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Parameterization of Applied General Equilibrium Models with Flexible Trade Specifications Based on the Armington, Krugman, and Melitz Models

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January 2013

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Keywords: applied general equilibrium; calibration; heterogeneous firms

JEL classification: C68, D58, F12, L16

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Abstract

Comparing simulation results obtained by applied general equilibrium (AGE) models using intra-industry trade specifications based on Armington, Krugman, and Melitz models may be important in evaluating trade-related economic policies today. This paper explains how the Armington-Krugman-Melitz supermodel developed by Dixon and Rimmer can be parameterized, and demonstrates that only two kinds of additional information are required in order to extend a standard trade model to include Melitz-type monopolistic competition and heterogeneous firms. Further, it is shown how specifying too much additional information leads to violations of the model constraints, necessitating adjustment and reconciliation of the data. Once a Melitz-type model is parameterized, a Krugman-type model can also be parameterized using the calibrated values in the Melitz-type model without any additional data. One aim of this study is to motivate additional data collection and database development concerning the proportions of exporting firms established in each country. Sample code for the General Algebraic Modeling System (GAMS) has also been prepared to promote the innovative supermodel in the AGE community.

Keywords: applied general equilibrium; calibration; monopolistic competition; heterogeneous firms.

JEL Classification Numbers: C68, D58, F12, L16.

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1. Introduction

As the global economy has become increasingly interdependent, thousands of applied general equilibrium (AGE) analyses have been utilized to evaluate regional trade agreements and economic partnership arrangements, and some model builders have attempted to incorporate theoretical information on intra-industry trade to account for economies of scale and imperfect competition. In conventional AGE models of global trade, the so-called "Armington assumption" has been widely adopted to handle cross-hauling, which is often observed in real data, between developed economies that have similar technologies and factor endowments.[1] Since this can be regarded as an *ad hoc* approach and sometimes can cause embarrassing simulation results from its tendency to undervalue efficiency gains, some models such as Francois (1998) and Roson (2006) have introduced theoretical illustrations of product differentiation in their analytical models as presented in the pioneering work of Krugman.

Krugman (1980) focused on two sources of efficiency gains that result from reducing trade barriers: cost reductions brought by economies of scale and increased variety obtained through additional imports. In the steady advance of new trade theory that followed, one of the most successful extensions of his work had been done by Melitz (2003). He appended another source of efficiency gains, namely, the reallocation of resources resulting from endogenous productivity growth among heterogeneous firms. In the AGE research community, Zhai (2008) introduced a Melitz-type specification into an AGE model as an alternative to the Armington approach. Then, Balistreri and Rutherford (2012) prepared a comprehensive guide to the treatment of the three approaches by Armington, Krugman, and Melitz, and Dixon and Rimmer (2012) finally developed a generalized supermodel that includes those three types of model as special cases.

The purpose of this paper is to explain how the supermodel can be parameterized and to show that only two kinds of additional information are required in order to extend a standard trade model to include Melitz-type monopolistic competition and heterogeneous firms. Once a Melitz-type model is parameterized, a Krugman-type model can also be parameterized using the calibrated values in the Melitz-type model without any additional data. Contributing to the AGE community through promotion of the supermodel by providing actual sample code is an additional goal of this paper.

The paper is organized as follows. Sections 2 and 3 illustrate the Armington-Krugman-Melitz supermodel proposed by Dixon and Rimmer (2012) and how it can be calibrated. Then, Section 4 introduces a sample implementation of an AGE model that includes the supermodel in a more

[1] Armington (1969).

practical form. Finally, Section 5 concludes the paper.

2. The Armington-Krugman-Melitz Supermodel

In this section, we will review details of the supermodel developed by Dixon and Rimmer (2012), which includes the Armington, Krugman, and Melitz models as special cases.

Let us start with aggregator functions for imported products from firms indexed e operating in region j':

$$\widetilde{D}_{jj'} = \left\{\sum_e \alpha_{jj'}^T \widehat{D}_{ejj'}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)}; \quad (1)$$

and

$$O_j + C_j = \varphi_j^T \left\{\sum_{j'} \widetilde{D}_{jj'}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)}, \quad (2)$$

where

$\widehat{D}_{ejj'}$ is the distribution (trade flow) of commodity from firm e operating in region j' to region j;

$\widetilde{D}_{jj'}$ is the quantity of commodity distributed from all firms operating in region j' to region j;

O_j is intermediate input;

C_j is consumption;

$\alpha_{jj'}^T$ is a positive parameter that reflects preference of j with respect to the region of origin j';

σ^T is the elasticity of substitution[2]; and

φ_j^T is the unit coefficient[3].

Economic agents in region j choose $\widehat{D}_{ejj'}$ to minimize the total purchase value of commodities subject to (1) and (2). This problem can be expressed as

$$\max \quad \sum_{j'} \sum_e \left(1 + \tau_{jj'}\right) \hat{p}_{ejj'} \widehat{D}_{ejj'}$$

$$\text{s.t.} \quad O_j + C_j = \varphi_j^T \left\{\sum_{j'} \alpha_{jj'}^T \sum_e \widehat{D}_{ejj'}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)}, \quad (3)$$

where

$\hat{p}_{ejj'}$ is the sales price of commodity exclusive of transportation margin and import tariff;

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[2] Notice that the same substitution elasticity σ^T is utilized in Equations (1) and (2).

[3] Dixon and Rimmer (2012) do not include this parameter in their aggregator formulation. We will set φ_j^T to unity when we calibrate a model in Section 3. Balistreri and Rutherford (2012) also refer to the treatment of this parameter.

and

$\tau_{jj'}$ is the rate of transportation margin and import tariff.

Equation (3) is derived by substituting (1) into (2). Setting the Lagrange multiplier for (3) as p_j^M, we get the following first-order condition (FOC):

$$\left(1+\tau_{jj'}\right)\hat{p}_{ejj'} = \alpha_{jj'}^T p_j^M \left(\varphi_j^T\right)^{(\sigma^T-1)/\sigma^T} \left(\frac{O_j+C_j}{\hat{D}_{ejj'}}\right)^{1/\sigma^T}, \quad (4)$$

where p_j^M represents the market price of commodity inclusive of transportation margin and import tariff.

Aggregate total profit of all firms operating in region j' can be expressed as

$$\pi_{j'} = \sum_{e\in J(jj')} \sum_j \hat{\pi}_{ejj'} - \sum_e \mu_{j'}^K p_{j'}^W Q_{j'}, \quad (5)$$

where

$J(jj')$ is the set of active firms that sell products on the j-j' link;

$\mu_{j'}^K$ is the fixed cost, measured in units of gross output (composite input), necessary to establish a firm in region j';

$p_{j'}^W$ is the wholesale price of products; and

$Q_{j'}$ is gross output produced in region j'.

Next, let $\hat{\pi}_{ejj'}$ be the contribution of firm e operating in region j' to the total profit from its sales to j:

$$\hat{\pi}_{ejj'} = \hat{p}_{ejj'}\hat{D}_{ejj'} - p_{j'}^W \hat{Q}_{ejj'}. \quad (6)$$

Here, $\hat{Q}_{ejj'}$ is gross output produced by firm e in region j' and sold in region j.

Assuming that the transformation of gross output $\hat{Q}_{ejj'}$ to regional distribution $\hat{D}_{ejj'}$ follows

$$\hat{D}_{ejj'} = \max\left\{\hat{\psi}_{ejj'}\left(\hat{Q}_{ejj'} - \mu_{jj'}^M Q_{j'}\right), 0\right\}, \quad (7)$$

we can rewrite (6) as

$$\hat{\pi}_{ejj'} = \hat{p}_{ejj'}\hat{D}_{ejj'} - \frac{p_{j'}^W}{\hat{\psi}_{ejj'}}\hat{D}_{ejj'} - \mu_{jj'}^M p_{j'}^W Q_{j'}, \quad (8)$$

where

$\hat{\psi}_{ejj'}$ is the productivity of firm e in region j' selling its products to j; and

$\mu_{jj'}^M$ is the fixed cost, measured in units of gross output (composite input), necessary to make sales on the j-j' link.

Firm e in region j' chooses the price and quantity of sales in region j to maximize $\hat{\pi}_{ejj'}$. Then the sales price exclusive of transportation margin and import tariff is marked up as

$$\hat{p}_{ejj'} = \left(\frac{1}{1+\varepsilon}\right)\frac{p_{j'}^W}{\hat{\psi}_{ejj'}}, \quad (9)$$

where ε is related to the elasticity of substitution σ^T such that $\varepsilon = -1/\sigma^T$.

Using (4) and (9), we can rewrite (8) as

$$\hat{\pi}_{ejj'} = -\varepsilon\left(\frac{1}{1+\varepsilon}\right)^{1-\sigma^T}\left(\frac{p^w_{j'}}{\hat{\psi}_{ejj'}\varphi^T_j}\right)^{1-\sigma^T}\left(O_j + C_j\right)\left(\frac{\alpha^T_{jj'}p^M_j}{1+\tau_{jj'}}\right)^{\sigma^T} - \mu^M_{jj'}p^w_{j'}Q_{j'}. \quad (10)$$

Therefore, (5) becomes

$$\pi_{j'} = -\varepsilon\left(\frac{1}{1+\varepsilon}\right)^{1-\sigma^T}\sum_{e\in J(jj')}\sum_j\left(\frac{p^w_{j'}}{\hat{\psi}_{ejj'}\varphi^T_j}\right)^{1-\sigma^T}\left(O_j + C_j\right)\left(\frac{\alpha^T_{jj'}p^M_j}{1+\tau_{jj'}}\right)^{\sigma^T}$$
$$-\sum_j \widetilde{M}_{jj'}\mu^M_{jj'}p^w_{j'}Q_{j'} - M_{j'}\mu^K_{j'}p^w_{j'}Q_{j'}, \quad (11)$$

where

$\widetilde{M}_{jj'}$ is the number of active firms operating in j' that sell products on the j-j' link; and

$M_{j'}$ is the number of firms registered in j'.

Next, transformation of total gross output $Q_{j'}$ can be expressed as

$$\sum_{e\in J(jj')}\sum_j\frac{\hat{D}_{ejj'}}{\hat{\psi}_{ejj'}} = \left(1 - \sum_j \widetilde{M}_{jj'}\mu^M_{jj'} - M_{j'}\mu^K_{j'}\right)Q_{j'}. \quad (12)$$

Equation (12) replaces the transformation part of gross output into domestic goods and exports in standard AGE models.

Melitz (2003) defines the relation between the average productivity of active firms $\psi_{jj'}$ and the minimum productivity required to operate on the j-j' link $\tilde{\psi}_{jj'}$ as

$$\psi_{jj'} = \left(\frac{\zeta}{\zeta-\sigma^T+1}\right)^{1/(\sigma^T-1)}\tilde{\psi}_{jj'}, \quad (13)$$

where

ζ is a shape parameter related to productivity by $\zeta > \sigma^T - 1$.

In addition, the proportion of registered but inactive firms $\xi_{jj'}$, whose productivity is insufficient to meet the minimum requirement, is defined as

$$\xi_{jj'} = 1 - \tilde{\psi}_{jj'}{}^{-\zeta}$$
$$= 1 - \left(\frac{\zeta}{\zeta-\sigma^T+1}\right)^{\zeta/(\sigma^T-1)}\psi_{jj'}{}^{-\zeta}\ . \quad (14)$$

The minimum productivity required for a firm in region j' to export its products to region j is determined at the level that satisfies $\hat{\pi}_{ejj'} = 0$. Using (10), we obtain

$$\psi_{jj'} = \frac{\varepsilon^{1/(\sigma^T-1)}}{1+\varepsilon}\left\{\left(\varphi^T_j\right)^{\sigma^T-1}\left(O_j + C_j\right)\left(\alpha^T_{jj'}p^M_j\right)^{\sigma^T}\right\}^{1/(1-\sigma^T)}$$
$$\times\left\{\left(1+\tau_{jj'}\right)p^w_{j'}\right\}^{\sigma^T/(\sigma^T-1)}\left(\mu^M_{jj'}Q_{j'}\right)^{1/(\sigma^T-1)}. \quad (15)$$

Using (4), (13), and (15), we obtain the average productivity of active firms:

$$\psi_{jj'} = \left(\frac{\zeta}{\zeta-\sigma^T+1}\right)^{1/(\sigma^T-1)} \frac{\varepsilon^{1/(\sigma^T-1)}}{1+\varepsilon} \left(\frac{p_{j'}^w}{\hat{p}_{ejj'}}\right)^{\sigma^T/(\sigma^T-1)} \left(\frac{\mu_{jj'}^M Q_{j'}}{\widehat{D}_{ejj'}}\right)^{1/(\sigma^T-1)}. \quad (16)$$

Rewriting (11) using the average productivity of active firms $\psi_{jj'}$ and the number of active firms $\widetilde{M}_{jj'}$, we obtain

$$\pi_{j'} = -\varepsilon \left(\frac{1}{1+\varepsilon}\right)^{1-\sigma^T} \sum_j \widetilde{M}_{jj'} \left(\frac{p_{j'}^w}{\psi_{jj'}\varphi_j^T}\right)^{1-\sigma^T} \left(O_j + C_j\right) \left(\frac{\alpha_{jj'}^T p_j^M}{1+\tau_{jj'}}\right)^{\sigma^T}$$
$$- \sum_j \widetilde{M}_{jj'} \mu_{jj'}^M p_{j'}^w Q_{j'} - M_{j'} \mu_{j'}^K p_{j'}^w Q_{j'}. \quad (17)$$

The number of firms $M_{j'}$ is determined at the level that satisfies $\pi_{j'} = 0$. Using (4), (9), and (17), we obtain

$$\sum_j \widetilde{M}_{jj'} \mu_{jj'}^M p_{j'}^w Q_{j'} + M_{j'} \mu_{j'}^K p_{j'}^w Q_{j'} = -\varepsilon \sum_j \widetilde{M}_{jj'} \hat{p}_{ejj'} \widehat{D}_{ejj'}. \quad (18)$$

Finally, equations to be included in the model are summarized as follows:

$$O_j + C_j = \varphi_j^T \left\{\sum_{j'} \alpha_{jj'}^T \widetilde{M}_{jj'} D_{jj'}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)} \quad \perp p_j^M; \quad (19)$$

$$\left(1 + \tau_{jj'}\right) p_{jj'} = \alpha_{jj'}^T p_j^M \left(\varphi_j^T\right)^{(\sigma^T-1)/\sigma^T} \left(\frac{O_j + C_j}{D_{jj'}}\right)^{1/\sigma^T} \quad \perp D_{jj'}; \quad (20)$$

$$p_{jj'} = \left(\frac{1}{1+\varepsilon}\right) \frac{p_{j'}^w}{\psi_{jj'}} \quad \perp p_{jj'}; \quad (21)$$

$$\sum_{j'} \widetilde{M}_{j'j} \frac{D_{j'j}}{\psi_{j'j}} = \left(1 - \sum_{j'} \widetilde{M}_{j'j} \mu_{j'j}^M - M_j \mu_j^K\right) Q_j \quad \perp p_j^W; \quad (22)$$

$$\xi_{jj'} = 1 - \left(\frac{\zeta}{\zeta-\sigma^T+1}\right)^{\zeta/(\sigma^T-1)} \psi_{jj'}{}^{-\zeta} \quad \perp \xi_{jj'}; \quad (23)$$

$$\psi_{jj'} = \left(\frac{\zeta}{\zeta-\sigma^T+1}\right)^{1/(\sigma^T-1)} \frac{\varepsilon^{1/(\sigma^T-1)}}{1+\varepsilon} \left(\frac{p_{j'}^w}{p_{jj'}}\right)^{\sigma^T/(\sigma^T-1)} \left(\frac{\mu_{jj'}^M Q_{j'}}{D_{jj'}}\right)^{1/(\sigma^T-1)} \quad \perp \psi_{jj'}; \quad (24)$$

and

$$\left(\sum_{j'} \widetilde{M}_{j'j} \mu_{j'j}^M + M_j \mu_j^K\right) p_j^w Q_j = -\varepsilon \sum_{j'} \widetilde{M}_{j'j} p_{j'j} D_{j'j} \quad \perp M_j. \quad (25)$$

In some equations, $\widehat{D}_{ejj'}$, $\hat{p}_{ejj'}$, and $\hat{\psi}_{ejj'}$ are respectively replaced with the average distribution (trade flow) of the commodity by active firm $D_{jj'}$, the sales price of the commodity exclusive of transportation margin and import tariff $p_{jj'}$, and the average productivity of active firms $\psi_{jj'}$. The perpendicular symbol "⊥" shows the corresponding relationships between variables and equations. Equations (23) and (24) do not appear in either the Krugman- or Armington-type formulations. Equation (25) also is dropped from the Armington–type specification.

Melitz-type Formulation: In a Melitz-type formulation, the following two assumptions are made, in addition to (19) through (25):

$$\varepsilon = -\frac{1}{\sigma^T};$$

and

$$\widetilde{M}_{jj'} = \left(1 - \xi_{jj'}\right) M_{j'}.$$

Krugman-type Formulation: In a Krugman-type formulation, the following four relations are assumed, in addition to (19) through (22), and (25):

$$\mu^M_{jj'} = 0;$$

$$\varepsilon = -\frac{1}{\sigma^T};$$

$$\psi_{jj'} = 1;$$

and

$$\widetilde{M}_{jj'} = M_{j'} \qquad (\therefore \xi_{jj'} = 0).$$

Armington-type Formulation: In an Armington-type formulation, the following four relations are assumed, in addition to (19) through (22):

$$\mu^K_{j'} = \mu^M_{jj'} = 0;$$

$$\varepsilon = 0;$$

$$\psi_{jj'} = 1;$$

and

$$\widetilde{M}_{jj'} = M_{j'} = 1 \quad (\therefore \xi_{jj'} = 0).$$

3. Parameterization

In this section, we explain the calibration procedures for parameterizing the three types of model presented in Section 2. Then, we shall see that we need only two kinds of additional information to extend an Armington-type model to be a Melitz-type model. The first piece of information is on ζ (shape parameter related to productivity).[4] The other is information on one of the following: $D_{jj'}$ (average distribution of commodity by active firm in region j'); $\mu^M_{jj'}$ (fixed cost necessary to make sales on the j-j' link); or $\xi_{jj'}$ (proportion of registered but inactive firms). Furthermore, once a Melitz-type model is parameterized, a Krugman-type model can also be parameterized using the calibrated values in the Melitz-type without any additional data.[5] Therefore, we start by calibrating

[4] Balistreri *et al*. (2011) implemented structural estimation of this shape parameter for a Pareto distribution, as well as the Melitz-type bilateral fixed cost.

[5] For more issues related to parameterization, see Zhai (2008), Balistreri *et al*. (2011), and Balistreri and Rutherford

a Melitz-type model, and after that, we can verify the procedure for a Krugman-type model.

3.1 Calibration of a Melitz-type Model

To parameterize an Armington-type model, it is well known that the following kinds of information are required in advance: $p_j^W Q_j$ (gross output at wholesale price); $p_j^M O_j$ (intermediate input at market price inclusive of transportation cost and import tariff); $p_j^M C_j$ (consumption at market price inclusive of transportation cost and import tariff); σ^T (elasticity of substitution); $\tau_{jj'}$ (rate of transportation margin and import tariff); and trade flows at free-on-board prices or producer prices, such as "*VXWD*" or "*VXMD*" as presented in the Global Trade Analysis Project (GTAP) database.[6] In the present framework, the two types of trade flows at the different price levels become identical.[7] Let us refer to the data related to the trade flow values as "$TF_{jj'}$" here. $TF_{jj'}$ can be regarded as

$$TF_{jj'} = (1 - \xi_{jj'}) M_{j'} p_{jj'} D_{jj'}. \tag{26}$$

In addition to the information listed above, information on ζ as well as on $D_{jj'}$, $\mu_{jj'}^M$, or $\xi_{jj'}$ is necessary to include Melitz-type monopolistic competition and heterogeneous firms. Then, two of the latter three pieces of information, as well as $\mu_{j'}^K$ (fixed cost necessary to establish a firm in region j') and $M_{j'}$ (number of firms registered in region j'), can be derived and calibrated. In this process, initial values of other endogenous variables, which cannot be observed directly from the given data, $p_{jj'}$ (sales price of the commodity by a firm in region j' exclusive of transportation margin and import tariff) and $\psi_{jj'}$ (average productivity of active firms in region j') also are derived by setting $p_{j'}^W$ (wholesale price of commodity produced in region j') to unity following the usual custom of AGE modeling. After that, initial values of p_j^M (market price of the commodity inclusive of transportation margin and import tariff) and $\alpha_{jj'}^T$ (parameter reflecting the preference of j with respect to region of origin j') are derived and calibrated.

Since $\xi_{jj'}$ might be relatively more observable than $D_{jj'}$ and $\mu_{jj'}^M$, we presume information on $\xi_{jj'}$ is given.[8] Then, we obtain initial values of $\psi_{jj'}$ using (21):

$$\psi_{jj'} = (1 - \xi_{jj'})^{-1/\zeta} \left(\frac{\zeta}{\zeta - \sigma^T + 1}\right)^{1/(\sigma^T - 1)}. \tag{27}$$

From the value of $\psi_{jj'}$ obtained by (27), initial values of $p_{jj'}$ are also derived from (21) by setting $p_{j'}^W$ to unity.

Using (4), (16), and (17), and setting $\pi_{j'} = 0$, as well as $\widetilde{M}_{jj'} = (1 - \xi_{jj'}) M_{j'}$, we obtain

(2012).

[6] Hertel (1997).

[7] More precisely, trade flows that are dealt with here include both domestic goods ("*VDM*" in the GTAP database) and intraregional trade in the part $j = j'$.

[8] In some cases, the number of registered firms $M_{j'}$ may bring a scaling problem as well as quantity variables.

$$\mu_{j'}^{K} = \left(\frac{\sigma^T-1}{\zeta-\sigma^T+1}\right)\sum_j\left(1-\xi_{jj'}\right)\mu_{jj'}^{M}. \tag{28}$$

Hence, we find that $\mu_{j'}^{K}$ is a function of $\mu_{jj'}^{M}$ when $\xi_{jj'}$ is given: $\mu_{j'}^{K}\left(\mu_{jj'}^{M}\right)$.

Next, we can derive the following relation using (25) and (26):

$$\left\{\sum_j\left(1-\xi_{jj'}\right)\mu_{jj'}^{M} + \mu_{j'}^{K}\right\}M_{j'}p_{j'}^{w}Q_{j'} = -\varepsilon\sum_j TF_{jj'}.$$

Therefore, we obtain

$$M_{j'} = -\frac{\varepsilon\sum_j TF_{jj'}}{\left\{\sum_j\left(1-\xi_{jj'}\right)\mu_{jj'}^{M}+\mu_{j'}^{K}\right\}p_{j'}^{w}Q_{j'}}. \tag{29}$$

From (29), we find that $M_{j'}$ is a function of $\mu_{jj'}^{M}$ and $\mu_{j'}^{K}$ when $\xi_{jj'}$ is given: $M_{j'}\left(\mu_{jj'}^{M},\ \mu_{j'}^{K}\right)$.

Substituting (21) into (26), we get

$$TF_{jj'} = \left(1-\xi_{jj'}\right)M_{j'}\left(\frac{1}{1+\varepsilon}\right)\frac{p_{j'}^{w}}{\psi_{jj'}}D_{jj'}.$$

Therefore,

$$D_{jj'} = \frac{(1+\varepsilon)\psi_{jj'}TF_{jj'}}{\left(1-\xi_{jj'}\right)M_{j'}p_{j'}^{w}}. \tag{30}$$

Hence, we find that $D_{jj'}$ is a function of $M_{j'}$: $D_{jj'}\left(M_{j'}\right)$.

Plugging (24) into (23), we can derive

$$\mu_{jj'}^{M} = -\varepsilon\left(\frac{1}{1+\varepsilon}\right)^{1-\sigma^T}\left(1-\xi_{jj'}\right)^{(1-\sigma^T)/\zeta}\left(\frac{p_{jj'}}{p_{j'}^{w}}\right)^{\sigma^T}\frac{D_{jj'}}{Q_{j'}}. \tag{31}$$

From (31), we find that $\mu_{jj'}^{M}$ is a function of $D_{jj'}$ when $\xi_{jj'}$ is given: $\mu_{jj'}^{M}\left(D_{jj'}\right)$.

Finally, $\mu_{jj'}^{M}$, $\mu_{j'}^{K}$, $M_{j'}$, and $D_{jj'}$ can be calibrated simultaneously by solving the system of equations (28) through (31) when $\xi_{jj'}$ is given.

Once $\psi_{jj'}$, $p_{jj'}$, $\mu_{jj'}^{M}$, $\mu_{j'}^{K}$, $M_{j'}$, and $D_{jj'}$ are calibrated, we can derive initial values of p_j^M and parameter $\alpha_{jj'}^{T}$ as follows:

$$p_j^M = \frac{\sum_{j'}\left(1+\tau_{jj'}\right)p_{jj'}\left(1-\xi_{jj'}\right)M_{j'}D_{jj'}}{\sum_{j'}\left(1-\xi_{jj'}\right)M_{j'}D_{jj'}}; \tag{32}$$

and

$$\alpha_{jj'}^{T} = \frac{\left(1+\tau_{jj'}\right)p_{jj'}}{p_j^M}\left(\varphi_j^T\right)^{(1-\sigma^T)/\sigma^T}\left(\frac{D_{jj'}}{O_j+C_j}\right)^{1/\sigma^T}. \tag{33}$$

Equation (33) is derived from (4). In (33), φ_j^T is set to unity to maintain consistency with trade theories based on monopolistic competition.

Thus we have demonstrated that only two kinds of additional information are required to extend a standard trade model to include Melitz-type monopolistic competition and heterogeneous firms. Note that giving too much additional information will lead to violations of the model constraints, necessitating further adjustment and reconciliation of data.

3.2 Calibration of a Krugman-type Model

In a Krugman-type model, either parameter $\mu^K_{j'}$ (fixed cost necessary to establish a firm in region j') or initial values of $M_{j'}$ (number of firms registered in region j') can be derived and calibrated. Note that information on both parameters has been already obtained in the parameterization process of a Melitz-type model based on the same benchmark dataset.

As in the case of the Melitz-type specification, initial values of $p_{jj'}$ can be derived from (21) setting $p^W_{j'}$ to unity. Then, we can obtain either of $M_{j'}$ or $\mu^K_{j'}$ using (29) as follows:

$$M_{j'} = -\frac{\varepsilon \sum_j TF_{jj'}}{\mu^K_{j'} p^W_{j'} Q_{j'}}; \tag{34}$$

and

$$\mu^K_{j'} = -\frac{\varepsilon \sum_j TF_{jj'}}{M_{j'} p^W_{j'} Q_{j'}}. \tag{35}$$

In (34) or (35), the calibrated values of $\mu^K_{j'}$ or $M_{j'}$ obtained in a Melitz-type formulation can be utilized.

Next, we obtain $D_{jj'}$ from (30):

$$D_{jj'} = \frac{(1+\varepsilon)\psi_{jj'} TF_{jj'}}{M_{j'} p^W_{j'}}. \tag{36}$$

Finally, we can derive $\alpha^T_{jj'}$ from (33) by setting φ^T_j to unity as in the case of the Melitz-type formulation.

4. General Equilibrium Formulation

In the previous sections, we focused on only the trade-related aspects among regions. In this section, we formulate an AGE model that includes the Armington-Krugman-Melitz supermodel as a module. To avoid complexity and keep the explanations simple and clear, we address in this section the case of a single-sector model, in which all of the industries are assumed to be imperfectly competitive when Melitz- and Krugman-type models are adopted. A more sophisticated example that includes two sectors, imperfectly competitive manufacturing, and perfectly competitive primary industries and services is presented in the Appendices.

Value-Added: Producers in region j determine input levels of primary factors K_j (capital input) and L_j (labor input) to minimize cost subject to a Cobb-Douglas technology. The problem can be expressed as

$$\min \quad w_j^K K_j + w_j^L L_j$$

$$\text{s.t.} \quad Y_j = \varphi_j^Y K_j^{\alpha_j^Y} L_j^{1-\alpha_j^Y} \qquad \perp p_j^Y, \qquad (37)$$

where

w_j^K is capital rental rate;

w_j^L is wage rate;

p_j^Y is price index for value-added;

Y_j is value-added;

α_j^Y is share parameter; and

φ_j^Y is unit coefficient.

The FOCs for optimization are

$$w_j^K = \alpha_j^Y p_j^Y \left(\frac{Y_j}{K_j}\right) \qquad \perp K_j, \qquad (38)$$

and

$$w_j^L = \left(1 - \alpha_j^Y\right) p_j^Y \left(\frac{Y_j}{L_j}\right) \qquad \perp L_j. \qquad (39)$$

Gross Output: As in the case of value-added, producers in region j determine input levels of composite factors Y_j (value-added) and O_j (intermediate input) to minimize cost subject to a constant elasticity of substitution technology. The problem can be expressed as

$$\min \quad p_j^Y Y_j + p_j^M O_j$$

$$\text{s.t.} \quad Q_j = \varphi_j^Q \left\{ \alpha_j^Q Y_j^{(\sigma^Q-1)/\sigma^Q} + \left(1 - \alpha_j^Q\right) O_j^{(\sigma^Q-1)/\sigma^Q} \right\}^{\sigma^Q/(\sigma^Q-1)}$$

$$\perp p_j^Q, \qquad (40)$$

where

p_j^Q is the price index for gross output;

σ^Q is the elasticity of substitution;

α_j^Q is the share parameter; and

φ_j^Q is the unit coefficient.

p_j^M (market price of commodity inclusive of transportation margin and import tariff) and Q_j (gross output) are the same variables as presented in the previous sections. The FOCs for optimization are

$$p_j^Y = \alpha_j^Q p_j^Q \left(\varphi_j^Q\right)^{(\sigma^Q-1)/\sigma^Q} \left(\frac{Q_j}{Y_j}\right)^{1/\sigma^Q} \qquad \perp Y_j, \qquad (41)$$

and

$$p_j^M = \left(1-\alpha_j^Q\right) p_j^Q \left(\varphi_j^Q\right)^{(\sigma^Q-1)/\sigma^Q} \left(\frac{Q_j}{O_j}\right)^{1/\sigma^Q} \qquad \perp O_j. \qquad (42)$$

Household: The representative household in region j maximizes the level of composite consumption C_j subject to a household budget constraint, given as the total of factor income and tariff revenue transferred from the regional authority. This problem can be expressed as follows:

$$\max \quad C_j$$

$$\text{s.t.} \quad p_j^M C_j = w_j^K K_j + w_j^L L_j + T_j \qquad \perp \lambda_j, \qquad (43)$$

where

λ_j is marginal utility of income; and

T_j is tariff revenue, defined as

$$T_j \equiv \sum_{j'} \tau_{jj'}^M \left(1+\tau_{jj'}^T\right)\left(\frac{1}{1+\varepsilon}\right) p_{j'}^W \left(1-\xi_{jj'}\right) M_{j'} \frac{D_{jj'}}{\psi_{jj'}}.$$

Note that $\tau_{jj'}$ appeared previously and is now divided into $\tau_{jj'}^M$ (import tariff rate) and $\tau_{jj'}^T$ (transportation margin). The FOC for optimization is

$$\lambda_j p_j^M = 1 \qquad \perp C_j. \qquad (44)$$

Factor Market: The factor market clearing conditions are

$$K_j = \bar{K}_j \qquad \perp w_j^K, \qquad (45)$$

and

$$L_j = \bar{L}_j \qquad \perp w_j^L, \qquad (46)$$

where $\bar{K}_j$ and $\bar{L}_j$ are exogenously given endowments.

Others: Equations (19), (20), (22), and (25) require some additional modifications as follows:

$$O_j + C_j = \varphi_j^T \left\{\sum_{j'} \alpha_{jj'}^T \left(1-\xi_{jj'}\right) M_{j'} D_{jj'}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)} \qquad \perp p_j^M; \qquad (47)$$

$$\left(1+\tau_{jj'}^M\right)\left(1+\tau_{jj'}^T\right) p_{jj'} = \alpha_{jj'}^T p_j^M \left(\varphi_j^T\right)^{(\sigma^T-1)/\sigma^T} \left(\frac{O_j+C_j}{D_{jj'}}\right)^{1/\sigma^T} \qquad \perp D_{jj'}; \qquad (48)$$

$$\sum_{j'} \left(1-\xi_{j'j}\right) M_j \frac{D_{j'j}}{\psi_{j'j}} + \Gamma_j = \left\{1-\sum_{j'}\left(1-\xi_{j'j}\right) M_j \mu_{j'j}^M - M_j \mu_j^K\right\} Q_j$$

$$\perp p_j^W; \qquad (49)$$

and

$$\left\{\sum_{j'}(1-\xi_{j'j})\mu^M_{j'j} + \mu^K_j\right\} p^W_j Q_j = -\varepsilon \sum_{j'}(1-\xi_{j'j}) p_{j'j} D_{j'j} \qquad \perp M_j, \qquad (50)$$

where

Γ_j is interregional transportation supply defined with regional share parameter γ_j as

$$\Gamma_j \equiv \frac{\gamma_j}{p^W_j} \sum_{j'} \sum_{j''} \left(1 + \tau^M_{j'j''}\right)\left(1 + \tau^T_{j'j''}\right)\left(\frac{1}{1+\varepsilon}\right) p^W_{j''} \left(1 - \xi_{j'j''}\right) M_{j''} \frac{D_{j'j''}}{\psi_{j'j''}}.$$

Γ_j is included in (49) to satisfy the special treatment concerning interregional shipping supply by the transportation service sector required in the GTAP database.

Finally, a relation between p^Q_j (price index for gross output) and p^W_j (wholesale price) is added:

$$p^Q_j = p^W_j \qquad \perp Q_j. \qquad (51)$$

The system of an AGE model that includes the supermodel developed by Dixon and Rimmer (2012) is described by 18 equations consist of (21), (23), (24), and (37) through (51). Since Walras' Law holds, one of the market clearing conditions automatically holds. In this regard, for example, we drop (49) with respect to region 1, exogenously setting p^W_1 to unity.

Sample code for General Algebraic Modeling System (GAMS),[9] which includes two sectors, imperfectly competitive manufacturing based on the treatment shown above, and perfectly competitive primary industries and services, is given in the Appendices.

5. Concluding Remarks

Comparing simulation results obtained by AGE models based on the intra-industry trade specifications of Armington, Krugman, and Melitz may have considerable importance in evaluating trade-related economic policies today.

This paper explained how the Armington-Krugman-Melitz supermodel developed by Dixon and Rimmer (2012) can be parameterized, and clarified that only two kinds of additional information are required in order to extend a standard trade model to include Melitz-type monopolistic competition and heterogeneous firms. The required information must include the shape parameter related to productivity (ζ) and one of the following: the average distribution of commodity by active firm in region j' ($D_{jj'}$); the fixed cost necessary to make sales on the j-j' link ($\mu^M_{jj'}$); or the proportion of registered but inactive firms ($\xi_{jj'}$). Then, with two of the latter three pieces of information, as well as the fixed cost necessary to establish a firm in region j' ($\mu^K_{j'}$) and

[9] Brook *et al*. (1992).

the number of firms registered in region j' ($M_{j'}$), the parameters specific to a model based on monopolistic competition and economies of scale can be derived and calibrated. In addition, once a Melitz-type model is parameterized, a Krugman-type model can be parameterized using the calibrated values in the Melitz-type model without any additional data.

In this paper, we assumed that information on $\xi_{jj'}$ is available, since it should be relatively be more observable than $D_{jj'}$ and $\mu^M_{jj'}$. I hope this study will motivate additional data collection and database development concerning the proportions of exporting firms established in every country. Sample code for GAMS has also been prepared to promote the innovative supermodel in the AGE community. My next goal is to prepare an extension module for the GTAP models to make comprehensive trade analysis more accessible.

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Appendix A: Benchmark Data for a Three-Region, Two-Sector Model

The sample AGE model that includes Armington-Krugman-Melitz modules presented in Appendix B is based on an artificial dataset. The benchmark dataset consists of: input-output (I-O) tables (Table 1); value of trade flows at three different price levels (Tables 2 through 4); value of interregional shipping supply (Table 5); three types of substitution elasticities (Table 6); proportion of inactive firms (Table 7); and shape parameter $\zeta = 4.50$. The former three can be obtained from the GTAP database, and social accounting matrices (SAMs) can be constructed for each country (Table 8). Since we assume symmetric regions, I-O tables and SAMs are identical.

In the tables, r01 through r03 denote regions. s0x, AT0x, and CT0x are production sectors, where 01 implies imperfectly competitive manufacturing and 02 denotes perfectly competitive primary industries and services. C, E, M, Q, K, L, FM, HH, WT, and IS respectively denote consumption, exports, imports, gross output, capital, labor, firm, household, exports/imports, and interregional shipping.

	s01	s02	C	E-M	Q
s01	15000	4500	5250	-4750	20000
s02	4000	1500	1000	240	6740
K	600	240			
L	400	500			
Q	20000	6740			

Table 1: Input-Output Table for Each Region

		r01	r02	r03	Imports
r01	s01	10000	5000	5000	20000
	s02	3000	1000	1000	5000
r02	s01	5000	10000	5000	20000
	s02	1000	3000	1000	5000
r03	s01	5000	5000	10000	20000
	s02	1000	1000	3000	5000
Exports	s01	20000	20000	20000	
	s02	5000	5000	5000	

Table 2: Trade Flows at FOB Prices

		r01	r02	r03	Imports
r01	s01	10500	5500	5500	21500
	s02	3120	1060	1060	5240
r02	s01	5500	10500	5500	21500
	s02	1060	3120	1060	5240
r03	s01	5500	5500	10500	21500
	s02	1060	1060	3120	5240
Exports	s01	21500	21500	21500	
	s02	5240	5240	5240	

Table 3: Trade Flows at CIF Prices

		r01	r02	r03	Imports
r01	s01	11550	6600	6600	24750
	s02	3744	1378	1378	6500
r02	s01	6600	11550	6600	24750
	s02	1378	3744	1378	6500
r03	s01	6600	6600	11550	24750
	s02	1378	1378	3744	6500
Exports	s01	24750	24750	24750	
	s02	6500	6500	6500	

Table 4: Trade Flows at Tariff Inclusive Market Prices

r01	r02	r03
1740	1740	1740

Table 5: Interregional Shipping Supply

	σ^Q	σ^O	σ^T
s01	0.75	0.75	2.00
s02	0.75	0.75	2.00

Table 6: Substitution Elasticities

	r01	r02	r03
r01	0.20	0.60	0.60
r02	0.60	0.20	0.60
r03	0.60	0.60	0.20

Table 7: Proportion of Inactive Firms ($\xi_{jj'}$)

	Expenditures:	Activities		Commodities		Factors		Institutions		Trade		Total
Receipts:		AT01	AT02	CT01	CT02	K	L	FM	HH	WT	IS	TT
Activities	AT01			0						20000		20000
	AT02				0					5000	1740	6740
Commodities	CT01	15000	4500						5250			24750
	CT02	4000	1500						1000			6500
Factors	K	600	240									840
	L	400	500									900
Institutions	FM					840						840
	HH			3250	1260		900	840		0	0	6250
Trade	WT			20000	5000							25000
	IS			1500	240							1740
Total	TT	20000	6740	24750	6500	840	900	840	6250	25000	1740	

Table 8: Social Accounting Matrix for Each Region

Appendix B: GAMS Code for Three-Region, Two-Sector Model

This sample AGE model includes three regions and two sectors. Here, s01 is regarded as an imperfectly competitive manufacturing sector and 02 represents perfectly competitive primary industries and services. Switching between Armington-Krugman-Melitz modules is implemented in the model settings.

According to the expansion to a two-sector model, two types of aggregator functions, that define composite intermediate input and composite consumption (eqPOs and eqPCs), and FOCs (eqOOs and eqCC) are added to the model explained in Section 4. The correspondence between equations shown in Sections 2 through 4 and the GAMS code is as follows: (21), eqP; (23), eqXI; (24), eqPSI; (37), eqPY; (38), eqK; (39), eqL; (40), eqPQ; (41), eqY; (42), eqO; (43), eqLAMBDA; (44), eqC; (45), eqWK; (46), eqWL; (47), eqPM; (48), eqD; (49), eqPW; (50), eqM; and (51), eqQ.

```
$TITLE A Three-Region Two-Sector Static Applied General Equilibrium Model

$ONTEXT
   Includes Armington-Krugman-Melitz Supermodel
   s01: Increasing Returns to Scale
   s02: Constant Returns to Scale (Armington Type Demand System)
$OFFTEXT

* Model Setting ======================================================

SETS
   i                Activity & Commodity   /s01,s02/
   j                Economic Region        /r01*r03/;

ALIAS (i,ii,iii),(j,jj,jjj);

* Benchmark Data Set =================================================

TABLE
   TF04(i,j,jj)     Trade Flow at CIF Price (Incl. Tariff)
             r01     r02     r03
```

```
 s01.r01   11550    6600    6600
 s02.r01    3744    1378    1378
 s01.r02    6600   11550    6600
 s02.r02    1378    3744    1378
 s01.r03    6600    6600   11550
 s02.r03    1378    1378    3744;

TABLE
   TF03(i,j,jj)      Trade Flow at CIF Price
              r01     r02     r03
 s01.r01   10500    5500    5500
 s02.r01    3120    1060    1060
 s01.r02    5500   10500    5500
 s02.r02    1060    3120    1060
 s01.r03    5500    5500   10500
 s02.r03    1060    1060    3120;

TABLE
   TF01(i,j,jj)      Trade Flow at Market Price (FOB Price)
              r01     r02     r03
 s01.r01   10000    5000    5000
 s02.r01    3000    1000    1000
 s01.r02    5000   10000    5000
 s02.r02    1000    3000    1000
 s01.r03    5000    5000   10000
 s02.r03    1000    1000    3000;

TABLE
   V01(i,j)          Operating Surplus
              r01     r02     r03
 s01          600     600     600
 s02          240     240     240;

TABLE
   L01(i,j)          Wage and Salary
```

```
            r01     r02     r03
 s01        400     400     400
 s02        500     500     500;

TABLE
   O01(i,ii,j)      Intermediate at Market Price (Incl. Tariff)
         s01.r01 s02.r01
 s01       15000    4500
 s02        4000    1500

 +       s01.r02 s02.r02
 s01       15000    4500
 s02        4000    1500

 +       s01.r03 s02.r03
 s01       15000    4500
 s02        4000    1500;

TABLE
   C01(i,j)         Consumption at Market Price (Incl. Tariff)
            r01     r02     r03
 s01       5250    5250    5250
 s02       1000    1000    1000;

PARAMETER
   ISS0(j)          Interregional Shipping Supply at Market Price
/r01   1740
 r02   1740
 r03   1740/;

* Elasticities for CES Aggregators ==================================

PARAMETERS
   sigmaQ(i)        Factor Substitution Elasticity
/s01   0.75
```

```
 s02   0.75/

   sigmaO(i)        Commodity Substitution Elasticity (Intermediate)
/s01   0.75
 s02   0.75/

   sigmaT(i)        Import Substitution Elasticity
/s01   2.00
 s02   2.00/

* Other Data Set ======================================================

TABLE
   XIO(j,jj)        Proportion of Inactive Firms
        r01    r02    r03
 r01   0.20   0.60   0.60
 r02   0.60   0.20   0.60
 r03   0.60   0.60   0.20;

SCALAR
   zeta             Shape PARAMETER Related to Productivity   /4.50/;

* Derivation of Additional Data Set [A] ===============================

PARAMETERS
   KE(j)            Capital Endowment
   LE(j)            Labor Endowment
   Y0(i,j)          Value Added
   Q0(i,j)          Gross Output
   qoppa(j)         Regional Share of Interregional Shipping Supply
   tauT(i,j,jj)     Rate of Interreginal Shipping Margin
   tauM(i,j,jj)     Import Tariff Rate
   epsilon          Price Markup Rate
   TF0(i,j,jj)      Core Trade Flow (Incl. Domestic Product);
KE(j)= SUM(i,V01(i,j));
```

```
LE(j)= SUM(i,L01(i,j));
Y0(i,j)= V01(i,j)+L01(i,j);
Q0(i,j)= SUM(ii,O01(ii,i,j))+Y0(i,j);
qoppa(j)= ISS0(j)/SUM(jj,ISS0(jj));
tauT(i,j,jj)= (TF03(i,j,jj)-TF01(i,j,jj))/TF01(i,j,jj);
tauM(i,j,jj)= (TF04(i,j,jj)-TF03(i,j,jj))/TF03(i,j,jj);
epsilon= -1/sigmaT("s01");
TF0(i,j,jj)= (1+epsilon$(ORD(i) EQ 1))*TF01(i,j,jj);

OPTION DECIMALS= 8;
DISPLAY
   KE,LE,Y0,Q0,qoppa,tauT,tauM,epsilon,TF0;

* Parameterization of Melitz Type =====================================

* Derivation of Additional Data Set [B] ===============================

PARAMETERS
   PSI0(j,jj)       Average Productivity of Active Firms
   P0_M(i,j,jj)     Markup Price (Excl. Transportation Cost and Tariff);
PSI0(j,jj)= (1-XI0(j,jj))**(-1/zeta)
   *(zeta/(zeta-sigmaT("s01")+1))**(1/(sigmaT("s01")-1));
P0_M(i,j,jj)= ((1+epsilon)**(-1)/PSI0(j,jj))$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2);

DISPLAY
   PSI0,P0_M;

* Parameterization [A] ================================================

POSITIVE VARIABLES
   sampiM0(j,jj),sampiK0(j),M00(j),D00(j,jj);

EQUATIONS
   eq01(j,jj),eq02(j),eq03(j),eq04(j,jj);
```

```
eq01(j,jj)..
   sampiM0(j,jj) =E= (1-XI0(j,jj))**((1-sigmaT("s01"))/zeta)
   *(-epsilon)*(1+epsilon)**(sigmaT("s01")-1)*P0_M("s01",j,jj)
   **sigmaT("s01")*D00(j,jj)/Q0("s01",jj);
eq02(j)..
   sampiK0(j) =E= (sigmaT("s01")-1)/(zeta-sigmaT("s01")+1)
   *SUM(jj,(1-XI0(jj,j))*sampiM0(jj,j));
eq03(j)..
   M00(j) =E= -epsilon*SUM(jj,TF01("s01",jj,j))
   /((SUM(jj,(1-XI0(jj,j))*sampiM0(jj,j))+sampiK0(j))*Q0("s01",j));
eq04(j,jj)..
   D00(j,jj) =E= TF0("s01",j,jj)*PSI0(j,jj)/((1-XI0(j,jj))*M00(jj));

* Initialization of Variables ========================================

sampiM0.LO(j,jj)= 1e-10; sampiK0.LO(j)= 1e-10;
M00.LO(j)= 1e-10; D00.LO(j,jj)= 1e-10;

sampiM0.L(j,jj)= 1e-2;
sampiK0.L(j)= 1e-2;
M00.L(j)= 1e+2;
D00.L(j,jj)= 1e+3;

* Model Definition ===================================================

MODEL CALIBRATION
   /eq01.sampiM0,eq02.sampiK0,eq03.M00,eq04.D00/;

OPTIONS
   ITERLIM= 1e+8,
   RESLIM= 1e+8,
   LIMROW= 0,
   LIMCOL= 0,
   MCP= PATH;
```

```
SOLVE CALIBRATION USING MCP;

* Output ==========================================================

PARAMETERS
   sampiM(j,jj)      Fixed Cost of Operation on the j-jj Link
   sampiK_M(j)       Fixed Cost of Establishing a Firm in Region j
   M0(j)             Number of Registered Firms
   D0_M(i,j,jj)      Core Trade FLow by Firm (Incl. Domestic Product);
sampiM(j,jj)= sampiM0.L(j,jj);
sampiK_M(j)= sampiK0.L(j);
M0(j)= M00.L(j);
D0_M(i,j,jj)= D00.L(j,jj)$(ORD(i) EQ 1)+TF0(i,j,jj)$(ORD(i) EQ 2);

DISPLAY
   sampiM,sampiK_M,M0,D0_M;

* Derivation of Additional Data Set [C] ==============================

PARAMETERS
   PM0_M(i,j)        Composite Price of Commodity
   O0_M(i,ii,j)      Core Intermediate
   C0_M(i,j)         Core Consumption
   OO0_M(i,j)        Core Composite Intermediate
   CC0_M(j)          Core Composite Consumption
   PO0_M(i,j)        Price of Composite Intermediate
   PC0_M(j)          Price of Composite Consumption;
PM0_M(i,j)= SUM(jj,(1+tauM(i,j,jj))*(1+tauT(i,j,jj))*P0_M(i,j,jj)
   *((1-XI0(j,jj))*M0(jj)$(ORD(i) EQ 1)+1$(ORD(i) EQ 2))*D0_M(i,j,jj))
   /SUM(jj,((1-XI0(j,jj))*M0(jj)$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2))*D0_M(i,j,jj));
O0_M(i,ii,j)= O01(i,ii,j)/PM0_M(i,j);
C0_M(i,j)= C01(i,j)/PM0_M(i,j);
OO0_M(i,j)= SUM(ii,O0_M(ii,i,j));
```

```
CC0_M(j)= SUM(i,C0_M(i,j));
PO0_M(i,j)= SUM(ii,OO1(ii,i,j))/OOO_M(i,j);
PC0_M(j)= SUM(i,CO1(i,j))/CC0_M(j);

DISPLAY
   PMO_M,OO_M,CO_M,OOO_M,CCO_M,POO_M,PCO_M;

* Parameterization [B] ================================================

PARAMETERS
   alphaY(i,j)      Share of Capital Input
   alphaO_M(i,ii,j) Share of Commodity (Intermediate)
   alphaC(i,j)      Share of Commodity (Consumption)
   alphaQ_M(i,j)    Share of Composite Factor
   alphaT_M(i,j,jj) Share of Commodity from Each Region
   phiY(i,j)        Unit Coefficient in Value Added Aggregator
   phiO_M(i,j)      Unit Coefficient in Composite Intermediate Aggregator
   phiC_M(j)        Unit Coefficient in Composite Consumption Aggregator
   phiQ_M(i,j)      Unit Coefficient in Gross Output Aggregator
   phiT_M(i,j)      Unit Coefficient in Commodity Aggregator;
alphaY(i,j)= VO1(i,j)/YO(i,j);
alphaO_M(i,ii,j)= PMO_M(i,j)*OO_M(i,ii,j)**(1/sigmaO(i))
   /SUM(iii,PMO_M(iii,j)*OO_M(iii,ii,j)**(1/sigmaO(iii)));
alphaC(i,j)= CO1(i,j)/SUM(ii,CO1(ii,j));
alphaQ_M(i,j)= YO(i,j)**(1/sigmaQ(i))
   /(YO(i,j)**(1/sigmaQ(i))+POO_M(i,j)*OOO_M(i,j)**(1/sigmaQ(i)));
alphaT_M(i,j,jj)= ((1+tauM(i,j,jj))*(1+tauT(i,j,jj))*PO_M(i,j,jj)/PMO_M(i,j)
   *((SUM(ii,OO_M(i,ii,j))+CO_M(i,j))/DO_M(i,j,jj))
   **(-1/sigmaT(i)))$(ORD(i) EQ 1)
   +((1+tauM(i,j,jj))*(1+tauT(i,j,jj))*PO_M(i,j,jj)
   *DO_M(i,j,jj)**(1/sigmaT(i))
   /SUM(jjj,(1+tauM(i,j,jjj))*(1+tauT(i,j,jjj))*PO_M(i,j,jjj)
   *DO_M(i,j,jjj)**(1/sigmaT(i))))$(ORD(i) EQ 2);
phiY(i,j)= YO(i,j)/(VO1(i,j)**alphaY(i,j)*LO1(i,j)**(1-alphaY(i,j)));
phiO_M(i,j)= OOO_M(i,j)
```

```
   /SUM(ii,alphaO_M(ii,i,j)*OO_M(ii,i,j)**((sigmaO(i)-1)/sigmaO(i)))
   **(sigmaO(i)/(sigmaO(i)-1));
phiC_M(j)= CCO_M(j)/PROD(i,CO_M(i,j)**alphaC(i,j));
phiQ_M(i,j)= QO(i,j)/((alphaQ_M(i,j)*YO(i,j)**((sigmaQ(i)-1)/sigmaQ(i))
   +(1-alphaQ_M(i,j))*OOO_M(i,j)**((sigmaQ(i)-1)/sigmaQ(i)))
   **(sigmaQ(i)/(sigmaQ(i)-1)));
phiT_M(i,j)= (SUM(ii,OO_M(i,ii,j))+CO_M(i,j))
   /SUM(jj,alphaT_M(i,j,jj)*((1-XIO(j,jj))*MO(jj)$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2))*DO_M(i,j,jj)**((sigmaT(i)-1)/sigmaT(i)))
   **(sigmaT(i)/(sigmaT(i)-1));

DISPLAY
   alphaY,alphaO_M,alphaC,alphaQ_M,alphaT_M,
   phiY,phiO_M,phiC_M,phiQ_M,phiT_M;

* Parameterization of Krugman Type ====================================

* Derivation of Additional Data Set [D] ===============================

PARAMETERS
   PO_K(i,j,jj)     Markup Price (Excl. Transportation Cost and Tariff)
   sampiK_K(j)      Fixed Cost of Establishing a Firm in Region j
   DO_K(i,j,jj)     Core Trade Flow by Firm (Incl. Domestic Product)
   PMO_K(i,j)       Composite Price of Commodity
   OO_K(i,ii,j)     Core Intermediate
   CO_K(i,j)        Core Consumption
   OOO_K(i,j)       Core Composite Intermediate
   CCO_K(j)         Core Composite Consumption
   POO_K(i,j)       Price of Composite Intermediate
   PCO_K(j)         Price of Composite Consumption;
PO_K(i,j,jj)= ((1+epsilon)**(-1))$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2);
sampiK_K(j)= -epsilon*SUM(jj,TFO1("s01",jj,j))/(MO(j)*QO("s01",j));
DO_K(i,j,jj)= (TFO("s01",j,jj)/MO(jj))$(ORD(i) EQ 1)
   +TFO(i,j,jj)$(ORD(i) EQ 2);
```

```
PMO_K(i,j)= SUM(jj,(1+tauM(i,j,jj))*(1+tauT(i,j,jj))*PO_K(i,j,jj)
   *((1-XIO(j,jj))*MO(jj)$(ORD(i) EQ 1)+1$(ORD(i) EQ 2))*DO_K(i,j,jj))
   /SUM(jj,((1-XIO(j,jj))*MO(jj)$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2))*DO_K(i,j,jj));
OO_K(i,ii,j)= OO1(i,ii,j)/PMO_K(i,j);
CO_K(i,j)= CO1(i,j)/PMO_K(i,j);
OOO_K(i,j)= SUM(ii,OO_K(ii,i,j));
CCO_K(j)= SUM(i,CO_K(i,j));
POO_K(i,j)= SUM(ii,OO1(ii,i,j))/OOO_K(i,j);
PCO_K(j)= SUM(i,CO1(i,j))/CCO_K(j);

DISPLAY
   PO_K,sampiK_K,DO_K,PMO_K,OO_K,CO_K,OOO_K,CCO_K,POO_K,PCO_K;

* Parameterization [C] ================================================

PARAMETERS
   alphaO_K(i,ii,j) Share of Commodity (Intermediate)
   alphaQ_K(i,j)    Share of Composite Factor
   alphaT_K(i,j,jj) Share of Commodity from Each Region
   phiO_K(i,j)      Unit Coefficient in Composite Intermediate Aggregator
   phiC_K(j)        Unit Coefficient in Composite Consumption Aggregator
   phiQ_K(i,j)      Unit Coefficient in Gross Output Aggregator
   phiT_K(i,j)      Unit Coefficient in Commodity Aggregator;
alphaO_K(i,ii,j)= PMO_K(i,j)*OO_K(i,ii,j)**(1/sigmaO(i))
   /SUM(iii,PMO_K(iii,j)*OO_K(iii,ii,j)**(1/sigmaO(iii)));
alphaQ_K(i,j)= YO(i,j)**(1/sigmaQ(i))
   /(YO(i,j)**(1/sigmaQ(i))+POO_K(i,j)*OOO_K(i,j)**(1/sigmaQ(i)));
alphaT_K(i,j,jj)= ((1+tauM(i,j,jj))*(1+tauT(i,j,jj))*PO_K(i,j,jj)/PMO_K(i,j)
   *((SUM(ii,OO_K(i,ii,j))+CO_K(i,j))/DO_K(i,j,jj))
   **(-1/sigmaT(i)))$(ORD(i) EQ 1)
   +((1+tauM(i,j,jj))*(1+tauT(i,j,jj))*PO_K(i,j,jj)
   *DO_K(i,j,jj)**(1/sigmaT(i))
   /SUM(jjj,(1+tauM(i,j,jjj))*(1+tauT(i,j,jjj))*PO_K(i,j,jjj)
   *DO_K(i,j,jjj)**(1/sigmaT(i))))$(ORD(i) EQ 2);
```

```
phiO_K(i,j)= OOO_K(i,j)
   /SUM(ii,alphaO_K(ii,i,j)*OO_K(ii,i,j)**((sigmaO(i)-1)/sigmaO(i)))
   **(sigmaO(i)/(sigmaO(i)-1));
phiC_K(j)= CCO_K(j)/PROD(i,CO_K(i,j)**alphaC(i,j));
phiQ_K(i,j)= QO(i,j)/((alphaQ_K(i,j)*YO(i,j)**((sigmaQ(i)-1)/sigmaQ(i))
   +(1-alphaQ_K(i,j))*OOO_K(i,j)**((sigmaQ(i)-1)/sigmaQ(i)))
   **(sigmaQ(i)/(sigmaQ(i)-1)));
phiT_K(i,j)= (SUM(ii,OO_K(i,ii,j))+CO_K(i,j))
   /SUM(jj,alphaT_K(i,j,jj)*(MO(jj)$(ORD(i) EQ 1)+1$(ORD(i) EQ 2))
   *DO_K(i,j,jj)**((sigmaT(i)-1)/sigmaT(i)))**(sigmaT(i)/(sigmaT(i)-1));

DISPLAY
   alphaO_K,alphaQ_K,alphaT_K,phiO_K,phiC_K,phiQ_K,phiT_K;

* Parameterization of Armington Type ==================================

* Derivation of Additional Data Set [E] ===============================

PARAMETERS
   PMO_A(i,j)       Composite Price of Commodity
   OO_A(i,ii,j)     Core Intermediate
   CO_A(i,j)        Core Consumption
   OOO_A(i,j)       Core Composite Intermediate
   CCO_A(j)         Core Composite Consumption
   POO_A(i,j)       Price of Composite Intermediate
   PCO_A(j)         Price of Composite Consumption;
PMO_A(i,j)= SUM(jj,TF04(i,j,jj))/SUM(jj,TF01(i,j,jj));
OO_A(i,ii,j)= OO1(i,ii,j)/PMO_A(i,j);
CO_A(i,j)= CO1(i,j)/PMO_A(i,j);
OOO_A(i,j)= SUM(ii,OO_A(ii,i,j));
CCO_A(j)= SUM(i,CO_A(i,j));
POO_A(i,j)= SUM(ii,OO1(ii,i,j))/OOO_A(i,j);
PCO_A(j)= SUM(i,CO1(i,j))/CCO_A(j);

DISPLAY
```

```
   PM0_A,O0_A,C0_A,OO0_A,CC0_A,PO0_A,PC0_A;

* Parameterization [D] ================================================

PARAMETERS
   alphaO_A(i,ii,j) Share of Commodity (Intermediate)
   alphaQ_A(i,j)    Share of Composite Factor
   alphaT_A(i,j,jj) Share of Commodity from Each Region
   phiO_A(i,j)      Unit Coefficient in Composite Intermediate Aggregator
   phiC_A(j)        Unit Coefficient in Composite Consumption Aggregator
   phiQ_A(i,j)      Unit Coefficient in Gross Output Aggregator
   phiT_A(i,j)      Unit Coefficient in Gross Output Aggregator;
alphaO_A(i,ii,j)= PM0_A(i,j)*O0_A(i,ii,j)**(1/sigmaO(i))
   /SUM(iii,PM0_A(iii,j)*O0_A(iii,ii,j)**(1/sigmaO(iii)));
alphaQ_A(i,j)= Y0(i,j)**(1/sigmaQ(i))
   /(Y0(i,j)**(1/sigmaQ(i))+PO0_A(i,j)*OO0_A(i,j)**(1/sigmaQ(i)));
alphaT_A(i,j,jj)=  (1+tauM(i,j,jj))*(1+tauT(i,j,jj))
   *TF01(i,j,jj)**(1/sigmaT(i))
   /SUM(jjj,(1+tauM(i,j,jjj))*(1+tauT(i,j,jjj))
   *TF01(i,j,jjj)**(1/sigmaT(i)));
phiO_A(i,j)= OO0_A(i,j)
   /SUM(ii,alphaO_A(ii,i,j)*O0_A(ii,i,j)**((sigmaO(i)-1)/sigmaO(i)))
   **(sigmaO(i)/(sigmaO(i)-1));
phiC_A(j)= CC0_A(j)/PROD(i,C0_A(i,j)**alphaC(i,j));
phiQ_A(i,j)= Q0(i,j)/((alphaQ_A(i,j)*Y0(i,j)**((sigmaQ(i)-1)/sigmaQ(i))
   +(1-alphaQ_A(i,j))*OO0_A(i,j)**((sigmaQ(i)-1)/sigmaQ(i)))
   **(sigmaQ(i)/(sigmaQ(i)-1)));
phiT_A(i,j)= (SUM(ii,O0_A(i,ii,j))+C0_A(i,j))
   /SUM(jj,alphaT_A(i,j,jj)*TF01(i,j,jj)**((sigmaT(i)-1)/sigmaT(i)))
   **(sigmaT(i)/(sigmaT(i)-1));

DISPLAY
   alphaO_A,alphaQ_A,alphaT_A,phiO_A,phiC_A,phiQ_A,phiT_A;

* Parameterization Complete ==========================================
```

```
PARAMETERS
   epsilon          Price Markup Rate
   sampiK(j)        Fixed Cost of Establishing a Firm in Region j
   alphaO(i,ii,j)   Share of Commodity (Intermediate)
   alphaQ(i,j)      Share of Composite Factor
   alphaT(i,j,jj)   Share of Commodity From Each Region
   phiO(i,j)        Unit Coefficient in Composite Intermediate Aggregator
   phiC(j)          Unit Coefficient in Composite Consumption Aggregator
   phiQ(i,j)        Unit Coefficient in Gross Output Aggregator
   phiT(i,j)        Unit Coefficient in Commodity Aggregator;
epsilon= -1/sigmaT("s01");
sampiK(j)= sampiK_M(j);
alphaO(i,ii,j)= alphaO_M(i,ii,j);
alphaQ(i,j)= alphaQ_M(i,j);
alphaT(i,j,jj)= alphaT_M(i,j,jj);
phiO(i,j)= phiO_M(i,j);
phiC(j)= phiC_M(j);
phiQ(i,j)= phiQ_M(i,j);
phiT(i,j)= phiT_M(i,j);

* Formulation of the Model ============================================

POSITIVE VARIABLES
   Q(i,j)           Gross Output
   Y(i,j)           Value Added
   OO(i,j)          Composite Intermediate
   CC(j)            Composite Consumption
   K(i,j)           Capital Input
   L(i,j)           Labor Input
   O(i,ii,j)        Intermediate
   C(i,j)           Consumption
   D(i,j,jj)        Trade Flow (Incl. Domestic Product)
   PQ(i,j)          Price of Gross Output
   PY(i,j)          Price of Value Added
```

```
    PO(i,j)          Price of Composite Intermediate
    PC(j)            Price of Composite Consumption
    PW(i,j)          Producer Price
    P(i,j,jj)        Markup Price (Excl. Transportation Cost and Tariff)
    PM(i,j)          Composite Price of Commodity
    WK(j)            Rental Price of Capital
    WL(j)            Rental Price of Labor
    M(j)             Number of Registered Firms
    XI(j,jj)         Proportion of Inactive Firms
    PSI(j,jj)        Average Productivity of Active Firms
    LAMBDA(j)        Marginal Utility of Income;

EQUATIONS
    eqPQ(i,j)        Gross Output Aggregator
    eqPY(i,j)        Value Added Aggregator
    eqPO(i,j)        Composite Intermediate Aggregator
    eqPC(j)          Composite Consumption Aggregator
    eqPW(i,j)        Transformation of Gross Output
    eqP(i,j,jj)      Price Markup Rule
    eqPM(i,j)        Commodity Aggregator
    eqWK(j)          Capital Market Equilibrium
    eqWL(j)          Labor Market Equilibrium
    eqQ(i,j)         Dual Relation
    eqY(i,j)         Dual Relation
    eqOO(i,j)        Dual Relation
    eqCC(j)          Dual Relation
    eqK(i,j)         Dual Relation
    eqL(i,j)         Dual Relation
    eqO(i,ii,j)      Dual Relation
    eqC(i,j)         Dual Relation
    eqD(i,j,jj)      Dual Relation
    eqM(j)           Number of Registered Firms
    eqXI(j,jj)       Proportion of Inactive Firms (Melitz)
    eqPSI(j,jj)      Average Productivity of Active Firms (Melitz)
    eqLAMBDA(j)      Budget Constraint;
```

```
eqPQ(i,j)..
   phiQ(i,j)*((alphaQ(i,j)*Y(i,j)**((sigmaQ(i)-1)/sigmaQ(i))
   +(1-alphaQ(i,j))*OO(i,j)**((sigmaQ(i)-1)/sigmaQ(i)))
   **(sigmaQ(i)/(sigmaQ(i)-1)))-Q(i,j) =G= 0;
eqPY(i,j)..
   phiY(i,j)*K(i,j)**alphaY(i,j)*L(i,j)**(1-alphaY(i,j))-Y(i,j) =G= 0;
eqPO(i,j)..
   phiO(i,j)*SUM(ii,alphaO(ii,i,j)*O(ii,i,j)
   **((sigmaO(i)-1)/sigmaO(i)))**(sigmaO(i)/(sigmaO(i)-1))-OO(i,j) =G= 0;
eqPC(j)..
   phiC(j)*PROD(i,C(i,j)**alphaC(i,j))-CC(j) =G= 0;
eqPW(i,j)..
   Q(i,j)
   *(1-(SUM(jj,(1-XI(jj,j))*sampiM(jj,j))+sampiK(j))*M(j)$(ORD(i) EQ 1))
   -qoppa(j)/PW(i,j)*SUM((ii,jj,jjj),tauT(ii,jj,jjj)*PW(ii,jjj)
   *((1+epsilon)**(-1)*(1-XI(jj,jjj))*M(jjj)$(ORD(ii) EQ 1)/PSI(jj,jjj)
   +1$(ORD(ii) EQ 2))*D(ii,jj,jjj))$(ORD(i) EQ 2)
   -SUM(jj,(((1-XI(jj,j))*M(j)/PSI(jj,j))$(ORD(i) EQ 1)+1$(ORD(i) EQ 2))
   *D(I,jj,j))
   =G= 0;
eqP(i,j,jj)..
   (((1+epsilon)**(-1)/PSI(j,jj))$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2))*PW(i,jj)-P(i,j,jj) =G= 0;
eqPM(i,j)..
   phiT(i,j)
   *SUM(jj,alphaT(i,j,jj)*((1-XI(j,jj))*M(jj)$(ORD(i) EQ 1)
   +1$(ORD(i) EQ 2))*D(i,j,jj)**((sigmaT(i)-1)/sigmaT(i)))
   **(sigmaT(i)/(sigmaT(i)-1))-SUM(ii,O(i,ii,j))-C(i,j) =G= 0;
eqWK(j)..
   KE(j)-SUM(i,K(i,j)) =G= 0;
eqWL(j)..
   LE(j)-SUM(i,L(i,j)) =G= 0;
eqQ(i,j)..
   PQ(i,j)-PW(i,j) =G= 0;
```

```
eqY(i,j)..
   PY(i,j)-alphaQ(i,j)*PQ(i,j)*phiQ(i,j)**((sigmaQ(i)-1)/sigmaQ(i))
   *(Q(i,j)/Y(i,j))**(1/sigmaQ(i)) =G= 0;
eqOO(i,j)..
   PO(i,j)-(1-alphaQ(i,j))*PQ(i,j)*phiQ(i,j)**((sigmaQ(i)-1)/sigmaQ(i))
   *(Q(i,j)/OO(i,j))**(1/sigmaQ(i)) =G= 0;
eqCC(j)..
   LAMBDA(j)*PC(j)-1 =G= 0;
eqK(i,j)..
   WK(j)-alphaY(i,j)*PY(i,j)*Y(i,j)/K(i,j) =G= 0;
eqL(i,j)..
   WL(j)-(1-alphaY(i,j))*PY(i,j)*Y(i,j)/L(i,j) =G= 0;
eqO(i,ii,j)..
   PM(i,j)-alphaO(i,ii,j)*PO(ii,j)*phiO(ii,j)**((sigmaO(ii)-1)/sigmaO(ii))
   *(OO(ii,j)/O(i,ii,j))**(1/sigmaQ(ii)) =G= 0;
eqC(i,j)..
   PM(i,j)-alphaC(i,j)*PC(j)*CC(j)/C(i,j) =G= 0;
eqD(i,j,jj)..
   (1+tauM(i,j,jj))*(1+tauT(i,j,jj))*P(i,j,jj)
   -alphaT(i,j,jj)*PM(i,j)*phiT(i,j)**((sigmaT(i)-1)/sigmaT(i))
   *((SUM(ii,O(i,ii,j))+C(i,j))/D(i,j,jj))**(1/sigmaT(i)) =G= 0;
eqM(j)..
   (SUM(jj,(1-XI(jj,j))*sampiM(jj,j))+sampiK(j))*PW("s01",j)*Q("s01",j)
   =E= -epsilon*SUM(jj,(1-XI(jj,j))*P("s01",jj,j)*D("s01",jj,j));
eqXI(j,jj)..
   XI(j,jj) =E=
   1-(zeta/(zeta-sigmaT("s01")+1))**(zeta/(sigmaT("s01")-1))
   *PSI(j,jj)**(-zeta);
eqPSI(j,jj)..
   PSI(j,jj) =E= (zeta/(zeta-sigmaT("s01")+1))**(1/(sigmaT("s01")-1))
   *(-epsilon)**(1/(1-sigmaT("s01")))*(1+epsilon)**(-1)
   *(PW("s01",jj)/P("s01",j,jj))**(sigmaT("s01")/(sigmaT("s01")-1))
   *(sampiM(j,jj)*Q("s01",jj)/D("s01",j,jj))**(1/(sigmaT("s01")-1));
eqLAMBDA(j)..
   WK(j)*SUM(i,K(i,j))+WL(j)*SUM(i,L(i,j))
```

```
   +SUM((i,jj),tauM(i,j,jj)*(1+tauT(i,j,jj))*PW(i,jj)
   *((1+epsilon)**(-1)*(1-XI(j,jj))*M(jj)$(ORD(i) EQ 1)/PSI(j,jj)
   +1$(ORD(i) EQ 2))*D(i,j,jj))-PC(j)*CC(j) =G= 0;

* Initialization of Variables ========================================

Q.LO(i,j)= 1e-10; Y.LO(i,j)= 1e-10; OO.LO(i,j)= 1e-10;
CC.LO(j)= 1e-10; K.LO(i,j)= 1e-10; L.LO(i,j)= 1e-10;
O.LO(i,ii,j)= 1e-10; C.LO(i,j)= 1e-10; D.LO(i,j,jj)= 1e-10;
PQ.LO(i,j)= 1e-10; PY.LO(i,j)= 1e-10; PO.LO(i,j)= 1e-10;
PC.LO(j)= 1e-10;  PW.LO(i,j)= 1e-10; P.LO(i,j,jj)= 1e-10;
PM.LO(i,j)= 1e-10; WK.LO(j)= 1e-10; WL.LO(j)= 1e-10;
M.LO(j)= 1e-10; XI.LO(j,jj)= 1e-10; PSI.LO(j,jj)= 1e-10;
LAMBDA.LO(j)= 1e-10;

PW.FX("s01","r01")= 1;

* Model Definition ===================================================

MODEL ARMINGTON
  /eqPQ.PQ,eqPY.PY,eqPO.PO,eqPC.PC,eqPW.PW,eqP.P,eqPM.PM,
   eqWK.WK,eqWL.WL,eqQ.Q,eqY.Y,eqOO.OO,eqCC.CC,eqK.K,eqL.L,
   eqO.O,eqC.C,eqD.D,eqLAMBDA.LAMBDA/;

MODEL KRUGMAN
  /ARMINGTON,eqM.M/;

MODEL MELITZ
  /KRUGMAN,eqXI.XI,eqPSI.PSI/;

Q.L(i,j)= Q0(i,j);
Y.L(i,j)= Y0(i,j);
OO.L(i,j)= OOO_M(i,j);
CC.L(j)= CC0_M(j);
K.L(i,j)= V01(i,j);
```

```
L.L(i,j)= L01(i,j);
O.L(i,ii,j)= OO_M(i,ii,j);
C.L(i,j)= CO_M(i,j);
D.L(i,j,jj)= DO_M(i,j,jj);
PQ.L(i,j)= 1;
PY.L(i,j)= 1;
PO.L(i,j)= POO_M(i,j);
PC.L(j)= PCO_M(j);
PW.L(i,j)= 1;
P.L(i,j,jj)= PO_M(i,j,jj);
PM.L(i,j)= PMO_M(i,j);
WK.L(j)= 1;
WL.L(j)= 1;
M.L(j)= MO(j);
XI.L(j,jj)= XIO(j,jj);
PSI.L(j,jj)= PSIO(j,jj);
LAMBDA.L(j)= 1/PCO_M(j);

SOLVE MELITZ USING MCP;

PARAMETERS
   GDP_M(j)         Gross Domestic Product (Melitz)
   HEV_M(j)         Hicksian Equivalent Variations (Melitz);
GDP_M(j)= SUM(i,PY.L(i,j)*Y.L(i,j));
HEV_M(j)= PCO_M(j)*(CC.L(j)-CCO_M(j));

PARAMETERS
   epsilon          Price Markup Rate
   sampiM(j,jj)     Fixed Cost of Operation on the j-jj Link
   sampiK(j)        Fixed Cost of Establishing a Firm in Region j
   alphaO(i,ii,j)   Share of Commodity (Intermediate)
   alphaQ(i,j)      Share of Composite Factor
   alphaT(i,j,jj)   Share of Commodity from Each Region
   phiO(i,j)        Unit Coefficeint in Composite Intermediate Aggregator
   phiC(j)          Unit Coefficeint in Composite Consumption Aggregator
```

```
   phiQ(i,j)        Unit Coefficient in Gross Output Aggregator
   phiT(i,j)        Unit Coefficient in Commodity Aggregator;
epsilon= -1/sigmaT("s01");
sampiM(j,jj)= 0;
sampiK(j)= sampiK_K(j);
alphaO(i,ii,j)= alphaO_K(i,ii,j);
alphaQ(i,j)= alphaQ_K(i,j);
alphaT(i,j,jj)= alphaT_K(i,j,jj);
phiO(i,j)= phiO_K(i,j);
phiC(j)= phiC_K(j);
phiQ(i,j)= phiQ_K(i,j);
phiT(i,j)= phiT_K(i,j);

XI.FX(j,jj)= 0;
PSI.FX(j,jj)= 1;

Q.L(i,j)= Q0(i,j);
Y.L(i,j)= Y0(i,j);
OO.L(i,j)= OO0_K(i,j);
CC.L(j)= CC0_K(j);
K.L(i,j)= V01(i,j);
L.L(i,j)= L01(i,j);
O.L(i,ii,j)= O0_K(i,ii,j);
C.L(i,j)= C0_K(i,j);
D.L(i,j,jj)= D0_K(i,j,jj);
PQ.L(i,j)= 1;
PY.L(i,j)= 1;
PO.L(i,j)= PO0_K(i,j);
PC.L(j)= PC0_K(j);
PW.L(i,j)= 1;
P.L(i,j,jj)= P0_K(i,j,jj);
PM.L(i,j)= PM0_K(i,j);
WK.L(j)= 1;
WL.L(j)= 1;
M.L(j)= M0(j);
```

```
LAMBDA.L(j)= 1/PCO_K(j);

SOLVE KRUGMAN USING MCP;

PARAMETERS
   GDP_K(j)          Gross Domestic Product (Krugman)
   HEV_K(j)          Hicksian Equivalent Variations (Krugman);
GDP_K(j)= SUM(i,PY.L(i,j)*Y.L(i,j));
HEV_K(j)= PCO_K(j)*(CC.L(j)-CCO_K(j));

PARAMETERS
   epsilon           Price Markup Rate
   sampiM(j,jj)      Fixed Cost of Operation on the j-jj Link
   sampiK(j)         Fixed Cost of Establishing a Firm in Region j
   alphaO(i,ii,j)    Share of Commodity (Intermediate)
   alphaQ(i,j)       Share of Composite Factor
   alphaT(i,j,jj)    Share of Commodity from Each Region
   phiO(i,j)         Unit Coefficeint in Composite Intermediate Aggregator
   phiC(j)           Unit Coefficeint in Composite Consumption Aggregator
   phiQ(i,j)         Unit Coefficient in Gross Output Aggregator
   phiT(i,j)         Unit Coefficient in Commodity Aggregator;
epsilon= 0;
sampiM(j,jj)= 0;
sampiK(j)= 0;
alphaO(i,ii,j)= alphaO_A(i,ii,j);
alphaQ(i,j)= alphaQ_A(i,j);
alphaT(i,j,jj)= alphaT_A(i,j,jj);
phiO(i,j)= phiO_A(i,j);
phiC(j)= phiC_A(j);
phiQ(i,j)= phiQ_A(i,j);
phiT(i,j)= phiT_A(i,j);

M.FX(j)= 1;
XI.FX(j,jj)= 0;
PSI.FX(j,jj)= 1;
```

```
Q.L(i,j)= Q0(i,j);
Y.L(i,j)= Y0(i,j);
OO.L(i,j)= OO0_A(i,j);
CC.L(j)= CC0_A(j);
K.L(i,j)= V01(i,j);
L.L(i,j)= L01(i,j);
O.L(i,ii,j)= O0_A(i,ii,j);
C.L(i,j)= C0_A(i,j);
D.L(i,j,jj)= TF01(i,j,jj);
PQ.L(i,j)= 1;
PY.L(i,j)= 1;
PO.L(i,j)= PO0_A(i,j);
PC.L(j)= PC0_A(j);
PW.L(i,j)= 1;
P.L(i,j,jj)= 1;
PM.L(i,j)= PM0_A(i,j);
WK.L(j)= 1;
WL.L(j)= 1;
LAMBDA.L(j)= 1/PC0_A(j);

SOLVE ARMINGTON USING MCP;

PARAMETERS
   GDP_A(j)         Gross Domestic Product (Armington)
   HEV_A(j)         Hicksian Equivalent Variations (Armington);
GDP_A(j)= SUM(i,PY.L(i,j)*Y.L(i,j));
HEV_A(j)= PC0_A(j)*(CC.L(j)-CC0_A(j));

* Indicators ============================================================

DISPLAY
   GDP_M,GDP_K,GDP_A,HEV_M,HEV_K,HEV_A;

* Fin ===================================================================
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