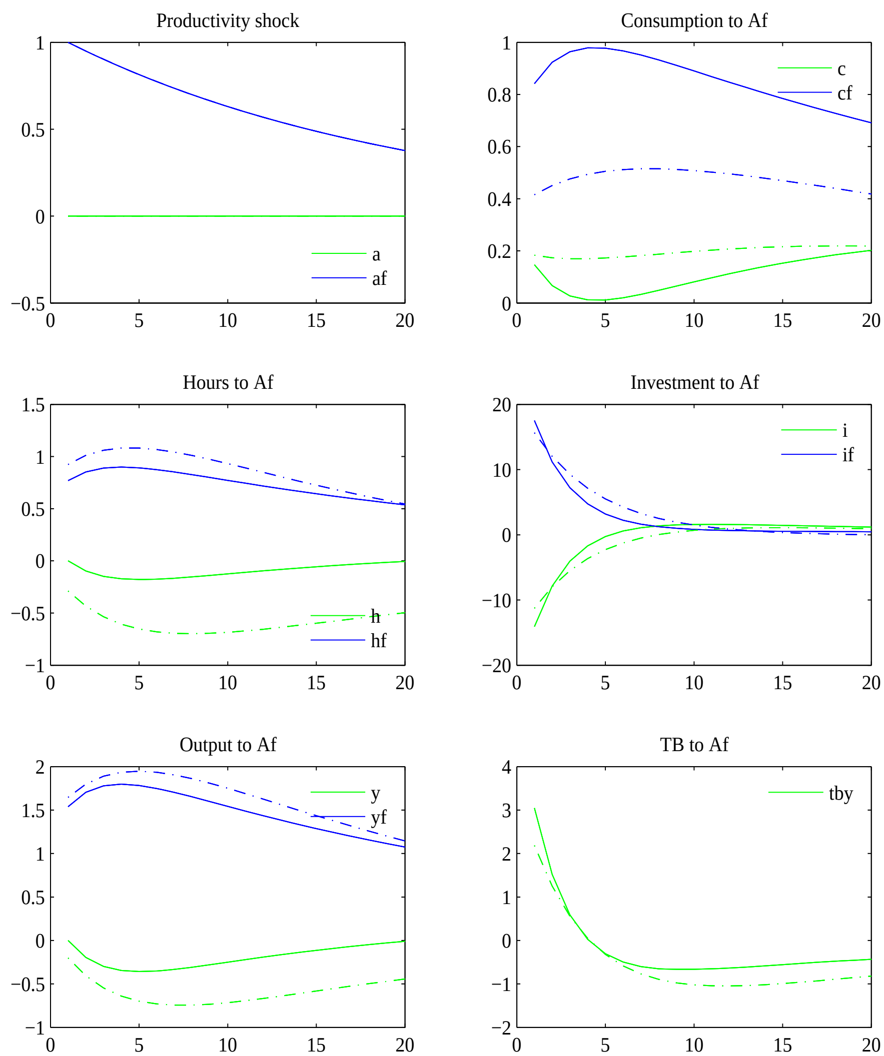
there is a positive cross-country correlation in investment in a bond economy model with time-to-build investment.

Table 4 presents the implied business cycle statistics when I vary the intertemporal elasticity of substitution in labor supply from 1.7 to 0.2, which are in the range of empirical studies. The cross-country correlation in consumption decreases when the elasticity is lower or labor supply becomes less responsive to shocks. As a result, when the elasticity is about $\frac{1}{3}$ the PKK puzzle is resolved, i.e., the cross-country correlation in consumption is smaller than that of output. Intuitively, restricted financial market structures limit risk-sharing capacity of consumers, hence lower correlation in consumption across countries. Moreover, when labor supply is less responsive to a country-specific productivity shocks and because of non-separability, the country-specific effect on consumption is lower. Therefore, the cross-country correlation in consumption becomes relatively weaker.

4 Conclusions

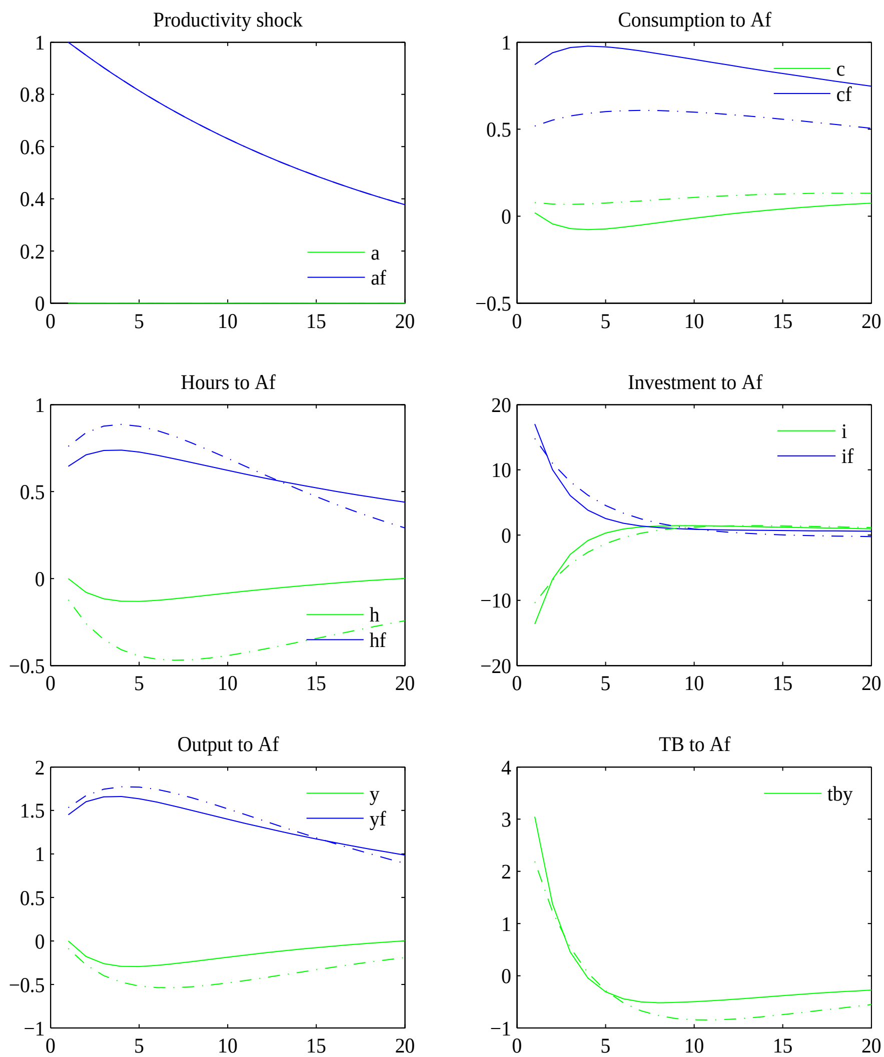
This paper quantitatively shows that the anomaly between models and empirical data in cross-country correlation in labor can be resolved by the introduction of non-separable GHH preferences. It also proposes solutions to anomalies in cross-country correlation in investment and the BKK puzzle.

Figure 1: **IRs: Complete Markets, One-time-to-build**



Solid line: GHH preference; Dashed line:Cobb−Douglas preference

Figure 2: **IRs: Bond Economy, One-time-to-build**



Solid line: GHH preference; Dashed line:Cobb−Douglas preference

Figure 3: **IRs: Complete Markets, Four-time-to-build**

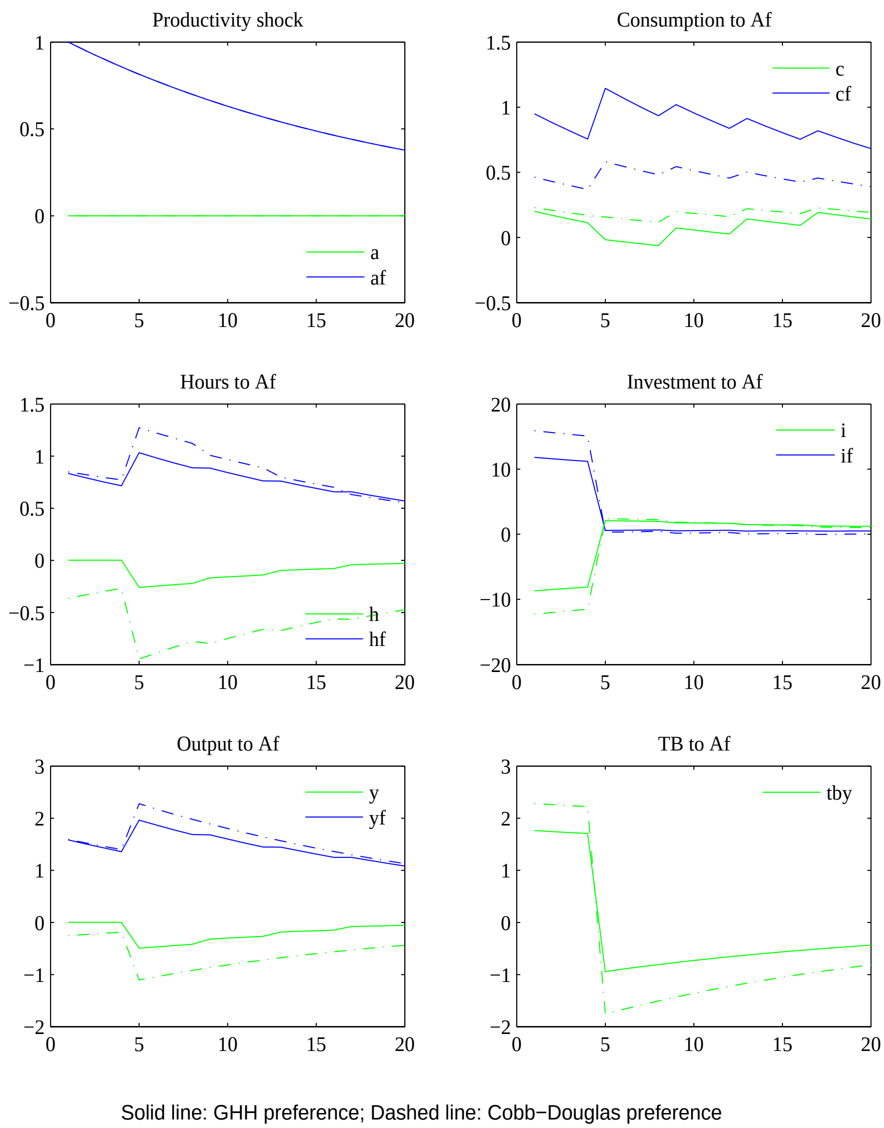
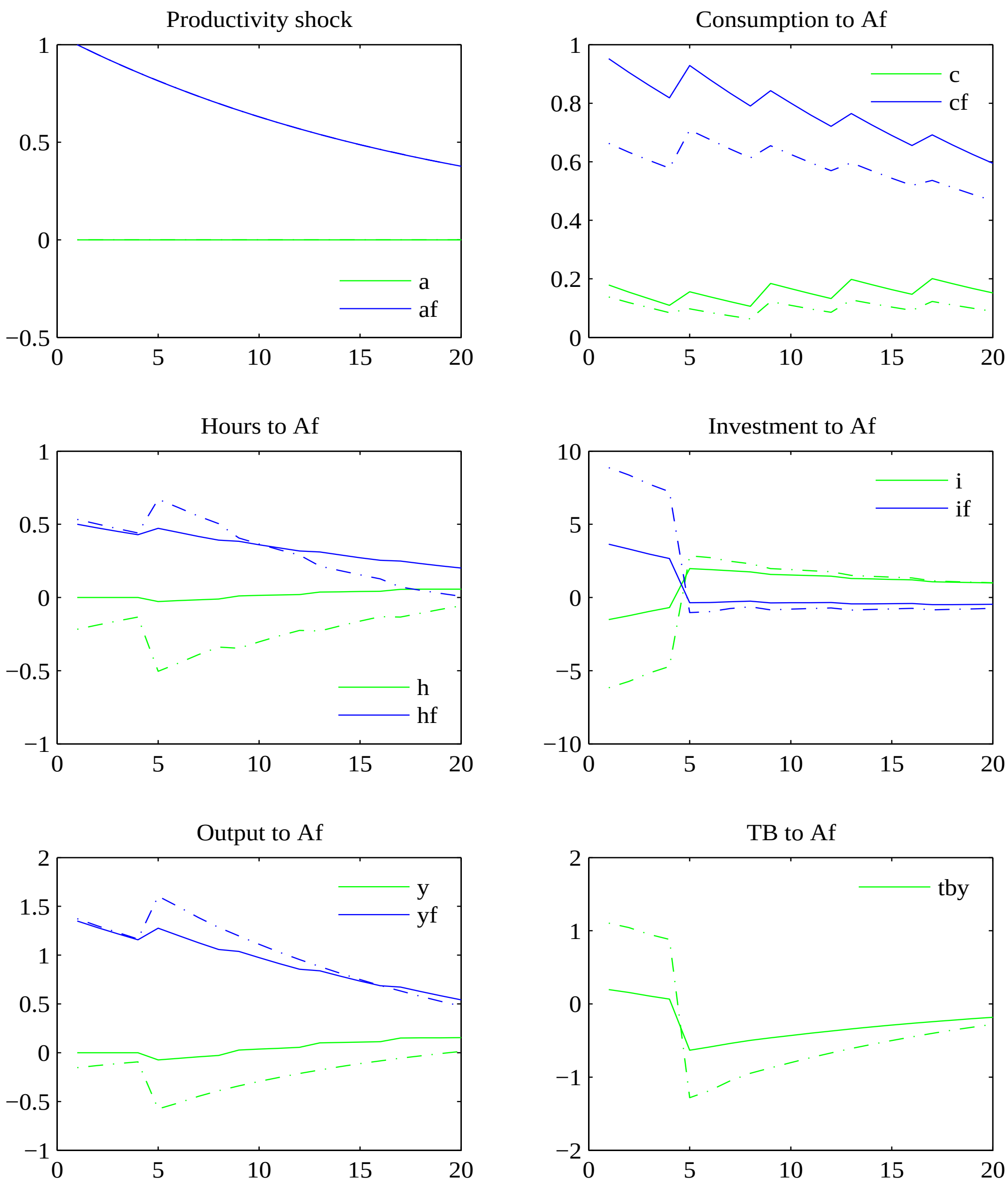


Figure 4: **IRs: Bond Economy, Four-time-to-build**



Solid line: GHH preference; Dashed line:Cobb−Douglas preference

Table 2: **Business Cycles Statistics: GHH Preferences**

		Economy with			
		One-time-to-build		Four-time-to-build	
Statistics	Data	CM	Bond	CM	Bond
Std.dev rel. to GDP					
Consumption	0.79	0.71	0.81	0.72	0.88
Investment	3.24	3.24	3.24	3.24	1.83
Employment	0.63	0.5	0.5	0.5	0.5
Net Exports/GDP	0.09	0.65	0.64	0.69	0.43
Domestic Comovement					
Corr. with GDP					
Consumption	0.87	0.96	0.96	0.96	0.91
Investment	0.93	0.5	0.5	0.43	0.7
Employment	0.86	1	1	1	1
Net Exports/GDP	-0.36	0.17	0.06	0.22	0.21
International Correlation					
Home and Foreign					
GDP	0.51	0.2	0.19	0.21	0.51
Consumption	0.32	0.68	0.28	0.69	0.72
Investment	0.29	-0.7	-0.7	-0.7	0.18
Employment	0.43	0.2	0.19	0.21	0.51

Notes: The statistics in the Data column are taken from Kehoe and Perri (2002), which are calculated from U.S. quarterly time series, 1970:1-1998:4 and an aggregate of 15 European countries. All relevant time series, except ratio of net exports to output, have been logged and HP-filtered. CM indicates a complete financial market economy while bond indicates an economy where people can only trade one-period non-contingent bond. The inter-temporal elasticity of substitution in labor supply is set to 1

Table 3: **Business Cycles Statistics: Cobb-Douglas Preferences**

		Economy with			
		One-time-to-build		Four-time-to-build	
Statistics	Data	CM	Bond	CM	Bond
Std.dev rel. to GDP					
Consumption	0.79	0.41	0.51	0.4	0.63
Investment	3.24	3.24	3.24	4.33	3.47
Employment	0.63	0.58	0.61	0.58	0.49
Net Exports/GDP	0.09	0.78	0.81	1.10	0.83
Domestic Comovement					
Corr. with GDP					
Consumption	0.87	0.7	0.58	0.7	0.7
Investment	0.93	0.51	0.51	0.27	0.41
Employment	0.86	0.93	0.86	0.93	0.79
Net Exports/GDP	-0.36	0.52	0.5	0.48	0.38
International Correlation					
Home and Foreign					
GDP	0.51	-0.45	-0.35	-0.5	0.04
Consumption	0.32	0.86	0.42	0.85	0.57
Investment	0.29	-0.76	-0.71	-0.88	-0.61
Employment	0.43	-0.87	-0.86	-0.9	-0.64

Note: The statistics in the Data column are taken from Kehoe and Perri (2002), which are calculated from U.S. quarterly time series, 1970:1-1998:4 and an aggregate of 15 European countries. All relevant time series, except ratio of net exports to output, have been logged and HP-filtered. CM indicates a complete financial market economy while bond indicates an economy where people can only trade one-period non-contingent bond.

Table 4: **Business Cycles Statistics: Sensitivity to Elasticity**

		ω					
Statistics	Data	1.58	2	3	4	5	6
Std.dev rel. to GDP							
Consumption	0.79	0.91	0.88	0.85	0.84	0.83	0.83
Investment	3.24	1.74	1.83	1.94	1.99	2.02	2.03
Employment	0.63	0.63	0.5	0.33	0.25	0.2	0.17
Net Exports/GDP	0.09	0.42	0.43	0.43	0.45	0.47	0.45
Domestic Comovement							
Corr. with GDP							
Consumption	0.87	0.93	0.91	0.88	0.87	0.85	0.85
Investment	0.93	0.66	0.7	0.73	0.75	0.75	0.76
Employment	0.86	1	1	1	1	1	1
Net Exports/GDP	-0.36	0.19	0.21	0.23	0.24	0.24	0.25
International Correlation							
Home and Foreign							
GDP	0.51	0.46	0.51	0.52	0.52	0.52	0.52
Consumption	0.32	0.76	0.72	0.59	0.52	0.48	0.46
Investment	0.29	0.1	0.18	0.27	0.31	0.33	0.35
Employment	0.43	0.46	0.51	0.52	0.52	0.52	0.52

Note: The statistics in the Data column are taken from Kehoe and Perri (2002), which are calculated from U.S. quarterly time series, 1970:1-1998:4 and an aggregate of 15 European countries. All relevant time series, except ratio of net exports to output, have been logged and HP-filtered. The model statistics are computed from an model economy with GHH preferences, non-contingent bond, and time-to-build investment. Omega is related to the intertemporal elasticity of substitution in labor supply: the higher omega is the lower the elasticity

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Appendix A: Theoretical Model

1 Solution Method

I solve the models by the perturbation method.[9] Particularly the set of optimality conditions of the economy can be expressed as follows:

$$E_t\{F(Y_{t+1}, Y_t, X_{t+1}, X_t)\} = 0 \tag{1.1}$$

E_t is the mathematical expectation operator conditional on information available at time t, Y_t is the vector of non-predetermined variables, and $X_t = [x_t^1, x_t^2]'$ is the state variable vector, x_t^1 is endogenous predetermined state variables while x_t^2 is exogenous state variables. Particularly, x_t^2 follows exogenous process given as:

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta}\bar{\sigma}\epsilon_{t+1} \tag{1.2}$$

where $\tilde{\eta}, \bar{\sigma}$ are given parameter. The solution of the optimal plan is of the form:

$$Y_t = g(X_t, \bar{\sigma}) \tag{1.3}$$

$$X_{t+1} = h(X_t, \bar{\sigma}) + \bar{\eta}\bar{\sigma}\epsilon_{t+1} \tag{1.4}$$

where $\bar{\eta} = [\emptyset, \tilde{\eta}]'$, these equations describe the policy and transition functions respectively. I compute a first order expansion of the two functions around the deterministic steady state.

2 Solving the bond economy model

The representative agent in Home country chooses sequences $\{c_t, l_t, k_{t+1}, b_{t+1}, s_{it}\}$ to solve the problem

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, l_t\big) \tag{2.5}$$

subject to

$$c_t + i_t + p_t^b b_{t+1} + \frac{\pi_b}{2}(b_{t+1})^2 = A_t(k_t)^\alpha (l_t)^{1-\alpha} + b_t \tag{2.6}$$

[9] For more details, see Schmitt-Grohe and Uribe (2004)

$$k_{t+1} = (1-\delta)k_t + s_{1t} - \frac{\phi}{2}k_t\left[\frac{s_{1t}}{k_t} - \delta\right]^2 \tag{2.7}$$

$$i_t = \sum_{i=1}^{4} \omega_i s_{it} \tag{2.8}$$

$$s_{j,t+1} = s_{j+1,t} \quad \text{for j = 1, 2, 3} \tag{2.9}$$

The Lagrangian associated with the Home agent's optimization problem can be written as:

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ U(.) + \lambda_t \left[A_t k_t^{\alpha} l_t^{1-\alpha} + b_t - c_t - \sum_{i=1}^{4} \omega_i s_{it} - p_t^b b_{t+1} - \frac{\pi_b}{2} b_{t+1}^2 \right] \\ + \lambda_t \nu_{2t}(s_{2t} - s_{1t+1}) + \lambda_t \nu_{3t}(s_{2t} - s_{1t+1}) + \lambda_t \nu_{4t}(s_{2t} - s_{1t+1}) \\ + \lambda_t q_t \left[(1-\delta)k_t + s_{1t} - \frac{\phi}{2} k_t \left[\frac{s_{1t}}{k_t} - \delta\right]^2 - k_{t+1} \right] \Bigg\}\end{aligned} \tag{2.10}$$

The optimality conditions associated with the Home representative agent's problem are:

$$U_c\big(c_t, l_t\big) = \lambda_t \tag{2.11}$$

$$\lambda_t \left(p_t^b + \pi_b b_{t+1}\right) = \beta \lambda_{t+1} \tag{2.12}$$

$$U_l\big(c_t, l_t\big) = -\lambda_t (1-\alpha) \frac{y_t}{l_t} \tag{2.13}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left(\frac{\alpha y_t}{k_t} + q_{t+1} \left[1 - \delta_k - \frac{\phi}{2} \left(\frac{s_{1t+1}}{k_t} - \delta_k \right)^2 + \left(\frac{s_{1t+1}}{k_t} - \delta_k \right) \frac{\phi s_{1t+1}}{k_t} \right] \right) \tag{2.14}$$

$$-\lambda_t \nu_{2t} + \beta E_t \left\{ \lambda_{t+1} q_{t+1} \left[1 - \phi \left(\frac{s_{1t+1}}{k_t} - \delta_k \right) \right] - \omega_1 \lambda_{t+1} \right\} = 0 \tag{2.15}$$

$$-\lambda_t \nu_{3t} + \beta E_t \left\{ \lambda_{t+1} \nu_{2t+1} - \omega_2 \lambda_{t+1} \right\} = 0 \tag{2.16}$$

$$-\lambda_t \nu_{4t} + \beta E_t \left\{ \lambda_{t+1} \nu_{3t+1} - \omega_3 \lambda_{t+1} \right\} = 0 \tag{2.17}$$

$$-\omega_4 \lambda_t + \lambda_t \nu_{4t} = 0 \tag{2.18}$$

$$c_t + i_t + p_t^b b_{t+1} + \frac{\pi_b}{2}(b_{t+1})^2 = A_t (k_t)^{\alpha} (l_t)^{1-\alpha} + b_t \tag{2.19}$$

$$k_{t+1} = (1-\delta)k_t + s_{1t} - \frac{\phi}{2}k_t\left[\frac{s_{1t}}{k_t} - \delta\right]^2 \tag{2.20}$$

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha} \tag{2.21}$$

$$s_{j,t+1} = s_{j+1,t} \quad \text{for j = 1, 2, 3} \tag{2.22}$$

Similar conditions for the Foreign representative agent.

The world market clearing condition for bonds is:

$$b_{t+1} + b_{t+1}^* = 0 \quad \text{bond economy} \tag{2.23}$$

In solving the system, I replace the budget constraint for Foreign representative agent:

$$c_t^* + i_t^* + p_t^b b_{t+1}^* + \frac{\pi_b}{2}(b_{t+1}^*)^2 = y_t^* + b_t^*; \quad \text{foreign country} \tag{2.24}$$

by the unified world resource constraint for the single produced good:

$$(y_t - c_t - i_t) + (y_t^* - c_t^* - i_t^*) = 0 \tag{2.25}$$

3 Solving the complete market model

When financial markets are complete, the competitive equilibrium is Pareto optimal. Hence, we can derive the equilibrium system using an equal weight planner problem. The planner maximizes the sum of expected lifetime utilities

$$U = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t U\big(c_t, l_t\big) + \beta^t U\big(c_t^*, l_t^*\big) \right\} \tag{3.26}$$

subject to:

$$c_t + i_t + c_t^* + i_t^* = A_t (k_t)^{\alpha} (l_t)^{1-\alpha} + A_t^* (k_t^*)^{\alpha} (l_t^*)^{1-\alpha} \tag{3.27}$$

$$k_{t+1} = (1-\delta)k_t + s_{1t} - \frac{\phi}{2} k_t \left[\frac{s_{1t}}{k_t} - \delta\right]^2 \tag{3.28}$$

$$k_{t+1}^* = (1-\delta)k_t^* + s_{1t}^* - \frac{\phi}{2} k_t^* \left[\frac{s_{1t}^*}{k_t^*} - \delta\right]^2 \tag{3.29}$$

$$i_t = \sum_{i=1}^{4} \omega_i s_{it} \tag{3.30}$$

$$i_t^* = \sum_{i=1}^{4} \omega_i s_{it}^* \tag{3.31}$$

$$s_{j,t+1} = s_{j+1,t} \quad \text{for j = 1, 2, 3} \tag{3.32}$$

$$s^*_{j,t+1} = s^*_{j+1,t} \quad \text{for j} = 1,\ 2,\ 3 \tag{3.33}$$

The Lagrangian associated with the social planner's optimization problem can be set up similarly as the one in the bond economy above. The optimality conditions associated with this problem are straightforward.