INSTITUTE OF DEVELOPING ECONOMIES

IDE Discussion Papers are preliminary materials circulated
to stimulate discussions and critical comments

IDE DISCUSSION PAPER No. 283

A Model of Economic Growth with Saturating Demand

Kozo KUNIMUNE*

March 2011

Abstract

This study presents a model of economic growth based on saturating demand, where the demand for a good has a certain maximum amount.

In this model, the economy grows not only by the improvement in production efficiency in each sector, but also by the migration of production factors (labor in this model) from demand-saturated sectors to the non-saturated sector.

It is assumed that the production of a brand-new good will begin after all the existing goods are demand-saturated. Hence, there are cycles where the production of a new good emerges followed by the demand saturation of that good.

The model then predicts that should the growth rate be stable and positive in the long run, the above-mentioned cycle must become shorter over time. If the length of cycles is constant over time, the growth rate eventually approaches zero because the number of goods produced grows.

Keywords: Economic Growth, Economic Development, Demand Saturation

JEL classification: O1, O4

---

* Director, International Economics Studies Group, Development Studies Center, IDE (kozo_kunimune@ide.go.jp)

The Institute of Developing Economies (IDE) is a semigovernmental, nonpartisan, nonprofit research institute, founded in 1958. The Institute merged with the Japan External Trade Organization (JETRO) on July 1, 1998. The Institute conducts basic and comprehensive studies on economic and related affairs in all developing countries and regions, including Asia, the Middle East, Africa, Latin America, Oceania, and Eastern Europe.

INSTITUTE OF DEVELOPING ECONOMIES (IDE), JETRO
3-2-2, WAKABA, MIHAMA-KU, CHIBA-SHI
CHIBA 261-8545, JAPAN

A Model of Economic Growth with Saturating Demand

1. Introduction

Although there are already a variety of development theories that have been proposed, I would like to present another one in this paper. An aspect of development which I would like to focus here is that the amount of consumption of a single product is limited. This is most clear in the consumption of agricultural products, but it also applies to most other products. Moreover, some products are more basic than others which people want before less basic ones. For example, once people eat enough, they want more cloths; and once they possess enough cloths, then they need housing, and so on.

The model in this paper is built in accordance with the following observations:

(1) People want to have more basic products first, then more luxurious ones.

(2) There exists a maximum amount of consumption of an individual product.

In this paper I regard an economy as going through the following transition:

(1) Only the most basic product is produced and consumed in the initial stage.

(2) Once the people get enough of the most basic product, the demand for it stops growing and thereafter remains flat. In effect the demand for the product has become saturated, or what I call in this paper the "saturation of demand".

(3) The production of the second basic product starts and grows until the saturation of demand.

(4) The production of the third basic product starts, and so the cycle continues.

In this economy, the growth of total production rests on the smooth migration of production factors from the matured (or saturated) sector to the growing sector, in addition to the improvement of production efficiency in the growing sector.

In building this model, I will first set forth some simplifying assumptions.

(1) The utility function is nonlinear in the sense that people are eager to have the basic good until it is saturated, but thereafter no longer need any extra unit of the good whatsoever.

(2) Production efficiency improves according to exogenously given parameters.

(3) Other simplifications which will be presented in section 2.

These assumptions are used here as a starting point, and I would expect them to be refined and relaxed in the course of future study.

In section 2, I set forth the assumptions and terms of my model. We then look at the big picture of the long-run economic growth rate in Section 3. Section 4 presents the condition for solving the model. However, as will be seen, the algebraic solution of the model turns out to be difficult. Hence, I make use of an approximation technique in Section 5 to solve the model. I present my concluding remarks in Section 6.

2. Assumptions and Terms

Below are the simplifying assumptions for building the model.

(1) Total population is normalized to 1 which is assumed not to change over time. Labor

supply is inelastic and also normalized to 1.

(2) Production only needs labor. The efficiency of production, or labor productivity, improves at a certain positive rate which is fixed for each product (it can differ by product).

(3) The demand for a product stops growing when it reaches 1. It is assumed here that "saturation of demand" is common to all products.

(4) At the initial stage ($t = 0$), it is assumed that only the simplest product is produced and that the demand for it is already saturated. An instant after that point, the production of the second most needed product will start.

(5) The movement of production factor is accompanied by friction[1]. I will introduce a fixed (exogenous) parameter (θ) which signifies efficiency of factor movement.

(6) As stated in the Introduction, the utility function is nonlinear and very much peculiar. I do not show the explicit form of the utility function in this paper. It is implied in the assumption of saturating demand.

Based on the above assumptions, I will use the following terms:

(1) t is an index for time, which is continuous and is assumed $t \geq 0$.

(2) k is an index for products, which is discrete and is assumed $k = 0,1,2,\ldots$

(3) h_t^k is the amount of labor input for the production of k at time t.

(4) y_t^k is the amount of the production of k at time t.

(5) g_k is the potential rate of increase in the productivity of k. I assume the actual increase in productivity also depends on the efficiency of factor movement. See the next notation.

---

[1] The source of the friction may be government regulations or immature financial intermediations.

(6) θ is the efficiency of factor movement. I assume $0 < \theta \leq 1$, where $\theta = 1$ means no friction in the factor movement. For the product where demand is already saturated, the improvement of productivity is attained by getting rid of redundant labor. Hence, the actual rate of increase in productivity of the saturated product will be $g_k \theta$.

(7) T_k is the time at which the demand for k is saturated; and I define $F(k) \equiv T_k - T_{k-1}$ as the length of time that passes from the start of the production of k until the demand for it is saturated.

(8) Y_t is total production at time t.

3. The Rate of Economic Growth in the Big Picture

Looking at the big picture, I will focus on those points where all of the existing products are saturated and calculate average growth rates between them.

When t = T_{k-1}, there exist k products whose indexes are 0 to k-1, and all are saturated. The total production at the time is:

$$Y_{T_{k-1}} = k$$

When t = T_k, there exist k+1 products whose indexes are 0 to k, and all are saturated. The total production at the time is:

$$Y_{T_k} = k + 1$$

Since the length of time that elapsed between T_{k-1} and T_k was defined as F(k), average growth rate should be:

$$\frac{\ln(Y_{T_k}) - \ln(Y_{T_{k-1}})}{T_k - T_{k-1}} = \frac{\ln(k+1) - \ln(k)}{F(k)}$$

It is apparent that the numerator of the above equation becomes lower as k increases. If F(k) does not change over time, the average growth rates will decrease, and in the far future, it will approach zero.

However, empirical evidence does not support the lowering of average growth in the long run. Although there are not many countries that have long enough time-series data usable for calculating very long-run growth rates, existing data shows that the average growth rates of developed countries are fairly stable and greater than zero.

Hence, in this model, F(k) is decreasing over time to be consistent with the empirical evidence. Furthermore, the rate at which F(k) decreases also matters. If it decreases slower than the rate of increase in the numerator of the above equation, then average growth rates have to approach zero. And if it decreases faster, then average growth rates have to increase over time. Both cases do not fit with the evidence. Let us consider this point more closely.

Taking the limitation of the numerator in the above equation:

$$\lim_{n\to\infty}[\ln(n+1) - \ln(n)] = 0$$

Assuming that $\lim_{n\to\infty} F(n) = 0$, then l'Hôpital's rule can be applied to the calculation of the limitation of F(n):

$$\lim_{n\to\infty}\frac{\ln(n+1)-\ln(n)}{F(n)}=\lim_{n\to\infty}\frac{-1}{n(n+1)F'(n)}$$

For the above value to converge to a finite non-zero value, we need

$$F'(n)\propto\frac{1}{n(n+1)}$$

Furthermore, if $F'(n) < 0$, then the value converges to a finite positive number, which is consistent with stable long-run average growth.

Thus, it is important to know much about the functional form of F(n), which will be the main purpose in the following section.

4. The Condition

In pursuit of the functional form of F(n), we need to focus respectively on (1) the production of saturated products and (2) the production of non-saturated product during the transitional period between T_{n-1} and T_n. We can then deduce (3) the condition necessary for getting the functional form of F(n). However, even with this condition, it turns out to be difficult to get the solution algebraically; therefore I will use an approximation technique in the next section to overcome this problem.

(1) Production of saturated products

Once the demand of a product is saturated, reducing labor input becomes the only way to improve productivity.

While assuming that productivity can potentially improve at the rate g_k , it is also assumed that there are frictions in factor movement which constrain improvement in the productivity of saturated products. Hence, in the model the rate of improvement of the saturated product is considered as $g_k \theta$, which also equals the rate of decrease in labor in the saturated sector.

Therefore, the amount of labor utilized in a saturated product can be written as:

(eq.1)

$$h_t^k = h_{T_{n-1}}^k e^{-g_k \theta (t - T_{n-1})}$$

The equation shows that utilized labor is decreasing over time. This redundant labor will be utilized in the production of a non-saturated product, which will be described next.

(2) Production of a non-saturated product

Unlike saturated products, production grows by increasing the input of labor in the unsaturated sector. There is also the contribution of improved productivity.

Since it is assumed that there is a constant labor supply of 1, labor input in this sector can be calculated by subtracting the total labor input of the saturated product sector from 1. This can be formulated as:

(eq.2)

$$h_t^n = 1 - \sum_0^{n-1} h_t^k$$

Using the above equation, the production function of the non-saturated product becomes:

(eq.3)

$$y_t^n = e^{g_n(t-T_{n-1})}(1 - \sum_0^{n-1} h_t^k)$$

(3) The condition for deriving the functional form of F(n)

Using equations 1 and 3 yields:

$$y_t^n = e^{g_n(t-T_{n-1})}(1 - \sum_0^{n-1} h_{T_{n-1}}^k e^{-g_k\theta(t-T_{n-1})})$$

The production described in the above equation continues to grow until it reaches 1, where the demand of the product is saturated. Hence, the condition we are seeking is:

(eq.4)

$$e^{g_n(t-T_{n-1})}\left(1-\sum_{0}^{n-1} h_{T_{n-1}}^{k}\ e^{-g_k\theta(t-T_{n-1})}\right)=1$$

By solving above equation in terms of t, we can get T_n, from which we get F(n) by subtracting T_{n-1}.

However, because it is very difficult to solve the equation algebraically, I will present an approximate solution to this problem in the next section.

5. An Approximate Solution

I will use the following approximation hereafter.

(eq.5)

$$\ln\left(\sum_{0}^{n-1} w_k * e^{G_k * T}\right) \cong \ln\left(\sum_{0}^{n-1} w_k * e^{G_k}\right) * T$$

Where,

$$\sum_{0}^{n-1} w_k = 1$$

The equation applies strictly when (1) T = 0, (2) T = 1, or (3) G_k is equal for all k.

Note that in the application of this approximation, T will become smaller and approach zero over time.

(1) Applying the approximation to the problem

The problem has the following relationship.

$$\sum_{0}^{n-1} h^{k}_{T_{n-1}} = 1$$

Hence, $h^{k}_{T_{n-1}}$ can be considered as the variable corresponding to w_k in the above approximation.

Before applying it, it would be convenient to define the new variable.

$$\bar{g}_{n-1} \equiv -\ln\left(\sum_{0}^{n-1} h^{k}_{T_{n-1}} * e^{-g_k}\right)$$

Applying the approximate relation in equation 5 yields:

(eq.6)

$$\sum_{0}^{n-1} h^{k}_{T_{n-1}} * e^{-g_k \theta(t-T_{n-1})} \cong e^{-\bar{g}_{n-1}\theta(t-T_{n-1})}$$

Substituting equation 6 into equation 4 leads to:

(eq.7)

$$e^{g_n(t-T_{n-1})} - e^{(g_n-\bar{g}_{n-1}\theta)(t-T_{n-1})} = 1$$

By rearranging the above equation, the result is:

(eq.8)

$$e^{(g_n+\bar{g}_{n-1}\theta)(t-T_{n-1})} = e^{g_n(t-T_{n-1})} + e^{\bar{g}_{n-1}\theta(t-T_{n-1})}$$

Equation 8 is further transformed by using the approximation again.

$$e^{(g_n+\bar{g}_{n-1}\theta)(t-T_{n-1})} = 2\left\{\frac{1}{2}e^{g_n(t-T_{n-1})} + \frac{1}{2}e^{\bar{g}_{n-1}\theta(t-T_{n-1})}\right\}$$

$$\cong 2e^{\ln\{\frac{1}{2}e^{g_n}+\frac{1}{2}e^{\bar{g}_{n-1}\theta}\}(t-T_{n-1})}$$

Taking the log of both sides of the above equation yields:

$$(g_n + \bar{g}_{n-1}\theta)(t - T_{n-1}) \cong \ln 2 + \ln\left\{\frac{1}{2}e^{g_n} + \frac{1}{2}e^{\bar{g}_{n-1}\theta}\right\}(t - T_{n-1})$$

$$\cong \ln 2 + [\ln\{e^{g_n} + e^{\bar{g}_{n-1}\theta}\} - \ln 2](t - T_{n-1})$$

Solving this in terms of T gives us T_n, i.e.:

$$T_n \cong \frac{\ln 2}{\ln 2 + g_n + \bar{g}_{n-1}\theta - \ln\left(e^{g_n} + e^{\bar{g}_{n-1}\theta}\right)} + T_{n-1}$$

Subtracting T_{n-1} from both side of the above equation leads to:

(eq.9)

$$F(n) \cong \frac{\ln 2}{\ln 2 + g_n + \bar{g}_{n-1}\theta - \ln\left(e^{g_n} + e^{\bar{g}_{n-1}\theta}\right)}$$

This gives us an approximate functional form of F(n). For practical purposes the relationship between $\bar{g}_{n-1}$ and $\bar{g}_n$ is also needed, which will be dealt with next.

(2) Approximate relation between $\bar{g}_{n-1}$ and $\bar{g}_n$

Again making use of the approximation, I transform the definition of $\bar{g}_n$ as follows:

$$\begin{aligned}
\bar{g}_n &= -\ln\left(\sum_0^n h^k_{T_n} * e^{-g_k}\right) \\
&= -\ln\{(\sum_0^{n-1} h^k_{T_n} * e^{-g_k}) + (1 - \sum_0^{n-1} h^k_{T_n})e^{-g_n}\} \\
&= -\ln\{(\sum_0^{n-1} h^k_{T_{n-1}}\ e^{-g_k\theta(T_n - T_{n-1})}\ e^{-g_k}) + (1 - \sum_0^{n-1} h^k_{T_{n-1}}\ e^{-g_k\theta(T_n - T_{n-1})})e^{-g_n}\} \\
&= -\ln\{(\sum_0^{n-1} h^k_{T_{n-1}}\ e^{-g_k\theta\{1+F(n)\}}) + (1 - \sum_0^{n-1} h^k_{T_{n-1}}\ e^{-g_k\,\theta F(n)})e^{-g_n}\} \\
&\cong -\ln\{e^{-\bar{g}_{n-1}\theta\{1+F(n)\}} + (1 - e^{-\bar{g}_{n-1}\theta F(n)})e^{-g_n}\}
\end{aligned}$$

If v(n) is defined as

$$v(n) \equiv e^{-\bar{g}_{n-1}\theta F(n)}$$

Then,

$$\bar{g}_n \cong -\ln\left[\, v(n)e^{-\bar{g}_{n-1}} + \{1 - v(n)\}e^{-g_n} \,\right]$$

Hence, $\bar{g}_n$ can be considered as a kind of averaging between the former $\bar{g}$ (i.e. $\bar{g}_{n-1}$) and the latest g (i.e. g_n).

(3) Comparative statistics

Looking briefly at the effects of change in the exogenous variables on F(n), g_n by differentiation is:

(eq.10)

$$\frac{d\,F(n)}{dg_n} = \frac{\ln 2\left(\frac{e^{g_n}}{e^{g_n} + e^{\bar{g}_{n-1}\theta}} - 1\right)}{\{\ln 2 + g_n + \bar{g}_{n-1}\theta - \ln(e^{g_n} + e^{\bar{g}_{n-1}\theta})\}^2} < 0$$

A similar differentiation for other exogenous variables leads to a similar result.

(eq.11)

$$\frac{d\,F(n)}{d\bar{g}_{n-1}} < 0$$

(eq.12)

$$\frac{d\,F(n)}{d\theta} < 0$$

The greater g_n is, the lower F(n) becomes. Similarly, The greater $\bar{g}_{n-1}$, the lower F(n); and the greater θ, the lower F(n).

(4) A simple example

Before presenting a simple calculation as an example. I would like to simplify things further by constraining exogenous parameters. I assume, that:

(1) Improvement in production efficiency is uniform for all products:

$g_k = g$ for all k

(2) There is no friction in factor movements:

$\theta = 1$

These assumptions yield:

$$F(n) = \frac{\ln 2}{g}$$

Accordingly, average growth rate lead to:

$$\frac{g\,\{\ln(n+1) - \ln(n)\}}{\ln 2}$$

Note that these are not approximate solutions but exact ones due to the first assumption for the exogenous parameters.

It is apparent that the average growth rate converges to zero when n approaches infinity. Therefore this simplification is unsatisfactory and inconsistent with the evidence.

6. Concluding Remarks

In this study I have built a simple model for economic development where the demands for individual products will become saturated.

An important conclusion is that the average growth rate will gradually decrease and approach zero if the length of time between the starting of production and the saturation of demand is common for all products. Hence, should the model be consistent with stable and positive economic growth in long run, the length of time must become shorter for products whose production starts later.

Since the model assumes exogenous parameters for the improvement in production efficiency, we cannot discuss endogenous mechanisms for explaining the change in production efficiency, a problem that needs to be dealt with in future study.

Some of the model's simplifying assumptions are also open for future studies. The assumption that demand becomes saturated when it reaches 1 for all products, and that initial productivity when production starts equals 1 for all products, these assumptions were convenient for making the model easy for solving. There is room for refining and relaxing these assumptions. Lastly, the nonlinear utility function implicitly assumed in the model needs to be qualified.