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A Trade-Consistent SCGE Model**

Asao ANDO* and Bo MENG**

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Keywords: SCGE model, Armington assumption, transport sector

JEL classification: C67, C68, R15

*Graduate School of Information Sciences, Tohoku University (E-mail: ando@m.tohoku.ac.jp)

** Research Fellow, IDE-JETRO (E-mail: mengbo@ide.go.jp)

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Spatial Price Equilibrium and the Transport Sector: a Trade-Consistent SCGE Model*

Asao ANDO[†] and Bo MENG[‡]

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Abstract

In spatial computable general equilibrium (SCGE) models, interregional trade ought to play an important role in determining the spatial price equilibrium. In contrast to a Walrasian world with homogeneous commodities, the existence of cross-hauling indicates that the same commodities with different origins are imperfect substitutes. Although the Armington assumption is commonly employed to describe substitutions, many of the existing models do not explicitly consider the transport sector, either by considering the iceberg costs, or by introducing a commodity pool that clears trade imbalances.

Model formulation should be consistent with the model assumption, and this paper presents a framework for an SCGE model that is compatible with the Armington assumption and explicitly considers transport activities. In the model, the trade coefficient takes the form of a potential function, and the equilibrium market price becomes similar to the price index of varietal goods in the context of new economic geography (NEG).

The features of the model are investigated by using the minimal setting, which comprises two non-transport sectors and three regions. Because transport costs are given exogenously to facilitate study of their impacts, commodity prices are also determined relative to them. The model can be described as a system of homogeneous equations, where an output in one region can arbitrarily be determined similarly as a price in the Walrasian equilibrium. The model closure is sensitive to formulation consistency so that homogeneity of the system would be lost by use of an alternative form of trade coefficients.

Keywords: SCGE model, Armington assumption, transport sector.

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[†]Graduate School of Information Sciences, Tohoku University (E-mail: ando@m.tohoku.ac.jp)

[‡]Institute of Developing Economies at the Japan External Trade Organization.

1 Introduction

According to traditional trade theory (e.g. Samuelson, 1953), the phenomenon known as “cross-hauling” or “two-way trade” should not appear under perfect competition. Contrary to the theory, however, it is quite common for a pair of countries to trade the same commodities with each other. Brander (1981) explains the existence of cross-hauling by introducing “imperfect competition” due to strategic interaction among firms into traditional trade theory. In addition to theoretical explanations, cross-hauling can also be interpreted from a statistical viewpoint, which incorporates several observations. (1) Every practical classification of a commodity involves great diversity in quality. (2) A country often represents a highly aggregated area. (3) Trade statistics capture transactions in a finite period, during which a country may seek supplies of a commodity from various countries for reasons of seasonality and other factors. The first of these observations explains the “intra-industry trade” of half-products that belong to the same category as the final products.

In many multi-regional models, trade coefficients are formulated as potential functions to accommodate cross-hauling. For example, a popular class of formulations takes the form of an entropy model, in which it is assumed that the quantities of interregional trade are positively related to supply capacities and negatively related to transport costs (see Amano and Fujita, 1970). However, the problem with such formulations is that they are based on analogies in physics or on statistical principles; they do not provide a theoretical explanation that is based on the behavior of firms or households. Hence, when statistical formulas are combined with deterministic ones, logical inconsistencies may be present in the model.¹

In the CGE literature, the standard way to make cross-hauling compatible with a perfectly competitive market is to employ the assumption introduced by Armington (1969). The assumption states that the same commodities produced in different regions are imperfect substitutes for each other. Within an SCGE model, explaining the existence of cross-hauling under imperfect competition appears to be a more realistic approach than explaining it under perfect competition. However, this approach requires additional information on industry agglomeration (number of firms) and economies of scale (magnitude of fixed costs) for model calibration, and such information is often difficult to obtain. This is particularly true when developing

¹Meng and Ando (2005) show that very similar potential-type trade coefficients can be derived from the deterministic decisions of firms or individuals under the multi-regional input-output framework without relying on the ambiguous concept of socio-physics.

economies or relatively small regions are studied. In fact, imperfect competition hardly explains the cross-hauling caused by the observations described above. Thus, in many SCGE models, perfect competition with the Armington assumption continues to be the most popular and standard set of assumptions.

The elasticity of substitution between the same goods originated in a pair of regions under the Armington assumption is called the Armington elasticity. The effects that this elasticity has on the trade coefficients, spatial price equilibrium (SPE), and model solutions have not yet been fully clarified. One reason is that existing SCGE models tend to treat the transport sector as an ordinary service sector or as an imaginary transport agency that requires no resources to produce transport services (see Miyagi and Honbu, 1993 and the GTAP model.²). However, transport conditions, particularly the freight rates, are a source of regional price differentials and should be consistent with the SPE system. It is difficult to explain how transport conditions affect trade patterns and the SPE system under given Armington elasticity, and *vice versa*, unless the unique features of transport are explicitly considered. Some authors do explicitly consider the behavior of transport firms.³

It is generally difficult to estimate the Armington elasticity when the numbers of regions and sectors are large. In many existing SCGE models, elasticity values are borrowed from the literature or simply given without adequate verification. Because simulations based on groundless elasticity of substitution may yield nonsensical results, it is quite important to understand the significance of the Armington assumption in depth in the context of SCGE models.⁴

An SCGE model for a world comprising R regions and $J-1$ non-transport sectors is presented in Section 2. The model incorporates imperfect substitution among origins by using the Armington assumption and explicitly considers the transport network and the behavior of transport firms. In Section 3, the mathematical structure of the proposed model and the computational algorithms to obtain the equilibrium are discussed. The numerical analyses for the minimal setting (3 regions and 2 non-transport sectors) are described in Section 4, with an emphasis

²Developed in 1992 by the Center for Global Trade Analysis at Purdue University (<http://www.gtap.agecon.purdue.edu/>).

³See Harker (1987), Haddad and Hewings (2001), and Macann (2005), for example. In particular, Harker (1987) introduced transport firms and networks into the framework of Takayama and Judge (1971). This makes the SPE models a specific antecedent to development of the SCGE model.

⁴For a similar reason, Lofgren and Robinson (2002), Florenz (2005) and Ando and Meng (2009) use an assumption of perfect substitution to avoid the Armington assumption in their CGE models.

on the effect that parameters and exogenous variables have on the spatial equilibrium. Finally, Section 5 provides the concluding remarks and a discussion of possible alternative settings.

2 The SCGE Model

In this section, the basic assumptions of the model are given, and the behaviors of individual economic agents (here, general industries, households, and transport firms) are described in detail. It will then be shown that (1) trade coefficients can be endogenously derived from the deterministic decisions of firms and households under the Armington assumption and (2) the conditions of spatial price equilibrium can be obtained from the cost-minimization behavior of transport firms. The general equilibrium conditions of the entire system are then summarized. A glossary of the symbols used in the formulas can be found in *Appendix A*.

2.1 Basic Assumptions

- (a) **Numbers of regions and sectors:** R regions and $J - 1$ non-transport sectors exist.
- (b) **Two factors of production:** Two factors, labor and physical capital, are considered; both are immobile across regions and sectors.⁵
- (c) **Three types of economic agents:** General (non-transport) industries, transport firms, and households are considered.
- (d) **Transport demand:** Demand for transport services is assumed to be derived from purchases of other commodities only.⁶ Transport services are supplied by the regions where the shipments originate, and all the transport costs are paid there.
- (e) **Final demand:** Final demand only comes from households' consumption expenditures, which are equivalent to total disposable income.
- (f) **Imperfect substitutes:** Commodities produced in different regions are imperfect substitutes for each other (Armington assumption).

⁵This assumption can be easily modified to facilitate mobile capital and labor.

⁶For simplicity, only freight transportation is handled by the transport sector. Passenger transportation is assumed to be included in the other services.

2.2 Behavior of Economic Agents

2.2.1 Non-transport Firms ($j \neq J$)

The (aggregate) production function of sector j in region s combines the two factor inputs; labor L_j^s and capital stock K_j^s of sector j in region s , and the intermediate inputs $x_{ij}^{r,s}$ of commodity i produced in region r as follows:

$$X_j^s = A_j^s \prod_{i \neq J} \left(\sum_r (x_{ij}^{r,s})^{-\rho_{ij}^s} \right)^{-\frac{\alpha_{ij}^s}{\rho_{ij}^s}} (L_j^s)^{\alpha_{L_j}^s} (K_j^s)^{\alpha_{K_j}^s}. \quad (1)$$

The upper level of the production function uses a Cobb-Douglas type technology, and the lower level for intermediate inputs from different regions employs a constant elasticity of substitution (CES) technology. X_j^s denotes the amount of output produced by industry j in region s , $\rho_{ij}^s (\geq -1)$ the substitution parameter, and A_j^s the scale parameter. The subscript J indicates the transport sector. The following is assumed for the parameters α_{ij}^s , $\alpha_{L_j}^s$ and $\alpha_{K_j}^s$:

Assumption 1 *The production function is linearly homogeneous for each region.*⁷

$$\sum_{i \neq J} \alpha_{ij}^s + \alpha_{L_j}^s + \alpha_{K_j}^s = 1.$$

Non-transport firms face the problem of choosing a combination of $\{x_{ij}^{r,s}, K_j^s, L_j^s\}$ to maximize their profits, which are described as follows:

$$\pi_j^s = p_j^s X_j^s - \sum_{i \neq J} \sum_r (p_i^r + c_i^{r,s}) x_{ij}^{r,s} - \omega_j^s L_j^s - \gamma_j^s K_j^s, \quad (2)$$

where p_j^s is the mill price (FOB) of commodity j in region s , $c_i^{r,s}$ the transport cost for shipping a unit commodity i from region r to s , and $p_i^r + c_i^{r,s}$ the delivered price (CIF) in region s of commodity i produced in region r . ω_j^s and γ_j^s are the wage rate and capital rent, respectively, paid by sector j in region s .

One of the first-order conditions in problem (2) can be written as:

$$\frac{\partial \pi_j^s}{\partial x_{ij}^{r,s}} = \alpha_{ij}^s p_j^s X_j^s \frac{(x_{ij}^{r,s})^{-\rho_{ij}^s - 1}}{\sum_r (x_{ij}^{r,s})^{-\rho_{ij}^s}} - (p_i^r + c_i^{r,s}) = 0. \quad (3)$$

The ratio of the above conditions for different origins, r and r' , is obtained as follows:

$$\frac{p_i^{r'} + c_i^{r',s}}{p_i^r + c_i^{r,s}} = \left(\frac{x_{ij}^{r',s}}{x_{ij}^{r,s}} \right)^{-\rho_{ij}^s - 1} \quad \text{or} \quad \left(\frac{p_i^{r'} + c_i^{r',s}}{p_i^r + c_i^{r,s}} \right)^{-\frac{1}{\rho_{ij}^s + 1}} = \frac{x_{ij}^{r',s}}{x_{ij}^{r,s}}.$$

⁷Under *Basic Assumption* (4), freight transport ($i = J$) does not constitute an intermediate input.

When the trade coefficient in sector j is defined on the basis of physical shipment, it can be written in terms of delivered prices.

$$t_{ij}^{rs} \equiv \frac{x_{ij}^{rs}}{\sum_r x_{ij}^{rs}} = \frac{(p_i^r + c_i^{rs})^{-\frac{1}{\rho_{ij}^s+1}}}{\sum_r (p_i^r + c_i^{rs})^{-\frac{1}{\rho_{ij}^s+1}}} \quad (4)$$

Note that the coefficient differs for each recipient sector j because of differences in the substitution parameters ρ_{ij}^s . Conversely, if all the economic activities share the same ρ_{ij}^s , the same set of trade coefficients will apply. Equivalence of substitution parameters can be extended to ρ_{ih}^s for household consumption. The above expressions can also be written with the elasticity of substitution $\sigma_{ij}^s = 1/(\rho_{ij}^s + 1)$, and so it is convenient to assume the following.

Assumption 2 *The substitution parameters and the elasticity of substitution in industrial sectors and households are equivalent, irrespective of the agents demanding the commodities: $\rho_{ij}^s \equiv \rho_{ih}^s \equiv \rho_i^s$ and $\sigma_{ij}^s \equiv \sigma_{ih}^s \equiv \sigma_i^s$.*

With this assumption, the trade coefficient is simplified as follows:

$$t_i^{rs} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}, \quad (4)'$$

which implies that the trade coefficient depends on delivered prices and a substitution parameter.⁸ The composite (market) price of commodity i in region s is then obtained as the average of delivered prices weighted by this coefficient:

$$q_i^s \equiv \sum_r (p_i^r + c_i^{rs}) t_i^{rs} = \frac{\sum_r (p_i^r + c_i^{rs})^{1-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}. \quad (5)$$

When the Chenery-Moses' assumption of competitive imports is introduced, the intermediate input in physical terms can be written with the regional input coefficient, a_{ij}^s , as follows:

$$x_{ij}^{rs} = t_i^{rs} a_{ij}^s X_j^s = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}} a_{ij}^s X_j^s. \quad (6)$$

Additionally, equation (3) is solved for α_{ij}^s as

$$\alpha_{ij}^s = \frac{p_i^r + c_i^{rs}}{p_j^s X_j^s} \frac{\sum_r (x_{ij}^{rs})^{1-\frac{1}{\sigma_i^s}}}{(x_{ij}^{rs})^{-\sigma_i^s}},$$

⁸This expression resembles Harker's (1987) potential function, but differs in that it does not include X_i^r representing regional production capacities.

which is independent of the originating region r . The second term of the right-hand side can be rewritten by substituting (6) into it:

$$\frac{\sum_r (x_{ij}^{rs})^{1-\frac{1}{\sigma_i^s}}}{(x_{ij}^{rs})^{-\sigma_i^s}} = \frac{\sum_r (p_i^r + c_i^{rs})^{1-\sigma_i^s}}{(p_i^r + c_i^{rs}) \sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}} a_{ij}^s X_j^s$$

Because of (5), the relationship between α_{ij}^s and a_{ij}^s can now be established.

$$\alpha_{ij}^s = \frac{a_{ij}^s X_j^s \sum_r (p_i^r + c_i^{rs})^{1-\sigma_i^s}}{p_j^s X_j^s \sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}} = \frac{q_i^s}{p_j^s} a_{ij}^s \quad \text{or} \quad a_{ij}^s = \frac{p_j^s}{q_i^s} \alpha_{ij}^s \quad (7)$$

Regarding the factor inputs, the first-order conditions for the profit maximization problem (2) are obtained as follows:

$$\alpha_{Lj}^s = \frac{\omega_j^s L_j^s}{p_j^s X_j^s} \quad \text{and} \quad \alpha_{Kj}^s = \frac{\gamma_j^s K_j^s}{p_j^s X_j^s}, \quad (8)$$

which implies that the Cobb-Douglas parameters, α_{Lj}^s and α_{Kj}^s , are exactly the regional factor input coefficients in monetary terms.⁹ Then the relationships between the physical and monetary coefficients are straightforward:

$$a_{Lj}^s \equiv \frac{L_j^s}{X_j^s} = \frac{p_j^s}{\omega_j^s} \alpha_{Lj}^s \quad \text{and} \quad a_{Kj}^s \equiv \frac{K_j^s}{X_j^s} = \frac{p_j^s}{\gamma_j^s} \alpha_{Kj}^s. \quad (9)$$

2.2.2 Households

The source of income for households is the gross regional domestic product V^s , which comprises wage and rent payments:

$$V^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s,$$

where regions are assumed to be closed in terms of factor income. For simplicity, the firms and their capital stocks are owned by the households of the region where they are located. Further, since no governmental activities, such as taxes or income transfer, are present in the model, total household disposable income W^s naturally coincides with V^s in each region.

The aggregate utility of households in region s is also given as a nested CES function based on y_i^{rs} , the amount of commodity i produced in region r and consumed in region s . The households' problem is thus to choose the commodity bundle $\{y_i^{rs}\}$ that maximizes utility under the budget constraint.

$$\max_{y_i^{rs}} U^s = \prod_{i \neq J} \left(\sum_r (y_i^{rs})^{-\rho_{ih}^s} \right)^{-\frac{\beta_i^s}{\rho_{ih}^s}}, \quad (10)$$

$$\text{s.t.} \quad \sum_{i \neq J} \sum_r (p_i^r + c_i^{rs}) y_i^{rs} = W^s, \quad (11)$$

⁹By similar reasoning, α_{ij}^s represents the regional input coefficient in monetary terms.

where W^s is regional household disposable income, $\rho_{ih}^s (\geq -1)$ the substitution parameter, and β_i^s the parameter of the Cobb-Douglas utility. Analogous to *Assumption 1*, linear homogeneity of the utility function is assumed for β_i^s .

Assumption 3 *The utility function is linearly homogeneous, with $\sum_{i \neq J} \beta_i^s = 1$.*

One of the first-order conditions for this utility maximization problem is obtained with the Lagrange multiplier λ^s attached to the constraint (11):

$$\beta_i^s U^s \frac{(y_i^{rs})^{-\rho_{ih}^s - 1}}{\sum_r (y_i^{rs})^{-\rho_{ih}^s}} - \lambda^s (p_i^r + c_i^{rs}) = 0. \quad (12)$$

By taking the proportion of this equation for two different origins, r and r' , the following result is obtained:

$$\frac{p_i^{r'} + c_i^{r's}}{p_i^r + c_i^{rs}} = \left(\frac{y_i^{r's}}{y_i^{rs}} \right)^{-\rho_{ih}^s - 1} \quad \text{or} \quad \left(\frac{p_i^{r'} + c_i^{r's}}{p_i^r + c_i^{rs}} \right)^{-\frac{1}{\rho_{ih}^s + 1}} = \frac{y_i^{r's}}{y_i^{rs}}. \quad (13)$$

By introducing the elasticity of substitution σ_{ih}^s in place of ρ_{ih}^s , the trade coefficient for household consumption can be expressed by the delivered prices:

$$t_{ih}^{rs} \equiv \frac{y_i^{rs}}{\sum_r y_i^{rs}} = \frac{(p_i^r + c_i^{rs})^{-\sigma_{ih}^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_{ih}^s}}. \quad (14)$$

This equation is quite similar to (4) for the industrial inputs, except for the elasticity. Under *Assumption 2*, a simple mathematical principle indicates that the intermediate inputs and the final consumption share the same set of trade coefficients:

$$t_i^{rs} = \frac{x_{ij}^{rs}}{\sum_r x_{ij}^{rs}} = \frac{y_i^{rs}}{\sum_r y_i^{rs}} = \frac{\sum_j x_{ij}^{rs} + y_i^{rs}}{\sum_r (\sum_j x_{ij}^{rs} + y_i^{rs})} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}. \quad (15)$$

Multiplying y_i^{rs} with the first-order condition (12) and summing by region, the following expression is obtained:

$$\beta_i^s U^s = \lambda^s \sum_r (p_i^r + c_i^{rs}) y_i^{rs}.$$

Further summation with respect to $i \neq J$ yields the following:

$$U^s \sum_{i \neq J} \beta_i^s = \lambda^s \sum_{i \neq J} \sum_r (p_i^r + c_i^{rs}) y_i^{rs} = \lambda^s W^s.$$

Thus under *Assumption 3*, the Lagrange multiplier is equivalent to the average utility of income, so that $\lambda^s = U^s / W^s$, and the first-order condition (12) can now be rearranged:

$$\beta_i^s W^s = \frac{(p_i^r + c_i^{rs}) \sum_r (y_i^{rs})^{-\rho_{ih}^s}}{(y_i^{rs})^{-\rho_{ih}^s - 1}} = \frac{\sum_r (p_i^r + c_i^{rs})^{\frac{\rho_{ih}^s}{\rho_{ih}^s + 1}}}{(p_i^r + c_i^{rs})^{-\frac{1}{\rho_{ih}^s + 1}}} y_i^{rs} = \frac{\sum_r (p_i^r + c_i^{rs})^{1 - \sigma_i^s}}{(p_i^r + c_i^{rs})^{-\sigma_i^s}} y_i^{rs}. \quad (16)$$

Additionally, from (5) and (15), the proportion of composite price and trade coefficient coincides with the fraction in the right-hand side of the above:

$$\frac{q_i^s}{t_i^{rs}} = \frac{\sum_r (p_i^r + c_i^{rs})^{1-\sigma_i^s} \sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s} (p_i^r + c_i^{rs})^{-\sigma_i^s}} = \frac{\sum_r (p_i^r + c_i^{rs})^{1-\sigma_i^s}}{(p_i^r + c_i^{rs})^{-\sigma_i^s}}.$$

Accordingly, equation (16) can be arranged to calculate the aggregate consumption $y_i^s \equiv \sum_r y_i^{rs}$:

$$y_i^s = \frac{y_i^{rs}}{t_i^{rs}} = \frac{\beta_i^s W^s}{q_i^s}. \quad (17)$$

2.2.3 Transport Firms ($j = J$)

Under *Basic Assumption* (d), all demand for this sector is derived from purchases of other commodities. Non-transport firms can determine output levels to maximize their profits, but transport firms are required to provide transport services entailed by demand for other commodities and services. Thus, transport firms seek to minimize costs given the required level of services.

For convenience, the following assumption concerning payment for transport services is introduced:

Assumption 4 *Transport costs are paid at the origin. This also applies to purchases by the transport sector itself. However, transport firms do not recognize the imputed costs that accompany their own purchases from the regions in which they are located.*¹⁰

Total transport demands originating in region s , in monetary terms, are given by the left-hand side of the following formula:

$$\sum_{i \neq J} \sum_r c_i^{sr} (\sum_j x_{ij}^{sr} + y_i^{sr}) \leq p_J^s X_J^s. \quad (18)$$

Under *Assumption 4*, these demands will be fulfilled by transport firms in region s , and their monetary output will be $p_J^s X_J^s$, where the production function (1) also applies to X_J^s of the transport firms.¹¹ The cost to provide services required may then be written as follows:¹²

$$C_J^s = \sum_{i \neq J} \sum_{r \neq s} (p_i^r + c_i^{rs}) x_{iJ}^{rs} + \sum_{i \neq J} p_i^s x_{iJ}^{ss} + \omega_J^s L_J^s + \gamma_J^s K_J^s. \quad (19)$$

¹⁰Transport costs that accompany intra-regional purchases by transport firms are paid to those same firms. Thus such payments can be deducted from the total cost of rendering the transport services required.

¹¹Due to the difficulty in defining the physical units of transport services, their outputs are measured by the freight revenues. Thus the monetary output $p_J^s X_J^s$ is treated as an inseparable variable.

¹²The function (19) omits the cost of intra-regional transportation, $c_i^{ss} x_{iJ}^{ss}$. This is essentially an imputed cost that will be cancelled out when the total cost is redefined to reflect the intra-regional transportation.

Then the problem is to choose values of $\{x_{iJ}^{rs}, K_J^s, L_J^s\}$ that minimize the total cost (19) while satisfying transport demands (18).

If μ^s denotes the Lagrange multiplier attached to (18), then the first-order conditions for interregional and intra-regional inputs under *Assumption 2* become, respectively, as follows:

$$\begin{aligned} (p_i^r + c_i^{rs}) &= \mu^s \alpha_{iJ}^s p_J^s X_J^s \frac{(x_{iJ}^{rs})^{-\rho_i^s - 1}}{\sum_r (x_{iJ}^{rs})^{-\rho_i^s}}, \\ (p_i^s + \mu^s c_i^{ss}) &= \mu^s \alpha_{iJ}^s p_J^s X_J^s \frac{(x_{iJ}^{ss})^{-\rho_i^s - 1}}{\sum_r (x_{iJ}^{rs})^{-\rho_i^s}}. \end{aligned} \quad (20)$$

Similarly as in the case of non-transport firms, the trade coefficient of transport sector is obtained, using the elasticity of substitution, as follows:

$$\begin{aligned} t_{iJ}^{rs} &= \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_{r \neq s} (p_i^r + c_i^{rs})^{-\sigma_i^s} + (p_i^s + \mu^s c_i^{ss})^{-\sigma_i^s}} \quad (r \neq s), \\ t_{iJ}^{ss} &= \frac{(p_i^s + \mu^s c_i^{ss})^{-\sigma_i^s}}{\sum_{r \neq s} (p_i^r + c_i^{rs})^{-\sigma_i^s} + (p_i^s + \mu^s c_i^{ss})^{-\sigma_i^s}}. \end{aligned} \quad (21)$$

This implies that transport firms regard $\mu^s c_i^{ss}$ as the transport cost associated with their intra-regional inputs. Then the composite price calculated from t_{iJ}^{rs} takes the following form:

$$q_i^s = \sum_{r \neq s} (p_i^r + c_i^{rs}) t_{iJ}^{rs} + (p_i^s + \mu^s c_i^{ss}) t_{iJ}^{ss} = \frac{\sum_{r \neq s} (p_i^r + c_i^{rs})^{1-\sigma_i^s} + (p_i^s + \mu^s c_i^{ss})^{1-\sigma_i^s}}{\sum_{r \neq s} (p_i^r + c_i^{rs})^{-\sigma_i^s} + (p_i^s + \mu^s c_i^{ss})^{-\sigma_i^s}} \quad (22)$$

Clearly $\mu^s = 1$ is a sufficient condition for the q_i^s calculated from (22) to coincide with the value from (5).¹³ If $\mu^s = 1$ holds, then the trade coefficients (21) will also become coincident with (15). The relationship between monetary and physical input coefficients can be easily established in a way similar to that used with non-transport firms (7):

$$a_{iJ}^s = \frac{p_J^s}{q_i^s} \alpha_{iJ}^s. \quad (23)$$

Likewise, a relationship analogous to (9) also applies to the factor inputs of transport firms.

2.3 Market Clearance

Aside from the conditions arising from the behavior of individual agents, there are conditions to be satisfied to clear the market. In the above, the first-order conditions of nested CES production and utility functions in a multi-regional setting are consistent with the interregional input-output framework. Thus the price and output equations of the input-output system are the first to be examined.

¹³An economic interpretation of μ^s is that it is the cost increase accruing from a unit increase in the monetary output $p_J^s X_J^s$. Naturally, such a cost increase should equal one monetary unit in equilibrium.

2.3.1 Price Equations

The price equations correspond to column sums of the input-output table. Three patterns of equations must be considered. These correspond to the non-transport and transport sectors and to the households. The equation for non-transport sectors may be written as follows:

$$\begin{aligned} p_j^s X_j^s &= \sum_{i \neq J} \sum_r (p_i^r + c_i^{rs}) x_{ij}^{rs} + \omega_j^s L_j^s + \gamma_j^s K_j^s \\ &= \sum_{i \neq J} \sum_r (p_i^r + c_i^{rs}) t_i^{rs} a_{ij}^s X_j^s + \omega_j^s a_{Lj}^s X_j^s + \gamma_j^s a_{Kj}^s X_j^s, \end{aligned} \quad (24)$$

where the Chenery-Moses' assumption (6) is used to derive the last expression. Incorporating (5) along with the relations:

$$a_{Lj}^s X_j^s = \frac{\alpha_{LJ}^s p_J^s X_J^s}{\omega_j^s} \quad \text{and} \quad a_{Kj}^s X_j^s = \frac{\alpha_{KJ}^s p_J^s X_J^s}{\gamma_j^s},$$

which are straightforward from (9), and dividing both sides by $p_j^s X_j^s$, (24) becomes:¹⁴

$$1 = \sum_{i \neq J} \frac{q_i^s}{p_j^s} a_{ij}^s + \alpha_{Lj}^s + \alpha_{Kj}^s. \quad (25)$$

The price equation for the transport sector is analogous to that for the non-transport sector as far as $\mu^s = 1$ holds, but the price of transport services is inseparable from the monetary outputs. Hence, the price equation for the transport sector can be described in terms of the monetary coefficients only, and no information other than *Assumption 1* can be obtained. In contrast, the following equation holds concerning the household consumption:

$$W^s = \sum_{i \neq J} \frac{\beta_i^s W^s}{q_i^s} \sum_r (p_i^r + c_i^{rs}) t_i^{rs}. \quad (26)$$

Similar to the transport sector, households' demand is determined by disposable income. When the monetary coefficients are used, this equation simplifies to $\sum_{i \neq J} \beta_i = 1$ (*Assumption 3*). In short, no additional condition can be derived from the price equations corresponding to the transport and household sectors.

2.3.2 Output Equations

Output equations correspond to row sums of the input-output table. Under the Chenery-Moses' assumption (6), the output levels for non-transport sectors can be measured in physical terms

¹⁴When the physical input coefficients a_{ij}^s for endogenous sectors are eliminated by using (7), the expression (25) reduces to *Assumption 1*. Thus, to make the price equations meaningful, it is reasonable to consider coefficients a_{ij}^s exogenous.

as follows:

$$X_i^r = \sum_s \sum_j t_i^{rs} a_{ij}^s X_j^s + \sum_s t_i^{rs} y_i^s = \sum_s t_i^{rs} \left\{ \sum_j a_{ij}^s X_j^s + \frac{\beta_i^s W^s}{q_i^s} \right\},$$

where y_i^s is households' physical demand for commodity i in region s . However, the problem is that the physical coefficients a_{ij}^s cannot be used for the transport sector. Thus the above expression must be rewritten with the monetary coefficients:

$$X_i^r = \sum_s \frac{t_i^{rs}}{q_i^s} \left(\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s \right). \quad (27)$$

The output of the transport sector is defined as the total transport costs accruing from purchases of non-transport commodities originating in the region:

$$p_J^r X_J^r = \sum_{i \neq J} \sum_s \frac{c_i^{rs} t_i^{rs}}{q_i^s} \left(\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s \right). \quad (28)$$

2.3.3 Factor Market and Final Demand

According to *Basic Assumption* (b), capital rent and wage rate are determined from their respective values of marginal products:

$$\omega_j^s = \alpha_{Lj}^s \frac{p_j^s X_j^s}{L_j^s} \quad \text{and} \quad \gamma_j^s = \alpha_{Kj}^s \frac{p_j^s X_j^s}{K_j^s}. \quad (29)$$

The above equations apply to both non-transport and transport sectors. In the present model, households' consumption is the only final expenditure. Its total amount coincides with the total factor payments in the region.

$$W^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s. \quad (30)$$

3 The Model Structure and Solution Procedure

In this section, the equilibrium conditions and endogenous variables in the model are first compared, and the mathematical structure of the model is illustrated using a world consisting of three regions. It will be shown that the output equations are reduced to a system of homogeneous equations, which prescribes the solution procedure.

3.1 Equations and Variables

The endogenous variables, major exogenous variables, and parameters in the model are summarized in Table 1, where the values in parentheses indicate the counts of the relevant variables.

Because the price of transport is inseparable from its output, the product $(p_j^r X_j^r)$ is considered as an independent variable in the sector. For the minimal system of three regions ($R = 3$) and two non-transport sectors ($J = 3$), the total number of endogenous variables is 72, which includes t_i^{rs}, q_i^s , and $\alpha_{ij}^s (i, j \neq J)$. This system will be used for explanatory purposes and numerically analyzed.

Table 1: Variables and parameters

Endogenous variables	$X_{j(\neq J)}^s (R(J-1)), p_j^s X_j^s (R),$ $p_{j(\neq J)}^s (R(J-1)), W^s (R),$ $\omega_j^s (RJ), \gamma_j^s (RJ).$
	$t_i^{rs} (R^2(J-1)), q_i^r (R(J-1)),$ $\alpha_{ij(\neq J)}^s (R(J-1)^2)$
Exogenous variables	$K_j^s, L_j^s, c_i^{rs}.$
Parameters	$a_{ij(\neq J)}^s, \alpha_{iJ}^s, \alpha_{Kj}^s, \alpha_{Lj}^s, \beta_i^s, \sigma_i^s$ (or ρ_i^s).

As mentioned in the previous section, the price equation adds no information when the Cobb-Douglas parameters α_{ij}^s are exogenously given so as to satisfy *Assumption 1*. In contrast, specifying the exogenous factor inputs, L_j^s and K_j^s , is equivalent to determining the physical input coefficients, a_{Lj}^s and a_{Kj}^s . Thus it is not reasonable to determine those physical coefficients independently. As for the intermediate inputs in non-transport sectors, it is possible to provide physical technology exogenously and thus to regard the monetary coefficient as being endogenous.

The equilibrium conditions controlling the endogenous variables can be written as summarized in Table 2. In the minimal system, the number of respective equations will be 72, which coincides with the number of endogenous variables.

3.2 System of Homogeneous Equations

The model is composed of a system of nonlinear simultaneous equations. However, the equations are not uniformly interconnected. Several blocks of equations can be identified that are relatively independent from other blocks. In essence, the model calculations proceed in the following manner.

- (a) The FOB prices p_i^s can be solved independently from (25) in conjunction with the definitions of trade coefficients (15) and CIF prices (5).

Table 2: Equilibrium conditions

	Equations	Numbers
Price system (25)	$\sum_{i \neq J} \frac{q_i^s}{p_j^s} a_{ij}^s = 1 - \alpha_{Lj}^s - \alpha_{Kj}^s$	$R(J-1)$
Trade coefficient (15)	$t_{i(\neq J)}^{rs} = \frac{(p_i^r + c_i^{rs})^{-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}}$	$R^2(J-1)$
Composite price (5)	$q_{i(\neq J)}^s = \sum_r (p_i^r + c_i^{rs}) t_i^{rs}$	$R(J-1)$
Monetary input coeffs. (7)	$\alpha_{ij}^s = \frac{q_i^s}{p_j^s} a_{ij}^s \quad (i, j \neq J)$	$R(J-1)^2$
Non-transport sector (27)	$X_i^r = \sum_s \frac{t_i^{rs}}{q_i^s} (\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s)$	$R(J-1)$
Transport sector (28)	$p_J^r X_J^r = \sum_{i \neq J} \sum_s \frac{c_i^{rs} t_i^{rs}}{q_i^s} (\sum_j \alpha_{ij}^s p_j^s X_j^s + \beta_i^s W^s)$	R
Factor prices (29)	$\omega_j^s = \alpha_{Lj}^s p_j^s X_j^s / L_j^s, \gamma_j^s = \alpha_{Kj}^s p_j^s X_j^s / K_j^s$	$2RJ$
Households (30)	$W^s = \sum_j \omega_j^s L_j^s + \sum_j \gamma_j^s K_j^s$	R

- (b) The monetary input coefficients for non-transport sectors can be calculated by (7) once the FOB and CIF prices are determined.
- (c) The output equations (27) and (28) are solved simultaneously to obtain the outputs $X_{i \neq J}^r$ and $p_J^r X_J^r$.
- (d) The factor prices ω_j^s and γ_j^s are derived directly from (29), and the final demand expenditure W^s is calculated from these factor prices.

The important parts in the solution procedure are then (a) and (c) in the above, where (a) is solved by non-linear minimization with respect to p_i^s :

$$\begin{aligned}
& \min_{p_i^s} \sum_{j \neq J, s} (1 - \alpha_{Lj}^s - \alpha_{Kj}^s - \sum_{i \neq J} \frac{q_i^s}{p_j^s} a_{ij}^s)^2 \\
& \text{subject to } q_i^s = \frac{\sum_r (p_i^r + c_i^{rs})^{1-\sigma_i^s}}{\sum_r (p_i^r + c_i^{rs})^{-\sigma_i^s}} \quad (i = 1, \dots, J-1, s = 1, \dots, R). \quad (31)
\end{aligned}$$

If the optimal value for the problem (31) becomes zero, the equilibrium FOB prices for non-transport sectors are simultaneously obtained.¹⁵

¹⁵The objective function corresponds to the sum of squared residuals for the equilibrium condition (25). In our formulation, the problem is independent of the (physical) outputs, and can be simply solved by the quasi-Newton algorithm.

Regarding (c), let X and W , respectively, denote the column vectors of outputs (monetary outputs for the transport) and regional incomes:

$$X = (X_1^1 \dots X_{J-1}^1 p_J^1 X_J^1 | \dots | X_1^s \dots X_{J-1}^s p_J^s X_J^s | \dots | X_1^R \dots X_{J-1}^R p_J^R X_J^R)',$$

$$W = (W^1 \dots W^s \dots W^R)'$$

Further define a $RJ \times RJ$ matrix B_1 and a $RJ \times R$ matrix B_2 :

$$B_1 = \begin{pmatrix} B_1^{11} & \dots & B_1^{1s} & \dots & B_1^{1R} \\ \vdots & \ddots & \vdots & & \vdots \\ B_1^{r1} & \dots & B_1^{rs} & \dots & B_1^{rR} \\ \vdots & & \vdots & \ddots & \vdots \\ B_1^{R1} & \dots & B_1^{Rs} & \dots & B_1^{RR} \end{pmatrix} \quad \text{and} \quad B_2 = \begin{pmatrix} B_2^{11} & \dots & B_2^{1s} & \dots & B_2^{1R} \\ \vdots & \ddots & \vdots & & \vdots \\ B_2^{r1} & \dots & B_2^{rs} & \dots & B_2^{rR} \\ \vdots & & \vdots & \ddots & \vdots \\ B_2^{R1} & \dots & B_2^{Rs} & \dots & B_2^{RR} \end{pmatrix}, \quad (32)$$

with the cells B_1^{rs} and B_2^{rs} respectively defined as follows:

$$B_1^{rs} = \begin{pmatrix} (t_1^{rs}/q_1^s)\alpha_{11}^s p_1^s & \dots & (t_1^{rs}/q_1^s)\alpha_{1,J-1}^s p_{J-1}^s & (t_1^{rs}/q_1^s)\alpha_{1J}^s \\ \vdots & \ddots & \vdots & \vdots \\ (t_{J-1}^{rs}/q_{J-1}^s)\alpha_{J-1,1}^s p_1^s & \dots & (t_{J-1}^{rs}/q_{J-1}^s)\alpha_{J-1,J-1}^s p_{J-1}^s & (t_{J-1}^{rs}/q_{J-1}^s)\alpha_{J-1,J}^s \\ \sum_{i \neq J} (c_i^{rs} t_i^{rs}/q_i^s)\alpha_{i1}^s p_1^s & \dots & \sum_{i \neq J} (c_i^{rs} t_i^{rs}/q_i^s)\alpha_{i,J-1}^s p_{J-1}^s & \sum_{i \neq J} (c_i^{rs} t_i^{rs}/q_i^s)\alpha_{iJ}^s \end{pmatrix}, \quad (33)$$

$$B_2^{rs} = \left((t_1^{rs}/q_1^s)\beta_1^s \dots (t_{J-1}^{rs}/q_{J-1}^s)\beta_{J-1}^s \sum_{i \neq J} (c_i^{rs} t_i^{rs}/q_i^s)\beta_i^s \right)'. \quad (34)$$

Then the output equations (27) and (28) can be written in a matrix form, $X = B_1 X + B_2 W$, which can be solved by using the Leontief inverse:

$$X = (I - B_1)^{-1} B_2 W. \quad (35)$$

If each region is closed in terms of value added and final demand, it is possible to describe W^s as a function of outputs X_j^s by using (29):

$$W^s = \sum_j (\alpha_{Lj}^s + \alpha_{Kj}^s) p_j^s X_j^s. \quad (36)$$

Then the term $B_2 W$ in (35) can be rewritten as $B_3 X$ with another $RJ \times RJ$ matrix T_3 having a structure analogous to B_1 in (32). The cells of B_3 are defined as follows:

$$B_3^{rs} = \begin{pmatrix} \frac{t_1^{rs}}{q_1^s} \beta_1^s (\alpha_{L1}^s + \alpha_{K1}^s) p_1^s & \dots & \frac{t_1^{rs}}{q_1^s} \beta_1^s (\alpha_{L,J-1}^s + \alpha_{K,J-1}^s) p_{J-1}^s & \frac{t_1^{rs}}{q_1^s} \beta_1^s (\alpha_{LJ}^s + \alpha_{KJ}^s) \\ \vdots & \ddots & \vdots & \vdots \\ \frac{t_{J-1}^{rs}}{q_{J-1}^s} \beta_{J-1}^s (\alpha_{L1}^s + \alpha_{K1}^s) p_1^s & \dots & \frac{t_{J-1}^{rs}}{q_{J-1}^s} \beta_{J-1}^s (\alpha_{L,J-1}^s + \alpha_{K,J-1}^s) p_{J-1}^s & \frac{t_{J-1}^{rs}}{q_{J-1}^s} \beta_{J-1}^s (\alpha_{LJ}^s + \alpha_{KJ}^s) \\ \sum_{i \neq J} \frac{c_i^{rs} t_i^{rs}}{q_i^s} \beta_i^s (\alpha_{L1}^s + \alpha_{K1}^s) p_1^s & \dots & \sum_{i \neq J} \frac{c_i^{rs} t_i^{rs}}{q_i^s} \beta_i^s (\alpha_{L,J-1}^s + \alpha_{K,J-1}^s) p_{J-1}^s & \sum_{i \neq J} \frac{c_i^{rs} t_i^{rs}}{q_i^s} \beta_i^s (\alpha_{LJ}^s + \alpha_{KJ}^s) \end{pmatrix}. \quad (37)$$

Accordingly, (35) can be rewritten as a homogeneous system of linear equations.

$$(I - B_1 - B_3)X = 0. \quad (38)$$

The above equation always has the trivial solution $X = 0$. For a non-trivial solution to exist, the matrix $(I - B_1 - B_3)$ must be singular. Very much like the Walrasian equilibrium, where one equation becomes redundant, the present system (38) is also linearly dependent. In this case, however, all the prices are determined relative to the exogenously given transport costs so that a numéraire cannot be specified. Instead, one physical output in a region can be arbitrarily chosen. In contrast, no meaningful solution exists for the system when $(I - B_1 - B_3)$ is non-singular,

4 Numerical Analyses

4.1 Benchmark Equilibrium

As the benchmark case, the minimal world ($R = 3$ and $J = 3$) is considered. The regions are symmetric in parameters and exogenous variables, whose values are summarized as follows:

- Factor allocation: $L_i^s = K_i^s = 100$ ($i = 1, 2$), $L_3^s = K_3^s = 40$.
- Transport costs: c_i^{rs} common to both non-transport goods as shown in Figure 1.
- Input coefficients: $a_{ij}^s = 0.25$ ($j = 1, 2$), $\alpha_{i3} = 0.25$
- Factor inputs: $\alpha_{Li}^s = \alpha_{Ki}^s = 0.2$ ($i = 1, 2$), $\alpha_{L3}^s = \alpha_{K3}^s = 0.25$.
- Final demand structure and Armington elasticity: $\beta_i^s = 0.5$, $\sigma_i^s = 1.0$ ($i = 1, 2$).

Since one output in a region can arbitrarily be specified, X_1^1 is set at 10, and the corresponding line is deleted from (38). The endogenous variables for the benchmark case are summarized in Table 3.

Table 3: The benchmark solution.

$p_1^r = 0.8217$	$q_1^r = 0.9860$	$X_1^r = 10$	$\omega_1^r = 0.01643$	$\gamma_1^r = 0.01643$
$p_2^r = 0.8217$	$q_2^r = 0.9860$	$X_2^r = 10$	$\omega_2^r = 0.01643$	$\gamma_2^r = 0.01643$
$p_3^r X_3^r = 3.2868$		$W^r = 8.2170$	$\omega_3^r = 0.02054$	$\gamma_3^r = 0.02054$

4.2 Symmetric Modifications

First we consider cases in which parameters and exogenous variables are modified one at a time. This is intended to see their effects on the solution while maintaining the symmetric setting.

- (1) Labor allocations L_i^r are halved: $L_i^s = 50 (i = 1, 2), L_3^s = 20$,
- (2) Factor inputs for good 1 are changed: $\alpha_{L1}^r = 0.15, \alpha_{K1}^r = 0.25$.
- (3) Technology to produce good 1 is changed: $\begin{pmatrix} a_{11}^s & a_{12}^s \\ a_{21}^s & a_{22}^s \end{pmatrix} = \begin{pmatrix} 0.10 & 0.20 \\ 0.30 & 0.20 \end{pmatrix}$.
- (4) Transport costs for good 2 are doubled: $c_2^{rs} = \begin{pmatrix} 0.20 & 0.40 & 0.40 \\ 0.40 & 0.20 & 0.40 \\ 0.40 & 0.40 & 0.20 \end{pmatrix}$.
- (5) The elasticity of good 2 is increased: $\sigma_2^r = 5.0$.¹⁶
- (6) Both goods are highly substitutive: $\sigma_1^r = \sigma_2^r = 20$.
- (7) Both goods are barely substitutive: $\sigma_1^r = \sigma_2^r = 0.1$.

The solutions of individual cases are summarized in Table 4, where only the variables that differ from the benchmark case are described. First it is clear that halving the labor allocations only makes the wages double to compensate for the labor scarcity. This will not affect the other markets. Similarly, the change in factor inputs in sector 1, case (2), affects the factor prices in that sector. In case (3), the changes in production technology in a sector affects the prices

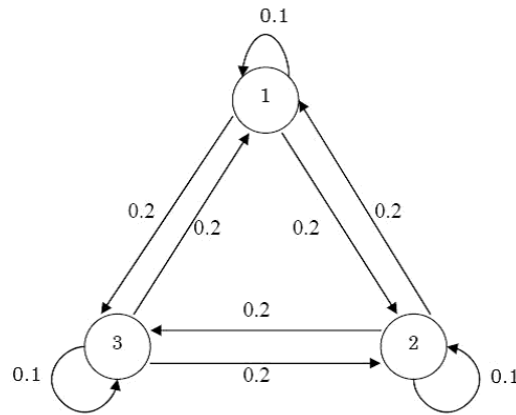


Figure 1: The study regions with the benchmark transport costs c_i^{rs} ($i = 1, 2$).

¹⁶ $\sigma_i^s = \infty$ (or $\rho_i^s = -1$) represents perfect substitution, which is supposed for Walrasian equilibrium, and $\sigma_i^s = 0$ (or $\rho_i^s = \infty$) implies a production or utility function of Leontief's type.

Table 4: Summary of symmetric modifications.

Case (1)	p_1^r : unch.	q_1^r : unch.	X_1^r : unch.	$\omega_1^r = 0.03287$	γ_1^r : unch.
	p_2^r : unch.	q_2^r : unch.	X_2^r : unch.	$\omega_2^r = 0.03287$	γ_2^r : unch.
	$p_3^r X_3^r$: unchanged		W^r : unch.	$\omega_3^r = 0.04109$	γ_3^r : unch.
Case (2)	p_1^r : unch.	q_1^r : unch.	X_1^r : unch.	$\omega_1^r = 0.01233$	$\gamma_1^r = 0.02054$
	p_2^r : unch.	q_2^r : unch.	X_2^r : unch.	ω_2^r : unch.	γ_2^r : unch.
	$p_3^r X_3^r$: unchanged		W^r : unch.	ω_3^r : unch.	γ_3^r : unch.
Case (3)	$p_1^r = 0.3236$	$q_1^r = 0.4851$	X_1^r : unch.	$\omega_1^r = 0.00647$	$\gamma_1^r = 0.00647$
	$p_2^r = 0.3236$	$q_2^r = 0.4851$	$X_2^r = 12$	$\omega_2^r = 0.00777$	$\gamma_2^r = 0.00777$
	$p_3^r X_3^r = 3.5597$		$W^r = 4.628$	$\omega_3^r = 0.02225$	$\gamma_3^r = 0.02225$
Case (4)	$p_1^r = 1.2311$	$q_1^r = 1.3961$	X_1^r : unch.	$\omega_1^r = 0.02462$	$\gamma_1^r = 0.02462$
	$p_2^r = 1.2311$	$q_2^r = 1.5585$	$X_2^r = 9.465$	$\omega_2^r = 0.02331$	$\gamma_2^r = 0.02331$
	$p_3^r X_3^r = 4.7493$		$W^r = 11.960$	$\omega_3^r = 0.02968$	$\gamma_3^r = 0.02968$
Case (5)	$p_1^r = 0.7959$	$q_1^r = 0.9602$	X_1^r : unch.	$\omega_1^r = 0.01592$	$\gamma_1^r = 0.01592$
	$p_2^r = 0.7959$	$q_2^r = 0.9500$	$X_2^r = 10.053$	$\omega_2^r = 0.01600$	$\gamma_2^r = 0.01600$
	$p_3^r X_3^r = 3.1919$		$W^r = 7.9803$	$\omega_3^r = 0.01995$	$\gamma_3^r = 0.01995$
Case (6)	$p_1^r = 0.5516$	$q_1^r = 0.6619$	X_1^r : unch.	$\omega_1^r = 0.01103$	$\gamma_1^r = 0.01103$
	$p_2^r = 0.5516$	$q_2^r = 0.6619$	X_2^r : unch.	$\omega_2^r = 0.01103$	$\gamma_2^r = 0.01103$
	$p_3^r X_3^r = 2.2064$		$W^r = 5.5159$	$\omega_3^r = 0.01379$	$\gamma_3^r = 0.01379$
Case (7)	$p_1^r = 0.8322$	$q_1^r = 0.9986$	X_1^r : unch.	$\omega_1^r = 0.01664$	$\gamma_1^r = 0.01664$
	$p_2^r = 0.8322$	$q_2^r = 0.9986$	X_2^r : unch.	$\omega_2^r = 0.01664$	$\gamma_2^r = 0.01664$
	$p_3^r X_3^r = 3.3288$		$W^r = 8.322$	$\omega_3^r = 0.02081$	$\gamma_3^r = 0.02081$

Note: "unch.": unchanged from the benchmark results.

of both goods and factors. The direction of movement will depend on the way the technology changes; in the illustrated case, all the prices decrease except for those related to transport, and so do the incomes. An increase in shipping costs of good 2, case (4), will raise the prices of both goods and of the factors, but those in sector 1 increase more. However, the transport sector gains most from the fare increases, and this leads to higher incomes.

The remaining three cases are designed to investigate the effects of Armington elasticity. In case (5), only good 2 becomes more substitutive. The prices of both goods, the factor prices, and incomes decrease, but the reduction in the factor prices are slightly moderate for the substitutive good 2. Cases (6) and (7) show the equilibria obtained when both goods are highly substitutive or barely substitutive, respectively. Prices and income are lower when goods

are highly substitutive, and prices and income are higher when goods are barely substitutive. This seems to imply that people can enjoy higher income when regions are less competitive. However, when nominal income is divided by the FOB price, $W^r/p_i^r = 10$ is obtained regardless of elasticity condition. This indicates that the Armington elasticity will not affect the real incomes in the symmetric equilibria.

The trade coefficient matrices, $T_i = \{t_i^{rs}\}$, for the benchmark equilibrium and for the cases (6) and (7) can be summarized as follows:

$$T_i = \begin{pmatrix} 0.3566 & 0.3217 & 0.3217 \\ 0.3217 & 0.3566 & 0.3217 \\ 0.3217 & 0.3217 & 0.3566 \end{pmatrix}, T_i^{(6)} = \begin{pmatrix} 0.8968 & 0.0516 & 0.0516 \\ 0.0516 & 0.8968 & 0.0516 \\ 0.0516 & 0.0516 & 0.8968 \end{pmatrix}, T_i^{(7)} = \begin{pmatrix} 0.3322 & 0.3322 & 0.3322 \\ 0.3322 & 0.3356 & 0.3322 \\ 0.3322 & 0.3322 & 0.3356 \end{pmatrix}.$$

It is clear that the domestic demand becomes dominant, as compared with the benchmark pattern, when the goods are highly substitutive (case (6)), and the trade pattern levels out when the production and utility functions are nearly Leontief type (case (7)).

4.3 Asymmetric Cases

In this subsection, we examine how solutions are affected by the introduction of some typical regional asymmetries. The following specific modifications are applied to the benchmark case:

(8) Asymmetric factor allocation: $L_i^1 = K_i^2 = 100$, $K_i^1 = L_i^2 = L_i^3 = K_i^3 = 50$ ($i = 1, 2$),
 $L_3^1 = K_3^2 = 40$, $K_3^1 = L_3^2 = L_3^3 = K_3^3 = 20$.

(9) Region 3 is remote from other regions: $c_i^{rs} = \begin{pmatrix} 0.10 & 0.20 & 0.40 \\ 0.20 & 0.10 & 0.40 \\ 0.40 & 0.40 & 0.10 \end{pmatrix}$ ($i = 1, 2$).

(10) Region 1 and 2 are more substitutive: $\sigma_i^1 = \sigma_i^2 = 5.0$ ($i = 1, 2$).

The equilibrium results are summarized in Table 5, the construction of which is similar to Table 4, but each cell in the latter is divided into three sub-cells to represent the regional distribution of each variable. First, the factors are allocated asymmetrically among regions in case (8). As in case (1), differences in factors affect the corresponding factor prices only, and the rest of the variables, including product prices and incomes, are unchanged. Such outcomes are a direct consequence of *Basic Assumption* (b) about immobile factors, and the outcomes are not sustained when this assumption is relaxed.

The remaining two cases represent intrinsically asymmetric equilibria. In case (9), region 3 is located relatively far from the other regions. This causes higher prices and higher incomes

Table 5: Summary of asymmetric examples.

Case	p_1^1	p_1^2	p_1^3	q_1^1	q_1^2	q_1^3	X_1^1	X_1^2	X_1^3	ω_1^1	ω_1^2	ω_1^3	γ_1^1	γ_1^2	γ_1^3
	p_2^1	p_2^2	p_2^3	q_2^1	q_2^2	q_2^3	X_2^1	X_2^2	X_2^3	ω_2^1	ω_2^2	ω_2^3	γ_2^1	γ_2^2	γ_2^3
	$p_3^1 X_3^1$		$p_3^2 X_3^2$		$p_3^3 X_3^3$		W^1	W^2	W^3	ω_3^1	ω_3^2	ω_3^3	γ_3^1	γ_3^2	γ_3^3
(8)	unchanged		unchanged		unchanged		unch.		0.03287	0.03287	0.03287	unch.	0.03287		
	unchanged		unchanged		unchanged		unch.		0.03287	0.03287	0.03287	unch.	0.03287		
	unchanged				unchanged		unch.		0.04109	0.04109	0.04109	unch.	0.04109		
(9)	1.1918	1.2527	1.4301	1.5032	unch.		9.192	0.02384		0.02303	0.02384		0.02303		
	1.1918	1.2527	1.4301	1.5032	unch.		9.192	0.02384		0.02303	0.02384		0.02303		
	4.4548			5.2619		11.761		11.842		0.02784		0.03289		0.02784	
(10)	0.7793	0.7896	0.9352	0.9475	unch.		8.046	0.01559		0.01271	0.01559		0.01271		
	0.7793	0.7896	0.9352	0.9475	unch.		8.046	0.01559		0.01271	0.01559		0.01271		
	3.0669			2.6432		7.768		6.404		0.01917		0.01652		0.01917	

than in the benchmark case. Comparing regions, the product prices will be higher in the remote region, and the nominal income must also be higher to afford them. However, mixed results are obtained regarding the factor prices: those in region 3 are higher in the transport sector, but slightly lower in non-transport sectors. Meanwhile, case (10) illustrates the effects of asymmetric elasticity, where region 3 is relatively less substitutive. In contrast to case (3), both prices and incomes are lower than in the benchmark case. However, the product prices in region 3 are higher than those in other regions, despite its lower income resulting from lower factor prices there. These results suggest that people in the less substitutive region may be worse off than those in more substitutive regions.

The trade coefficient matrices for cases (9) and (10) are as follows:

$$T_i^{(9)} = \begin{pmatrix} 0.3641 & 0.3341 & 0.3175 \\ 0.3341 & 0.3641 & 0.3175 \\ 0.3019 & 0.3019 & 0.3650 \end{pmatrix} \quad \text{and} \quad T_i^{(10)} = \begin{pmatrix} 0.4678 & 0.2730 & 0.3225 \\ 0.2730 & 0.4678 & 0.3225 \\ 0.2592 & 0.2592 & 0.3550 \end{pmatrix}.$$

It is clear that the purchases from the remote region 3 in case (9) are less than those from nearer regions, and so region 3 relies more heavily on domestic products. For case (10), the substitutive regions purchase more domestic products while region 3 purchases goods more evenly from all the regions. Since the transport costs are paid at the origin under *Basic Assumption* (d), the sales of the transport sector in region 3 are higher in case (9) and lower in case (10) than those of other regions. In other words, both transport improvement and increased substitutability in a poorer region will narrow the differences among the regions, but

both product prices and incomes will decline at the same time. ¹⁷

5 Concluding Remarks

In this paper, a simple SCGE model that explicitly considers the transport sector was formulated for a closed world. The model can be divided into two separable blocks: a set of non-linear price equations and a multi-regional input-output system. The input-output system becomes a set of homogeneous equations, and when its matrix representation includes a singular matrix, $(I - B_1 - B_3)$ in (38), it attains a non-trivial solution. Such systems are known to be sensitive to the consistency of formulas. In fact, if (4)' is replaced by a formula based on negative exponentials:

$$t_i^{rs} = \frac{\exp(-\delta_i^s(p_i^r + c_i^{rs}))}{\sum_r \exp(-\delta_i^s(p_i^r + c_i^{rs}))},$$

then the non-linear problem (31) will not converge to zero, and $(I - B_1 - B_3)$ will be non-singular. ¹⁸

Several cases, both symmetric and asymmetric, were compared with the benchmark results, but many other cases can also be studied. For example, regional differences in technology a_{ij}^s and preference β_i^s were not shown. It was shown that the effects of changes in factor allocations and factor inputs are limited to adjustments of corresponding factor prices. This conclusion is directly tied to the assumption that factors are immobile. One possible way to relax this assumption is to assume that labor is mobile among sectors, and capital is mobile among regions:

$$L^r = \sum_i L_i^r \quad \text{and} \quad K_i = \sum_r K_i^r.$$

Then the corresponding factor prices will be determined as ω^r for labor and γ_i for capital. In other words, the changes in factor inputs will propagate to other sectors or regions through factor reallocation.

The transport costs are exogenous and effectively work as the numéraire in the present model. Such a setup is convenient for evaluating the impacts of transport improvements, and it is impractical to determine prices of individual transport services diversified by the origin and destination pairs. One way to endogenize the prices of transport services is to consider some

¹⁷Transport improvement works similar to tariff reduction, except that tariff reduction induces a redistribution effect. Increased substitution may be regarded as globalization of commodity trade.

¹⁸The problem of formulation consistency can somehow be averted in an open system, where the existing residuals may be attributed to the rest of the world.

virtual traffic load, such as distance, associated with each OD pair, d^{rs} . In such a case, the price τ_i^r of transport services would be determined for each good, and this would add $R(J - 1)$ endogenous variables to the model. A more practical method would be to consider virtual traffic loads that also depend on the payload, d_i^{rs} , which corresponds to the fact that fuel consumption for carrying a good between two points depends not only on distance but also on weight. Then, the number of endogenous variables τ^r , the price of unit transport service rendered in region r , would be reduced to R . The relation between the transport cost and traffic load in such a model can be written as $c_i^{rs} = \tau^r d_i^{rs}$.

The purpose of this study is to emphasize the importance of the transport sector in determining a spatial price equilibrium, and to show that a model based on physical variables can be useful in deriving consistent formulations. Though the model presented here might be too simple to describe real world economies, it is hoped that the model will contribute to the understanding of the structure of SCGE models.

Appendix A: Glossary

Table A.1: Symbols in the Paper

Symbols	Definitions
X_j^s	production of industry j in region s
x_{ij}^{rs}	interregional input of good i from region r to region s , used in industry j
D_{ij}^s	composite intermediate input goods i of industry j in region s
L_j^s	labor input of industry j in region s
K_j^s	capital input of industry j in region s
π_j^s	profit of industry j in region s
p_j^s	FOB price of good j produced in region s
q_i^s	CIF price of good i used in region s
c_i^{rs}	transport cost for shipping goods i from region r to region s
U^s	utility of households in region s
y_i^{rs}	households' consumption of region s for good i produced in region r
y_i^s	composite final consumption of good i in region s
W^s	income of households in region s
A_j^s	scale parameter in production function
ρ_{ij}^s	substitution parameter used in production function
α_{ij}^s	regional input coefficient of intermediate goods, measured in monetary terms
α_{Lj}^s	regional input coefficient of labor, measured in monetary terms
α_{Kj}^s	regional input coefficient of capital, measured in monetary terms
ω_j^s	wage rate of industry j in region s
γ_j^s	capital rent of industry j in region s
a_{ij}^s	regional input coefficient of intermediate goods, measured in physical terms
t_i^{rs}	trade coefficient (physical term)
ρ_{ih}^s	substitution parameter used in utility function s
β_i^s	final demand parameter (monetary term)
σ_i^s	parameter representing the elasticity of substitution s

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