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Search, Matching, and Self-Organization of a Marketplace

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Abstract

In many developing countries, clusters of small shops are the typical market-place. We investigate an economic model in which, between buyers and sellers in a marketplace, a circular causality including the search process produces agglomeration forces, given the initial location of the marketplace location exogenously in a linear city. We conclude that initial number of buyers and sellers is important in forming a large marketplace.

Keywords: circular causality; market place

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1 Introduction

The marketplace, known in a traditional form as the market square or bazaar, is the oldest trading system in human history. These kinds of marketplaces now play a diminished role in developed countries following the emergence of big business. Yet even today in many developing countries, the main actors in trade remain numerous small buyers and sellers. Marketplaces support transactions between these small traders, and although marketplaces might appear simple, they nonetheless play a key role in trade within and between developing countries (Bellandi and Lombardi 2012; Ding 2012; Geertz, Geertz, and Rosen 1979; Ito 2011; Iwasaki 2012). The aim of this paper is to re-examine the nature of marketplace, for the first time from the perspective of the new economic geography. Here we focus not on the transaction space of the marketplace itself, but on the trading system behind the physical space.

As the world's largest marketplace for daily necessities, Yiwu China Commodity City (Yiwu Market) symbolizes the vitality of marketplaces. Yiwu Market is located within Yiwu, in the middle of Zhejiang Province, China. Thirty years ago, this marketplace was a street market with a mere 700 booths. By 2012, however, Yiwu Market had grown into a huge marketplace with a total floor area of 4.7 million square meters and 70,000 booths. If one were to stay at each booth for three minutes and devote eight hours per day to purchasing, it would take more than a year to visit the entire market.

For hundreds of years, Yiwu has historically been host to long-distance traders. In 1982, when the Yiwu government established a regular market, these traders became the major buyers and sellers in this marketplace.

Since 1982 the transaction volume of Yiwu Market has increased explosively, by a factor of more than 1300. At first, commodities traded in the marketplace were all purchased from factories in Zhejiang Province and Guangdong Province, where rural industrialization advanced earliest in China; and all the commodities were then sold in the domestic Chinese market. Now, however, 10% of commodities sold at Yiwu Market are purchased directly from foreign merchants and 65% of the commodities are exported.

At first most traders were from Yiwu, but over time, traders from other regions came to constitute the majority. Moreover, the change in the number of booths—705 in 1982, 16,000 in 1992, 42,000 in 2002, and 70,000 in 2011—shows that the number of sellers is continuing to rise. Data on the number of buyers are incomplete, but data on the daily number of visitors are available for four years: 10,000 in 1990, 110,000 in 1998, 160,000 in 2002, and 214,000 in 2007. Thus we can see that the number of buyers increased along with the number of sellers. The number of foreign buyers has continuously increased. In 2007, a total of 260,000 foreign buyers visited Yiwu. Many of them reside permanently in Yiwu and have established offices in the city. From 2007 to 2011, the number of foreign resident offices in Yiwu increased from 1,340 to 3,080.

With increased numbers of buyers and sellers having come to Yiwu, many marketplaces dealing in the same types of commodities have shrunk or even disappeared. This trend first appeared in areas surrounding Yiwu. For example, the Qiaotou Market in Wenzhou, Zhejiang Province, was the largest button market in Asia in the late 1980s and early 1990s. In the mid-2000s, however, all 400 local button factories operated booths at Yiwu Market, and the numbers of booths at Qiaotou Market decreased from 4,000 to a mere 500. A similar situation has also occurred in Taizhou, Zhejiang Province. Luqiao China Daily-Necessities City in Taizhou was China's largest plastic goods market in the 1990s. With the growth of Yiwu Market, however, transaction volume of the market in Taizhou decreased from 11.6 billion yuan (2001) to 9.05 billion yuan (2004). Meanwhile, 300 Taizhou firms have made Yiwu Market their major sales channel, where half of the plastic products produced in Taizhou are sold.

The emergence of Yiwu Market also had a significant impact on marketplaces in other developing countries. Ito (2011) reported a typical case where a Kenyan buyer, who had previously made purchases in marketplaces in Dubai, began going directly to Yiwu Market for purchasing. Through an analysis of data on foreign resident offices , Ding (2012: Chapter 6) pointed out that the daily necessities traders in Dubai and Hong Kong (most of whom operate booths in marketplaces) have begun to shift to Yiwu. Iwasaki (2012) reported an interesting phenomenon where many apparel businesses in Iran—who previously sold garments through Bazar-e Bozorg in Teheran—have given up production, and now go to Yiwu (and Guangzhou) to purchase garments directly.

A key factor that attracts buyers and sellers to agglomerate in Yiwu is its great variety of commodities. At Yiwu Market, commodities are rigorously classified by industry and location, thus making search costs in the market comparatively low and facilitating a greater variety of commodities. In 1998, the market was classified into 16 zones, where 30,000 types of commodities in 28 industries were traded. Thereafter, the number of commodity types continued to increase to more than 100,000 in 2002, 320,000 in 2004, and 1,700,000 in 2011.

The purpose of our model is to clarify the mechanism by which buyers and sellers attract each other. Circular causality between consumers and firms or between intermediate goods suppliers and their customers is employed in New Economic Geography, (e.g., Fujita, Krugman and Venables 1999) in multi-regional general equilibrium models to examine the emergence of the core-periphery structure. However, circular causality including the search process has not been examined. To make this complex problem tractable, we focus on marketplace formation in the spatial economy of a linear spatial economy with exogenously determined locations of marketplaces.

The formation of marketplaces without circular causality is well understood. By solving social planners' optimization problem, Wang (1990) has shown that a unique equilibrium marketplace that maximizes social welfare is lacated near the residences of buyers who prefer spatial factors. Wolinsky (1983) developed a monopolistic competition model of buyer searches, under the assumption that the marketplace and place of residence are separated; this model admitted a clustered market area and allowed for examination of whether a shop emerges outside the marketplace. Fischer and Harrington (1996) extended the search model of Wolinsky (1983) and derived the circumstances whereby some firms cluster but where many more are located away from the cluster as a result of the entry and exit of firms. Anderson and Renault (1999) derived the required model setup for the existence of equilibrium under monopolistic competition and other cases, rigorously examining Wollinsky (1986) from the view point of industrial organization. Konishi and Sandfort (2003) examined externalities produced by an anchor store with an established brand name. In a two-dimensional geographic space, Konishi (2005) used numerical analysis to examine the relationship between the number of stores at a shopping center, equilibrium price, market size, the probability of a buyer finding a purchasable commodity, and store profit.

The circular causality of our model emerges by the interaction between buyers and sellers. Many sellers in a marketplace attract buyers who live far from the marketplace and who expect good matching in marketplaces. Meanwhile, many buyers in a marketplace increase the capital returns of sellers through increased demand for products. However, sellers may hesitate to enter the marketplace because of the competition among them. Buyers may hesitate to make the trip to the marketplace.

The remainder of this paper is organized as follows: Section 2 constructs our model; Section 3 establishes and characterizes the spatial equilibrium; and Section 4 concludes the paper.

2 The Model

The economy is an ad hoc marketplace where goods are bought and sold. We restrict ourselves to transactions conducted in only a single marketplace in a linear city. There is a population L of consumers who reside along line $[0, L]$. The marketplace is located at the origin of the line (0). The distance between the marketplace and buyer $l \in [0, 1]$ is measured by lL.[1] We suppose that absentee capital owners posess, in total, N units of capital. We denote by $\lambda \in [0, 1]$ the share of capital employed by sellers in the marketplace. To produce a product, one unit of capital is used and the marginal cost is zero, which implies that sellers utilize increasing returns to scale technology. We assume that sellers can differentiate their products at no cost and also assume that there is no economy of scope; there is thus a one-to-one correspondence between firms and varieties. Hence, there is a continuum $[0, \lambda N]$ of horizontally differentiated products in the marketplace. Seller i offers one variety at price p_i.

The indirect utility of buyer l purchasing one unit of variety i at price p_i is given by:[2]

$$u_{li}(p_i) = y - p_i + \mu\epsilon_{li}$$

where $y - p_i$ is the amount of numéraire consumed, μ is a parameter expressing the heterogeneity of buyers' taste and $\epsilon_{li} \in [0, \lambda N]$ [3] is the expectation of a random variable that is identically and independently distributed across buyers and sellers, with a common density function f and the corresponding distribution function F. Hence, $\mu\epsilon_{li}$ is the consumer l's match value with variety i. As μ becomes larger, the impact of search on buyers becomes greater. In other words, buyers perceive varieties in the marketplace to be more heterogeneous. We assume that f is a continuous uniform distribution over convex sets:

$$f(x) = \frac{1}{\lambda N}, \qquad x \in [0, \lambda N]$$

where $x \equiv \epsilon_{lj} + (p^* - p_j)/\mu$, and p^* is the equilibrium price. According to Andersen and Renoult (1999), there exists an equilibrium under monopolistic competition if the density function f is log concave.[4] The continuous uniform distribution over convex sets is also

[1] Each buyer is bound to its place of residence, which means that the buyer cannot relocate.

[2] We assume that each buyer purchases only one variety and consumes one unit of the product. In reality, the major buyers in Yiwu Market are small merchants. For this reason, we believe that our definition of buyers is reasonable in the case of the buyers at Yiwu Market.

[3] In other words, we suppose that a greater number of varieties in the marketplace improves the upper value of matching.

[4] Log concave implies that $\log f(\alpha x + (1-\alpha)y) \geq \alpha \log f(x) + (1-\alpha)\log f(y), \alpha \in (0, 1)$ is satisfied.

log concave. Thus, an equilibrium may exist in this model.

Buyers incur a search (or sampling) cost c when checking a seller's product and price. The net utility of buyer l is given by $u_{li}(p_i) - kc$ if the buyer purchases product i at price p_i after visiting k sellers. In equilibrium, all sellers charge the same price p^*. Furthermore, the expected indirect utility of the buyer l who purchases a product in the marketplace is:

$$U(\lambda, l) = -p^* + \mu\epsilon^E - kc - tlL. \tag{1}$$

where ϵ^E is the expected match value. Otherwise buyer l receives positive utility, $V > 0$, by not traveling to the marketplace. We use θ to express a buyer who is indifferent between traveling to the marketplace and staying put, that is,

$$U(\lambda, l)\Big|_{l=\theta} = V$$

Hence, buyers $l \leq \theta$ will travel to the marketplace.

Assuming free entry and exit, and setting the zero profit condition yields the capital return:

$$r(\lambda, \theta) = p^* D(p^*, p^*) \tag{2}$$

where $D(p^*, p^*)$ represents the demand for a variety in equilibrium; the demand in equilibrium will be formulated later. We assume that the prevailing returns to capital outside the marketplace are $\bar{r}$. Accordingly, the owners of capital are indifferent between being employed by sellers inside and outside the marketplace if the following equation holds:

$$r(\lambda, \theta) = \bar{r}.$$

Following a well-established convintion in migration modeling, we focus on an adjustment process whereby the marketplace attracts (repuls) capital providing higher (lower) return to capital and higher (lower) utility for buyers:

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} r(\lambda, \theta) - \bar{r} \\ U(\lambda, \theta) - V \end{pmatrix} \tag{3}$$

We assume that the search process in the marketplace instantaneous, but that decisions on whether to utilize capital in the marketplace and whether to buy at the marketplace take longer.

2.1 Reservation value

Suppose that a buyer receives a best offer that provides utility $u_{lj}(p_j)$. If the buyer samples seller i and expects price p^* from that seller, the buyer will prefer to buy the product if $-p^*+\mu\epsilon_{li} > -p_j+\mu\epsilon_{lj}$ holds, which is equivalent to $x \equiv \epsilon_{lj}+(p^*-p_j)/\mu > \epsilon_{li}$. Hence, the utility gained by sampling an additional seller is given by $-p^*+\mu\epsilon_{li}+p_j-\mu\epsilon_{lj} = \mu(\epsilon_{li}-x)$. Furthermore, the expected marginal utility of searching an additional seller is expressed as $\mu g(x)$ and the value of x when the buyer stops searching, $\widehat{x}$, is given by

$$c/\mu = g(\widehat{x}), \qquad g(x) \equiv \int_x^{\lambda N} (\epsilon - x) f(\epsilon) d\epsilon$$

Here $\mu g(x)$ is the expected value of finding a better match than x. The expected marginal utility from an additional search exceeds the search cost if $x < \widehat{x}$, and vice versa. Solving $c/\mu = g(\widehat{x})$ yields

$$\widehat{x}(\lambda) = \lambda N - \sqrt{2\lambda N \frac{c}{\mu}}. \tag{4}$$

If μ goes to infinity or c goes to 0, we obtain $\widehat{x} = \lambda N$, which is the maximum value of ϵ_{li}. In other words, buyers never stop searching when the varieties are highly differentiated or the search cost is trivial. Furthermore we obtain:

$$\frac{\lambda N}{2} \gtreqless \frac{c}{\mu} \Leftrightarrow \widehat{x}(\lambda) \gtreqless 0. \tag{5}$$

If $\widehat{x} \leq 0$, buyers always purchase the first product found because the varieties are fairly standardized or the search cost is extremely high. To avoid the extreme case where buyers always purchase a product at the first seller visited, we assume that the following condition is satisfied:

$$\frac{\lambda N}{2} > \frac{c}{\mu}. \tag{6}$$

This implies $\widehat{x}(\lambda) > 0$. Otherwise, we will show that the indirect utility is negative when buyers visit the marketplace, which implies that no buyer travels to the market. Furthermore, from (4), we have $0 < \frac{\partial \widehat{x}(\lambda)}{\partial \lambda} < N$, which implies that the reservation value increases as the number of sellers increases, owing to the increased maximum match value. Furthermore, $\frac{\partial \lambda N - \partial \widehat{x}(\lambda)}{\partial \lambda} > 0$ implies that the gap $\lambda N - \hat{x}(\lambda)$ increases as λ increases. Hence, the more sellers present in the marketplace, the earlier buyers will stop their search.

2.2 Prices

Suppose that, for a given number of sellers and buyers, all firms set price p^* except for firm i, given the number of firms and consumers. It is optimal for buyers, when sampling seller i, to use the search rule described in the previous subsection, as in the model of Anderson and Renault (1999).

After seller i is sampled, the probability that a buyer stays with seller i without performing any further search is:

$$\Pr[x > \widehat{x}(\lambda)] = 1 - F[\widehat{x}(\lambda) + \Delta],$$

where $\Delta \equiv (p_i(\lambda) - p^*(\lambda))/\mu$ is the standardized price premium of seller i. To determine the probability that seller i is sampled, we focus on the distribution function of another seller sampled before seller i but where no purchase was made, that is, $F[\widehat{x}(\lambda)]$. Seller i is sampled first with $1/(\lambda N)$, second with probability $F[\widehat{x}(\lambda)]/(\lambda N)$, third with probability $F(\widehat{x})^2/(\lambda N)$, and so on. Summing these probabilities, we obtain the total probability $\frac{1}{\lambda N}\frac{1-F[\widehat{x}(\lambda)]^{\lambda N}}{1-F[\widehat{x}(\lambda)]}$. Since λN is sufficiently large, we can rewrite the above probability as $\frac{1}{\lambda N}\frac{1}{1-F[\widehat{x}(\lambda)]}$. Therefore, the probability that seller i is sampled is $\frac{1}{\lambda N}\frac{1}{1-F[\widehat{x}(\lambda)]}$, and that the seller's offer is accepted, $1 - F[\widehat{x}(\lambda) + \Delta]$, which results in $\frac{1}{\lambda N}\left[1 - F[\widehat{x}(\lambda) + \Delta]\right]\frac{1}{1-F[\widehat{x}(\lambda)]}$. Since no buyer can sample all sellers, the number of buyers or the demand for seller i is

$$D(p_i(\lambda), p^*(\lambda)) = \frac{\hat{\theta} L}{\lambda N}\left\{1 - F[\widehat{x}(\lambda) + \Delta]\right\}\frac{1}{1 - F[\widehat{x}(\lambda)]}.$$

From (4), we obtain the probability that a buyer does not purchase a particular product:

$$F[\widehat{x}(\lambda)] = 1 - \sqrt{\frac{c/\mu}{\lambda N/2}}. \tag{7}$$

The derivative of demand for seller i with respect to p_i, evaluated at $p_i = p^*$, is obtained:[5]

$$\frac{\partial D(p^*, p^*)}{\partial p_i} = -\frac{\theta L}{\mu}\sqrt{\frac{\lambda N}{2c/\mu}} < 0. \tag{8}$$

Since all sellers charge the same price in equilibrium, demand for each seller is given by:

$$D(p^*, p^*) = \frac{\theta L}{\lambda N}. \tag{9}$$

The equilibrium demand $D(p^*, p^*)$ increases with the number of buyers and decreases with the number of sellers. Accordingly, we can derive $p^*(\lambda) = \frac{\mu\{1-F[\widehat{x}(\lambda)]\}}{f[\widehat{x}(\lambda)]}$ under monopolistic competition as in the appendix of Anderson and Renault (1999). Thus, the symmetric equilibrium price is[6]

[5] It is readily verified that $\frac{\partial[1-F(\widehat{x}+\Delta)]}{\partial p} = -\frac{f(\widehat{x})}{\mu}$. Hence, we can arrive at (8).

[6] $MR = D + p\frac{\partial D}{\partial p} = 0 \Leftrightarrow p_i = \frac{-D}{\partial D/\partial p_i}$

$p^* = \frac{\mu[1-F(\widehat{x})]}{f(\widehat{x})} = \frac{\mu[1-(1-\sqrt{2(\frac{c}{\mu})/\lambda N})]}{\frac{1}{\lambda N}} = \sqrt{2c\mu\lambda N}$

$$p^*(\lambda) = \sqrt{2c\mu\lambda N}. \tag{10}$$

The equilibrium price increases with the search cost, c, as in Proposition 1 of Anderson and Renault (1999), and also increases with product differentiation, μ, as in Proposition 2 of Anderson and Renault (1999). The equilibrium price (10) also increases with the number of sellers, λN.[7] According to the results of Anderson and Renault (1999), however, price is lower with an increasing number of sellers is increasing in comparison with when the number of sellers is finite or infinite. We consider the marginal increase among only sellers that are sufficiently large. As Fischer and Harrington (1996) point out, the difference between the upper matching value and the lower matching value corresponds to the degree of product heterogeneity. Supposing the number of sellers is infinite, we suppose $\epsilon_{li} \in [0, \lambda N]$. Thus, we obtain the above result. In our set-up, a higher matching value means greater product heterogeneity. In (10), buyers perceive that product heterogeneity increases with λ.

2.3 Number of visits

A buyer continues searching until finding a product such that $x > \widehat{x}(\lambda)$. Otherwise, the buyer will continue to search. Hence, the probability that a buyer stops searching on the first visit is $1 - F[\widehat{x}(\lambda)]$; second visit, $F[\widehat{x}(\lambda)]\{1 - F[\widehat{x}(\lambda)]\}$; third visit, $F[\widehat{x}(\lambda)]^2\{1 - F[\widehat{x}(\lambda)]\}$; and so on. We obtain the expected number of visits as

$$k^e = \{1 - F[\widehat{x}(\lambda)]\} \sum_{i=1}^{n} iF[\widehat{x}(\lambda)]^{i-1}.$$

Because the number of sellers is sufficiently large, we use (7) to obtain[8]

$$k^e(\lambda) = \frac{1}{1 - F[\widehat{x}(\lambda)]} = \sqrt{\frac{\lambda N/2}{c/\mu}}. \tag{11}$$

We find that the buyer samples more products if the variety is greater, if the cost of an additional search is lower, and if the products are perceived to be more heterogeneous.

[7] The segmentation of the marketplace by product reduces prices such that $p^* = \sqrt{2c\mu\lambda N/s}$, where s is the number of segments.

[8] It is straightforward, as follows:

$$\sum_{M=0}^{\infty} MF(\widehat{x})^{M-1} = \lim_{M\to\infty} \frac{MF(\widehat{x})^{M+1} - MF(\widehat{x})^M - F(\widehat{x})^M + 1}{(F(\widehat{x}) - 1)^2} = \frac{1}{[1 - F(\widehat{x})]^2}, \quad |F(\widehat{x})| < 1.$$

Under (6), we have $k^e(\lambda) > 1$. Furthermore, using (7), the expected number of sellers visited is the inverse of the probability that the buyer purchases a product and stops searching. For example, if the probability is 1/3, a buyer's expected number of sellers visited is 3.

Furthermore, using (4) and (11), we obtain:

$$p^*(\lambda) = \mu[\lambda N - \hat{x}(\lambda)] = 2ck^e(\lambda).$$

In other words, the price increases when a larger number of varieties can be purchased, $\lambda N - \hat{x}$ and with greater sampling, $k^e(\lambda)$.

Proposition 1 *A larger number of varieties that can be purchased is equivalent to broader sampling and increased product prices.*

3 Instantaneous equilibrium

Using (2), (9) and (10), the capital return is given by:

$$r(\lambda, \theta) = p^*(\lambda) D\left[p^*(\lambda), p^*(\lambda)\right] = \theta L \sqrt{\frac{2c\mu}{\lambda N}}. \tag{12}$$

When the number of sellers in a marketplace increases, the effect of decreasing demand dominates the effect of increasing price. That is, the capital return decreases as the number of sellers increases. However, capital returns increases because of higher demand when the number of customers is higher. Thus, more sellers are attracted to a larger pool of buyers. Using the implicit function theorem and (10), we obtain

$$\frac{\partial \lambda}{\partial \theta} = \frac{2\lambda}{\theta} \gtreqless 1 \Leftrightarrow 2 \gtreqless \theta/\lambda.$$

Thus, we can state the following proposition.

Proposition 2 *Home market magnification occurs when $2 > \theta/\lambda$.*

The condition for home market magnification is satisfied on 75 % of domain (λ, θ). Thus, if λ is not too small in comparison with θ, home market magnification occurs. Notice that no parameters except λ and θ have an effect on home market magnification.

Setting $r^*(\lambda, \theta) = \bar{r}$ yields:

$$\theta(\lambda) = \frac{\bar{r}}{L} \sqrt{\frac{\lambda N}{2c\mu}} \equiv \Psi_r(\lambda). \tag{13}$$

Since θ is a square root function of λ, the function emanates from the origin with gradually decreasing positive slope.

Since buyers purchase a product such that $\epsilon_{lj} \in [\widehat{x}, \lambda N]$, we obtain

$$\epsilon^E = \int_{\widehat{x}}^{\lambda N} \frac{\epsilon}{\lambda N - \widehat{x}} d\epsilon = \lambda N - \sqrt{\frac{c\lambda N}{2\mu}}.$$

Thus, substituting (10) and (11) into (1), the expected utility of buyer $l = \theta$ is given by:

$$U(\lambda, \theta) = -2\sqrt{2c\mu\lambda N} + \mu\lambda N - t\theta L. \tag{14}$$

The first term on the right-hand side of (14) is the sum of costs stemming from the price and sampling cost. If (6) is not satisfied, (14) implies $U(\lambda, \theta) < 0$ and so no buyer travels to the marketplace. If (6) is satisfied, we obtain $\partial U/\partial\lambda > 0$. That is, the larger market enables the attraction of buyers located further from the marketplace. Setting $U(\lambda, \theta) = V$ yields:

$$\theta(\lambda) = \frac{1}{tL}\Big(- 4\sqrt{2c\mu\lambda N} + \mu\lambda N - V\Big) \equiv \Psi_U(\lambda), \tag{15}$$

From (15), we have:

$$\frac{\partial\theta}{\partial\lambda} = \frac{1}{tL}\left(\mu N - \sqrt{\frac{2c\mu N}{\lambda}}\right), \qquad \frac{\partial^2\theta}{\partial\lambda^2} > 0.$$

It is readily verified that $\Psi_U(\lambda)$ is monotonically increasing and a convex function on $\lambda \in [0, 1]$; ++as λ increases,++ $\Psi_U(\lambda)$ initially decreases and then increases with a negative minimum value, whereas $\Psi_r(\lambda)$ is monotonically increasing and concave on $\Psi_r(\lambda) \in [0, 1]$ ($\lambda \in [0, 1]cut$). It is easily verified that $\Psi_r(0) > \Psi_U(0)$. Hence, there is a unique solution $(\lambda^*, \theta^*) \in (0, +\infty) \times (0, +\infty)$ of the system given by (13) and (15).

Solving (13) and (15), we obtain the unique solution (λ^*, θ^*) as follows:

$$\lambda^* = \frac{1}{8c\mu^3 N}\left[\overline{r}t + 4c\mu + \sqrt{(\overline{r}t + 4c\mu)^2 + 8c\mu^2 V}\right]^2, \tag{16}$$

$$\theta^* = \frac{\overline{r}}{4c\mu^2 L}\left[\overline{r}t + 4c\mu + \sqrt{(\overline{r}t + 4c\mu)^2 + 8c\mu^2 V}\right]. \tag{17}$$

We conduct a comparative analysis of λ^* and θ^* in the following section. Examining $\lambda^* \leq 1$ and $\theta^* \leq 1$, we obtain the sufficient condition for (λ^*, θ^*) as follows:

$$\min\left\{\left[\left(\frac{\mu L}{\overline{r}} - 2\right)\frac{2c\mu}{\overline{r}} - t\right] L, \left[\mu\sqrt{N} - \frac{\overline{r}t + 4c\mu}{\sqrt{2c\mu}}\right]\sqrt{N}\right\} > V. \tag{18}$$

In other words, an interior solution is avoided by the following: (i) the large opportunity cost of buyers visiting the marketplace, (ii) the large opportunity cost of capital returns, (iii) the large cost of commuting to the marketplace, and (iv) a small number of buyers and sellers.

4 Stability

4.1 Interior solutions

We now turn to examine the stability of an interior solution (λ^*, θ^*). Using (12) and (14), linearizing (3) on (λ^*, θ^*) yields:

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\theta} \end{pmatrix} = A \begin{pmatrix} \lambda - \lambda^* \\ \theta - \theta^* \end{pmatrix}, \qquad A \equiv \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} -\frac{\theta^* L}{\lambda^*}\sqrt{\frac{c\mu}{2\lambda^* N}} & L\sqrt{\frac{2c\mu}{\lambda^* N}} \\ \mu N - \sqrt{\frac{2c\mu N}{\lambda^*}} & -tL \end{pmatrix}.$$

Matrix A expresses the agglomeration forces and dispersion forces. The agglomeration forces correspond to a circular causality between sellers and buyers in this model. As the number of buyers increases, the demand for variety increases and thus higher capital returns are realized, as in $a_{12} > 0$. Hence, each seller has an incentive to offer products in the marketplace. In a_{21}, there are both agglomeration forces and dispersion forces: (i) the expected match value increases as the number of sellers increases, that is, μN (agglomeration force) in a_{21}; (ii) the selling price and sampling cost increase as the number of sellers increases, that is, $-\sqrt{2c\mu N/\lambda^*}$ (dispersion force) in a_{21}. Although the sign of a_{21} can be negative if c is large and λ^* is small, agglomeration force dominates dispersion force if $a_{21} > 0$, and vice versa. The other two dispersion forces arise from lower demand for variety due to competition among sellers, as in $a_{11} < 0$, and from the distance between a buyer's residence and the marketplace being longe, as in $a_{22} < 0$.

Using (15), the determinant of A is given by

$$|A| = \frac{L\sqrt{2c\mu}}{\sqrt{\lambda^* N}} \left(\frac{\theta^* tL}{2\lambda^*} - \mu N + \sqrt{\frac{2c\mu N}{\lambda^*}} \right). \tag{19}$$

Substituting (15) into (19), we obtain $|A| = -\frac{V}{2\lambda^*} - \frac{\mu N}{2} < 0$.[9] Hence, the interior solution (λ^*, θ^*) is a saddle point. The negative determinant implies that $a_{21} > 0$. Furthermore, because the second term in the brackets of (19) is negative, it dominates the

[9] For $|A| > 0 \Leftrightarrow V < -\mu\lambda^* N$ is necessary. If this condition is satisfied, the economy is stable.

other two terms. That is, positive feedback or circular causality between the agglomeration of buyer and sellers in a marketplace.

Furthermore, the trace of A is

$$Tr(A) = -\frac{\theta^* L\sqrt{c\mu}}{\lambda^*\sqrt{2\lambda^* N}} - tL < 0.$$

By some simple calculations, we also obtain $Tr(A)^2 - 4|A| > 0$, which implies that there exist real eigenvalues $\omega_1 > 0$ and

$$\omega_2 = \frac{1}{2}\left(-\frac{\theta L\sqrt{c\mu}}{\lambda^*\sqrt{2\lambda^* N}} - tL - \sqrt{\left(\frac{\theta L\sqrt{c\mu}}{\lambda^*\sqrt{2\lambda^* N}} + tL\right)^2 + 2\left(\frac{V}{\lambda^*} + \frac{\mu N}{2}\right)}\right) < 0. \qquad (20)$$

Solving $AV = \omega_2 V$ where $V \equiv [v, 1]'$, we obtain

$$v = \frac{\frac{L\sqrt{2c\mu}}{\sqrt{\lambda^* N}}}{\frac{\theta L\sqrt{c\mu}}{\lambda^*\sqrt{2\lambda^* N}} + \omega_2}.$$

Using (20), it is readily verify that $v < 0$, which implies that the stable saddle path of (λ^*, θ^*) is downward-sloping around (λ^*, θ^*). This implies that, if the economy shifts slightly from (λ^*, θ^*), circular causality acts in two directions: increases in buyers and sellers intensify the attraction of the marketplace for both; and decreases in buyers and sellers weaken the attraction of the marketplace for both. As a result, this process may maintain the agglomeration of buyers and sellers, or otherwise lead to the complete disappearance of the marketplace.

4.2 Corner solutions

Since the interior solution is not stable if the economy is not on the stable saddle path, we must find other equilibrium conditions. Focusing on the cases of $\lambda^* = 1$ and/or $\theta^* = 1$,[10] the corner solutions can be divided into four cases: (i) $\lambda^* = \theta^* = 1$, (ii) $\lambda^* = 1$ and $\theta^* \in [0, 1]$, (iii) $\lambda^* \in [0, 1]$ and $\theta^* = 1$, and (iv) $\lambda^* = \theta^* = 0$.

In case (i), we examine the stability conditions for $\lambda^* = \theta^* = 1$, given as follows:

$$r(1, 1) > \overline{r}, \qquad U(1, 1) > V.$$

[10] It is readily verified that the corner solutions of $\lambda^* = 0$ or $\theta^* = 0$ is unstable due to $\dot{\lambda} < 0$ if $\theta = 0$ or $\dot{\theta} < 0$ if $\lambda = 0$.

Hence, the corner solution $\lambda^* = \theta^* = 1$ is an equilibrium if and only if:

$$\overline{r} < \frac{L\sqrt{2c\mu}}{\sqrt{N}}, \qquad V < -2\sqrt{2c\mu N} + \mu N - tL. \tag{21}$$

In other words, the opportunity cost of renting capital $\overline{r}$ and the opportunity cost of visiting a marketplace, V, must be very low. Furthermore, the commuting cost t should be low enough to attract buyers to visit the marketplace. By some simple calculations, (21) can be rewritten as follows:

$$\frac{\overline{r}\sqrt{N}}{\sqrt{2c\mu}} < L < \frac{-2\sqrt{2c\mu N} + \mu N - V}{t}.$$

Hence, if L is very small, we have $\dot{\lambda} < 0$, which implies that some capital will not be used in the marketplace; if L is very large, we have $\dot{\theta} < 0$, which implies that some buyers who are located far away from the marketplace will not travel to the marketplace. In the above two cases, the corner solution $\lambda^* = \theta^* = 1$ is unstable and not an equilibrium.

In case (ii), the equilibrium conditions are (a) $\Psi_U(1) < 1$, and (b) $\Psi_r(1) < \Psi_U(1)$. Solving $\theta^* = \Psi_U(1)$, we obtain

$$\theta^* = \frac{1}{tL}\left(-2\sqrt{2c\mu N} + \mu N - V\right).$$

As such, $\Psi_U(1) < 1$ yields:

$$-2\sqrt{2c\mu N} + \mu N - tL < V \Leftrightarrow \frac{-2\sqrt{2c\mu N} + \mu N - V}{t} < L. \tag{22}$$

That is, a larger population satisfies (22). Furthermore, $\Psi_r(1) < \Psi_U(1)$ yields:

$$\sqrt{N}\left(\mu\sqrt{N} - \frac{\overline{r}t + 4c\mu}{\sqrt{2c\mu}}\right) > V. \tag{23}$$

In case (iii), the equilibrium conditions are (a) $\Psi_r^{-1}(1) < 1$ and (b) $\Psi_r^{-1}(1) < \Psi_U^{-1}(1)$. Solving $\lambda^* = \Psi_r^{-1}(1)$ yields:

$$\lambda^* = \frac{L}{\overline{r}}\sqrt{\frac{2c\mu}{N}}.$$

Solving $\Psi_r^{-1}(1) < 1$ yields:

$$L\sqrt{\frac{2c\mu}{N}} > \overline{r} \Leftrightarrow L < \frac{\overline{r}\sqrt{N}}{\sqrt{2c\mu}}. \tag{24}$$

[10] To satisfy the condition $\frac{\overline{r}\sqrt{N}}{\sqrt{2c\mu}} < \frac{-2\sqrt{2c\mu N}+\mu N-V}{t}$, the mass of capital N must be sufficiently large.

The smaller number of potential buyers L satisfies (24). Furthermore, $\Psi_r^{-1}(1) < \Psi_U^{-1}(1)$ yields

$$L\left[\left(\frac{\mu L}{\bar{r}} - 2\right)\frac{2c\mu}{\bar{r}} - t\right] > V. \tag{25}$$

This condition is not satisfied if the total number of buyers L is too small. Finally, in case (iv), using (14), we find that indirect utility, excluding the commuting cost, $U(\lambda, \theta) - t\theta L < 0$, becomes negative if $\lambda < \frac{8c}{\mu N}$. Thus, $(0, 0)$ is always stable. We can summarize the above results in the following proposition.

Proposition 3 *1. There exists an interior solution if (18) is satisfied. The economy has equilibria that displays one of the following three patterns: (i) if (21) is satisfied, there are two corner solutions ($\lambda^* = \theta^* = 0$ and $\lambda^* = \theta^* = 1$) and one interior solution; (ii) if (22) and (23) are satisfied, there are two equilibria ($\lambda^* = \theta^* = 0$ and $\lambda^* = 1, \theta^* \in (0, 1)$), and one interior solution; (iii) if (24) and (25) are satisfied, there are two equilibria ($\lambda^* = \theta^* = 0$ and $\lambda^* \in (0, 1)$, $\theta^* = 1$) and one interior solution. However, the interior solutions in cases (i), (ii), and (iii) are always saddle points, which are unstable.*

2. If (18) is not satisfied, there is only one corner solution $\lambda^* = \theta^* = 0$.

The three cases in the first part of the above proposition can be divided according to the potential number of buyers L. If the potential number of buyers L is relatively small, not all of the capital K is employed in the marketplace. However, if the potential number of buyers L is very large, some buyers choose not to visit the marketplace.

The last part of the above proposition is important because it means that the formation of a marketplace requires satisfying (18). For example, by lowering commuting costs to the market t or by increasing the potential size of buyers L, the condition (18) is satisfied. However, the market might not be established, depending upon the initial point (λ, θ). To explain this, the phase diagram in Fig. 1 is useful. The domain (λ, θ) is divided by the lines expressing a pair (λ, θ) such that $\dot{\lambda} = 0$ and $\dot{\theta} = 0$. In this figure, condition (18) is satisfied and then the interior solution exists in the figure. If the initial point is on $A(B)$, for example, where both θ and λ are relatively large and $\theta > \lambda$ ($\lambda > \theta$). Firstly, the number of buyers decreases (increases) and the number of sellers increases (decreases) and then both increase. Finally, buyers and sellers establish a large market: $\lambda = \theta = 1$. However, if the initial point is on $C(D)$, then θ or λ is too small. As the number of buyers decreases (increases) and the number of sellers increases (decreases), no trade will emerge in the marketplace: $\lambda = \theta = 0$.

Both A and B — and both C and D in the figure — are divided by the stable saddle path, which is expressed as the line passing the intersection point of two lines $\dot{\lambda} = 0$ and

$\dot{\theta} = 0$. The intersection point is a saddle point that shifts to the north-east by increasing t, V and $\bar{r}$, as follows:[11]

$$\frac{\partial \lambda^*}{\partial t} > 0, \qquad \frac{\partial \lambda^*}{\partial V} > 0, \qquad \frac{\partial \lambda^*}{\partial \bar{r}} > 0, \tag{26}$$

$$\frac{\partial \theta^*}{\partial t} > 0, \qquad \frac{\partial \theta^*}{\partial V} > 0, \qquad \frac{\partial \theta^*}{\partial \bar{r}} > 0. \tag{27}$$

Thus, by decreasing commuting costs, capital returns in other sectors or other regions and indirect utility when buyers do not travel to market widen the initial points which can become $\lambda = \theta = 1$. Though a simple calculation, we obtain

$$\frac{\partial \lambda^*}{\partial c} \gtreqless 0 \Leftrightarrow 4c\mu \gtreqless \bar{r}t, \qquad \frac{\partial \theta^*}{\partial c} < 0,$$
$$\frac{\partial \lambda^*}{\partial \mu} < 0, \qquad \frac{\partial \theta^*}{\partial \mu} < 0.$$

In other words, lowering costs in order to sample a seller's product causes the initial pair of λ and θ to be widened such that the $\lambda = \theta = 1$ case emerges if commuting costs are high. If products become more differentiated — that is, if μ increases- the initial pair of λ and θ that reach $\lambda = \theta = 1$ widens. Furthermore, we obtain

$$\frac{\partial \lambda^*}{\partial N} < 0, \qquad \frac{\partial \lambda^*}{\partial L} = \frac{\partial \theta^*}{\partial N} = 0 \quad \text{and} \quad \frac{\partial \theta^*}{\partial L} < 0. \tag{28}$$

These results can be summarized as the following proposition.

Proposition 4 *The domain for the initial state to become a marketplace in equilibrium widens as (1) transport costs are lowered, (2) opportunity costs to visit the marketplace become lower, (3) opportunity costs to operate in a marketplace are lowered, and (4) heterogeneities of taste increase. The influence of search costs is ambiguous.*

The four points in the above proposition suggest the possibility of forming a marketplace.

5 Conclusion

Ever since Wolinsky (1984), market formation involving a search process has been studied by considering the location of the retailer. However, the circular causality between retailes and buyers was not modeled in previous literature. This model can explain the formation

[11] Increasing λ^* does not shift the saddle point to a new position.

of a big marketplace in a developing economy as well as the difficulty in beginning to open such a marketplace.

Our findings can be summarized as follows: First, we found that a greater variety of products purchased implies more sampling by buyers and higher prices for a product type Indeed, the agglomeration of sellers provides higher prices for a variety, which is good for sellers. More sampling suggests attractiveness to buyers. Second, the large opportunity cost of buyers in visiting the marketplace, the large opportunity cost of capital returns, the large costs of commuting to the marketplace, and the potentially small number on visitors and sellers will constrain potential formation of a marketplace, even if these conditions are restricted in the economy. Furthermore, a marketplace is not formed if either buyers or sellers are small at the initial stage. Third, if buyer preferences are more heterogeneous or if the commuting costs to the marketplace is lowered, the potential initial number of buyers and sellers can be smaller for forming a marketplace. Fourth, the effects of search cost on forming a marketplace are ambiguous.

A limitation of our model is its geographic configuration. Extending the model to two regions might explain the emergence of a core-periphery pattern, as in New Economic Geography. Furthermore, our framework might be extended to incorporate referral into the search process, as in Arbatskaya and Konish (2012).

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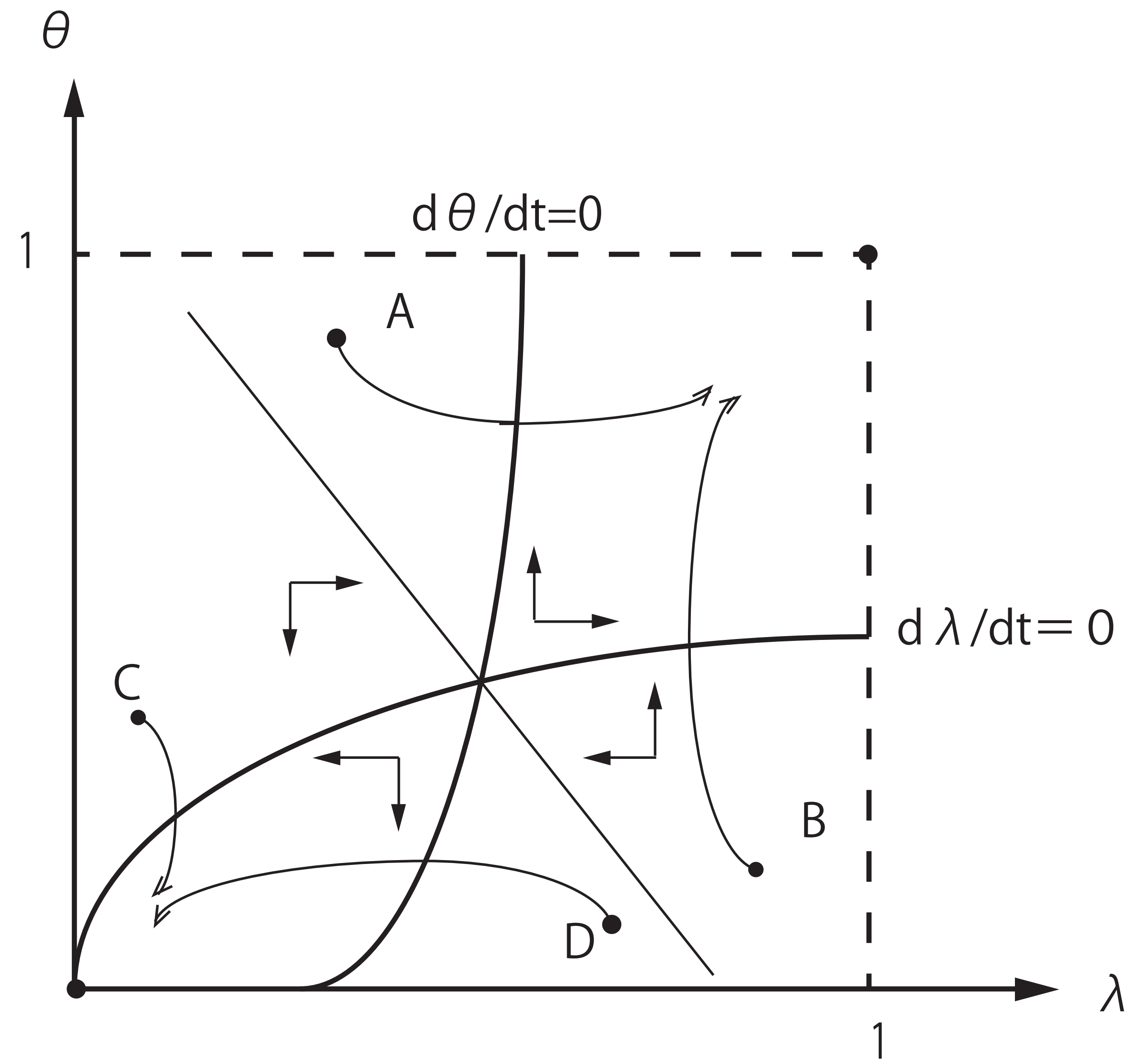

Figure 1