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Neutrality in the Choice of Number of Firms or Level of Fixed Costs in Calibrating an Armington-Krugman-Melitz Encompassing Module for Applied General Equilibrium Models

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March 2014

Abstract

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Keywords: applied general equilibrium; monopolistic competition; firm heterogeneity; calibration; neutrality

JEL classification: C63, C68, D58, F12, L11

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1. Introduction

As the global economy has become increasingly interdependent, thousands of applied general equilibrium (AGE) analyses have been utilized to evaluate regional trade agreements and economic partnership arrangements, and some model builders have attempted to incorporate theoretical information on intra-industry trade to account for economies of scale and imperfect competition. In conventional AGE models, the so-called "Armington assumption" has been widely adopted to handle cross-hauling, which is often observed in real data, between developed economies that have similar technologies and factor endowments.[1] Since this can be regarded as an *ad hoc* approach and sometimes can cause awkward simulation results from its tendency to underestimate efficiency gains, some models such as Francois and Roland-Holst (1997), Francois (1998), and Roson (2006) have introduced theoretical illustrations of product differentiation in their analytical models as presented in the pioneering work of Krugman.

Krugman (1980) focused on two sources of efficiency gains that result from reducing trade barriers: cost reductions brought by economies of scale and increased variety obtained through additional imports. In the steady advance of new trade theory that followed, one of the most successful extensions of his work had been done by Melitz (2003). He appended another source of efficiency gains, namely, the reallocation of resources resulting from endogenous productivity growth among heterogeneous firms. In the AGE research community, Zhai (2008) introduced a Melitz-type specification into an AGE model as an alternative to the Armington approach. Then, Balistreri and Rutherford (2012) prepared a comprehensive guide to the treatment of the three approaches by Armington, Krugman, and Melitz, and Dixon and Rimmer (2012) finally developed a generalized supermodel that includes those three types of model as special cases.

The supermodel, called "Armington-Krugman-Melitz encompassing (AKME) model", replaces the interregional trade part of a multi-regional AGE model that links gross output in a source region with absorption in a destination.[2] When one tries to introduce some kind of special factors in an AGE model, it is often the case that additional information or data has to be prepared. Contrary to this expectation, we found that extending a standard AGE model with an Armington-type trade specification to include Melitz-type monopolistic competition and heterogeneous firms basically requires only one kind of additional information, as far as a calibration procedure is utilized to parameterize a

[1] Armington (1969).
[2] In this meaning, we use the term "module" instead of "model".

model. To be a Krugman-type, nothing is needed. The purpose of this paper is to explain how an AKME module can be calibrated, and to clarify these facts. Our findings imply that model builders in applied economics could possibly be released from the time-consuming burden of data collection and reconciliation, and have chance to fully enjoy the featured properties of the theoretical models invented by Krugman (1980) and Melitz (2003) in practical policy evaluations at low cost. In this regard, cost performance of introducing an AKME module in a multi-regional AGE model is extremely high.

The reminder of this paper is organized as follows. Section 2 illustrates an AKME module based on Dixon and Rimmer (2012). Since they have not yet shown any concrete process of parameterization, Section 3 explains how the module can be calibrated and then shows two important characteristics of the Melitz-type specification. Section 4 concludes the paper.

2. The Armington-Krugman-Melitz Encompassing (AKME) Module

In this section, we review details of the supermodel proposed by Dixon and Rimmer (2012), which includes the Armington, Krugman, and Melitz models as special cases. While their original model is characterized by the dual approach, we take the primal approach in some part to learn the model from a different angle. Furthermore, every effort has been made to keep counterpart relationships between quantity and price variables clearly shown in equations. Hence, manipulations that may make the counterpart relationships unclear, such as substitution to derive a demand function, are avoided as much as possible, leaving first order conditions (FOCs) as they are.

Let us start with aggregator functions for imported products from firms indexed e operating in region r:

$$\tilde{Q}_{rs} = \left\{\delta_{rs}^{T} \sum_{e} \hat{Q}_{ers}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)}; \quad (1)$$

and

$$X_s + C_s = \theta_s^T \left\{\sum_r \tilde{Q}_{rs}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)}, \quad (2)$$

where

$\hat{Q}_{ers}$ is the distribution (trade flows) of commodity from firm e operating in region r to region s;

$\tilde{Q}_{rs}$ is the quantity of commodity distributed from all firms operating in region r

to region s;

X_s is intermediate demand for commodity in region s;

C_s is final demand for commodity in region s;

$\sigma^T > 1$ is the elasticity of substitution between varieties from different sources (firm e and region r);[3]

δ_{rs}^T is the weight parameter that reflects preference of region s with respect to the region of origin r; and

θ_s^T is the scaling factor of measuring units.[4]

Economic agents in region s choose $\hat{Q}_{ers}$ to minimize the total purchase value of commodities subject to (1) and (2). This problem can be expressed as

$$\min \quad \sum_e \sum_r (1+\tau_{rs}) \hat{p}_{ers} \hat{Q}_{ers}$$

$$\text{s.t.} \quad X_s + C_s = \theta_s^T \left\{ \sum_r \delta_{rs}^T \sum_e \hat{Q}_{ers}^{(\sigma^T-1)/\sigma^T} \right\}^{\sigma^T/(\sigma^T-1)}, \tag{3}$$

where

$\hat{p}_{ers}$ is the differentiated sales price of commodity produced by firm e operating in region r and sold to region s, exclusive of transportation margin and import tariff; and

τ_{rs} is the rate of transportation margin plus import tariff.

Equation (3) is derived by substituting (1) into (2). Setting the Lagrange multiplier for (3) as p_s^M, we get the following FOC:

$$(1+\tau_{rs})\hat{p}_{ers} = \delta_{rs}^T p_s^M (\theta_s^T)^{(\sigma^T-1)/\sigma^T} \left(\frac{X_s + C_s}{\hat{Q}_{ers}} \right)^{1/\sigma^T}. \tag{4}$$

Since the value of a Lagrange multiplier can be interpreted as the shadow price at the optimal solution, p_s^M represents the market price of commodity inclusive of transportation margin and import tariff.

Aggregate total profit of all firms operating in region r can be expressed as

$$\pi_r = \sum_{e \in R(rs)} \sum_s \hat{\pi}_{ers} - \sum_e p_r^W H_r, \tag{5}$$

where

$R(rs)$ is the set of active firms that sell products on the r-s link;

$\hat{\pi}_{ers}$ is the contribution of firm e operating in region r to the total profit from

[3] Notice that the same substitution elasticity σ^T is utilized in Equations (1) and (2).

[4] This parameter is needed to pass the replication test which verifies whether an AGE model can reproduce the state captured by the benchmark data when there is no policy change (the reference run). For example, think about the case we have a data set which includes expenditures for two kinds of commodity, 1 and 1, and total expenditure 2. If we assume a Cobb-Douglas type function to aggregate these two commodities to make a composite good, we need to equate 2 with $1^{0.5} \cdot 1^{0.5}$. In this example, the scaling factor $\theta = 2$ is required in order to satisfy $2 = \theta \cdot 1^{0.5} \cdot 1^{0.5}$.

its sales to region s;

p_r^w is the wholesale price of products; and

H_r is the fixed cost, measured in units of gross output (composite input), necessary to establish a firm in region r.

Next, let us see the relation between production and sales. Imagine that a fixed cost is required to a firm to establish a dealer section (sales department) to make sales of the product in a local market. In such case, a temporal profit $\hat{\pi}_{ers}$ can be expressed as

$$\hat{\pi}_{ers} = \hat{p}_{ers}\hat{Q}_{ers} - p_r^w \hat{Z}_{ers}, \tag{6}$$

using gross output $\hat{Z}_{ers}$ produced by firm e operating in region r and sold in region s. Note that we presume the fixed costs in this model are measured in units of gross output (composite input). The transformation of gross output $\hat{Z}_{ers}$ to regional trade flows $\hat{Q}_{ers}$ is assume to follow

$$\hat{Q}_{ers} = \max \left\{\hat{\varphi}_{ers}\left(\hat{Z}_{ers} - F_{rs}\right), 0\right\}. \tag{7}$$

Then, Equation (6) can be rewritten to

$$\hat{\pi}_{ers} = \hat{p}_{ers}\hat{Q}_{ers} - \frac{p_r^w}{\hat{\varphi}_{ers}}\hat{Q}_{ers} - p_r^w F_{rs}, \tag{8}$$

where

$\hat{\varphi}_{ers}$ is the productivity of firm e in region r selling its products to s; and

F_{rs} is the fixed cost, measured in units of gross output (composite input), necessary to make sales on the r-s link.

Firm e in region r chooses the price and quantity of sales in region s to maximize $\hat{\pi}_{ers}$. Then the sales price exclusive of transportation margin and import tariff is marked up as

$$\hat{p}_{ers} = \left(\frac{1}{1+\eta}\right)\frac{p_r^w}{\hat{\varphi}_{ers}}, \tag{9}$$

where η is related to the elasticity of substitution σ^T such that $\eta \equiv -1/\sigma^T$.

Using (4) and (9), we can rewrite (8) as

$$\hat{\pi}_{ers} = -\eta\left(\frac{1}{1+\eta}\right)^{1-\sigma^T}\left(\frac{p_r^w}{\hat{\varphi}_{ers}\,\theta_s^T}\right)^{1-\sigma^T}(X_s + C_s)\left(\frac{\delta_{rs}^T\, p_s^M}{1+\tau_{rs}}\right)^{\sigma^T} - p_r^w F_{rs}. \tag{10}$$

Therefore, (5) becomes

$$\pi_r = -\eta\left(\frac{1}{1+\eta}\right)^{1-\sigma^T}\sum_{e\in R(rs)}\sum_s\left(\frac{p_r^w}{\hat{\varphi}_{ers}\,\theta_s^T}\right)^{1-\sigma^T}(X_s + C_s)\left(\frac{\delta_{rs}^T\, p_s^M}{1+\tau_{rs}}\right)^{\sigma^T}$$
$$- \sum_s p_r^w \widetilde{N}_{rs} F_{rs} - p_r^w N_r H_r, \tag{11}$$

where

$\widetilde{N}_{rs}$ is the number of active firms operating in region r that sell products on the

r-s link; and

N_r is the number of firms registered in region r.

In Equation (11), all firms operating in region r are assumed to be identical that they have the same cost structure[5].

Next, transformation of total gross output Z_r can be expressed as

$$\sum_{e \in R(rs)} \sum_s \frac{\hat{Q}_{ers}}{\hat{\varphi}_{ers}} = Z_r - \sum_s \tilde{N}_{rs} F_{rs} - N_r H_r. \qquad (12)$$

By Equation (12), we assume that production is done by the industrial sector in which firms are operating, and the sector-wide production is divided and distributed through many dealers owned by firms. Then, Equation (12) replaces the transformation part of gross output into domestic goods and exports in standard AGE models.

Melitz (2003) defines the relation between the average productivity of active firms φ_{rs} and the cutoff productivity required at least to operate on the r-s link $\tilde{\varphi}_{rs}$ as

$$\varphi_{rs} = \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T - 1)} \tilde{\varphi}_{rs}, \qquad (13)$$

where γ is a shape parameter related to productivity such that $\gamma > \sigma^T - 1$.[6]

In addition, the proportion of registered but inactive firms $G_{rs} \in (0,1)$, whose productivity is insufficient to meet the minimum requirement, is defined as

$$\begin{aligned} G_{rs} &= 1 - \tilde{\varphi}_{rs}{}^{-\gamma} \\ &= 1 - \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{\gamma/(\sigma^T - 1)} \varphi_{rs}{}^{-\gamma}. \end{aligned} \qquad (14)$$

The cutoff productivity required at least for a firm in region r to sell its products to region s is determined at the level that satisfies $\hat{\pi}_{ers} = 0$. Using (10), we obtain

$$\begin{aligned} \tilde{\varphi}_{rs} = \frac{(-\eta)^{1/(1-\sigma^T)}}{1+\eta} &\left\{(\theta_s^T)^{\sigma^T - 1}(X_s + C_s)(\delta_{rs}^T\, p_s^M)^{\sigma^T}\right\}^{1/(1-\sigma^T)} \\ &\times \{(1 + \tau_{rs}) p_r^W\}^{\sigma^T/(\sigma^T - 1)} F_{rs}{}^{1/(\sigma^T - 1)}. \end{aligned} \qquad (15)$$

Using (4), (13), and (15), we obtain the average productivity of active firms:

$$\varphi_{rs} = \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T - 1)} \frac{(-\eta)^{1/(1-\sigma^T)}}{1+\eta} \left(\frac{p_r^W}{\hat{p}_{ers}}\right)^{\sigma^T/(\sigma^T - 1)} \left(\frac{F_{rs}}{\hat{Q}_{ers}}\right)^{1/(\sigma^T - 1)}. \qquad (16)$$

Rewriting (11) using φ_{rs} (the average productivity of active firms) and $\tilde{N}_{rs}$ (the number of active firms), we obtain

[5] As we will see later, all of the active (heterogeneous) firms are normalized by the average productivity (and thus become identical).

[6] For details, see Balistreri and Rutherford (2012).

$$\pi_r = -\eta \left(\frac{1}{1+\eta}\right)^{1-\sigma^T} \sum_s \widetilde{N}_{rs} \left(\frac{p_r^w}{\varphi_{rs}\,\theta_s^T}\right)^{1-\sigma^T} (X_s + C_s) \left(\frac{\delta_{rs}^T\, p_s^M}{1+\tau_{rs}}\right)^{\sigma^T}$$
$$- \sum_s p_r^w \widetilde{N}_{rs} F_{rs} - p_r^w N_r H_r. \qquad (17)$$

N_r (the number of registered firms) is determined at the level that satisfies $\pi_r = 0$. Using (4), (9), and (17), we obtain

$$\sum_s p_r^w \widetilde{N}_{rs} F_{rs} + p_r^w N_r H_r = -\eta \sum_s \hat{p}_{ers} \widetilde{N}_{rs} \hat{Q}_{ers}. \qquad (18)$$

Finally, equations that form an AKME module are summarized as follows:

$$X_s + C_s = \theta_s^T \left\{\sum_r \delta_{rs}^T \widetilde{N}_{rs} Q_{rs}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T-1)} \qquad \perp p_s^M; \quad (19)$$

$$(1+\tau_{rs}) p_{rs} = \delta_{rs}^T p_s^M (\theta_s^T)^{(\sigma^T-1)/\sigma^T} \left(\frac{X_s + C_s}{Q_{rs}}\right)^{1/\sigma^T} \qquad \perp Q_{rs}; \quad (20)$$

$$p_{rs} = \left(\frac{1}{1+\eta}\right) \frac{p_r^w}{\varphi_{rs}} \qquad \perp p_{rs}; \quad (21)$$

$$\sum_s \widetilde{N}_{rs} \frac{Q_{rs}}{\varphi_{rs}} = Z_r - \sum_s \widetilde{N}_{rs} F_{rs} - N_r H_r \qquad \perp p_r^W; \quad (22)$$

$$G_{rs} = 1 - \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{\gamma/(\sigma^T-1)} {\varphi_{rs}}^{-\gamma} \qquad \perp G_{rs}; \quad (23)$$

$$\varphi_{rs} = \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T-1)} \frac{(-\eta)^{1/(1-\sigma^T)}}{1+\eta} \left(\frac{p_r^w}{p_{rs}}\right)^{\sigma^T/(\sigma^T-1)} \left(\frac{F_{rs}}{Q_{rs}}\right)^{1/(\sigma^T-1)}$$
$$\perp \varphi_{rs}; \quad (24)$$

and

$$p_r^w \left(\sum_s \widetilde{N}_{rs} F_{rs} + N_r H_r\right) = -\eta \sum_s p_{rs} \widetilde{N}_{rs} Q_{rs} \qquad \perp N_r. \quad (25)$$

In some equations, $\hat{Q}_{ers}$, $\hat{p}_{ers}$, and $\hat{\varphi}_{ers}$ are respectively replaced with Q_{rs} (the average flows of traded commodity by active firm), p_{rs} (the differentiated sales price by firm exclusive of transportation margin and import tariff), and φ_{rs} (the average productivity of active firms). The perpendicular symbol "⊥" shows the corresponding relationships between variables and equations.[7] Equations (23) and (24) do not appear in either a Krugman- or Armington-type specification. Equation (25) also is dropped from an Armington–type specification. The module is included in the interregional trade part of a multi-regional AGE model that links Z_r (gross output in a source region r) with $(X_s + C_s)$ (absorption in a destination region s).

Melitz-type Specification: In a Melitz-type specification, the following two settings apply,

[7] To make it consistent with the original model presented by Melitz (2003), p_r^W should be given exogenously, endogenizing Z_r instead.

in addition to (19) through (25):

$$\eta = -\frac{1}{\sigma^T};$$

and

$$\tilde{N}_{rs} = (1 - G_{rs})N_r.$$

Krugman-type Specification: In a Krugman-type, the following four relations apply, in addition to (19) through (22), and (25):

$$F_{rs} = 0;$$

$$\eta = -\frac{1}{\sigma^T};$$

$$\varphi_{rs} = 1;$$

and

$$\tilde{N}_{rs} = N_r \qquad (\therefore G_{rs} = 0).$$

Armington-type Specification: In an Armington-type, the following four relations apply, in addition to (19) through (22):

$$F_{rs} = H_r = 0;$$

$$\eta = 0;$$

$$\varphi_{rs} = 1;$$

and

$$\tilde{N}_{rs} = N_r = 1 \qquad (\therefore G_{rs} = 0).$$

3. Parameterization

In this section, we explain the calibration procedures for parameterizing the Melitz- and Krugman-types of the module presented in Section 2, focusing on parameters and initial values of endogenous variables that are specific to the Melitz- and Krugman-types. Then, we shall see that basically we need just one kind of additional information in order to extend an Armington-type module to be the Melitz-type. It is information on γ (the shape parameter related to productivity).[8] Then, initial levels of G_{rs} (the proportion of inactive firms) and N_r (the number of registered firms), or parameter values of F_{rs} (fixed cost

[8] Balistreri, *et al*. (2011) implemented structural estimation of this shape parameter for a Pareto distribution, as well as the Melitz-type bilateral fixed cost and unobserved iceberg trade costs.

necessary to make sales on the r-s link) and H_r (fixed cost necessary to establish a firm in region r), can be set freely to any preferred value, just as the cases we derive quantities from given value data assuming some of the initial prices to be unity. Furthermore, a Krugman-type module can be parameterized without any additional data.[9] Let us start by calibrating a Melitz-type module, followed by the procedure for a Krugman-type.

3.1 Matching Theory with Data

While the trade specification by Armington (1969) assumed that varieties are differentiated by region of origin, the monopolistic competition models presented by Krugman (1980) and Melitz (2003) assume that an importer assesses variety expansion regardless of its source. These imply, as Ardelean (2006) has pointed out, an Armington-type specification eliminates the variety expansion channel of larger exporters fixing the number of varieties so that an exporter grows only through the intensive margin, while Krugman- and Melitz-types predict that the rate of variety expansion is proportional to the growth in the volume of exports so that an exporter grows only through the extensive margin.[10]

In the implementation process of an AGE model, we need to match the theoretical features shown above with benchmark data. There are two possible approaches as Hertel (2009) has shown. One way is to assume the existence of unobserved (iceberg) trade costs to fill the gap between observed and calculated trade flows given as a solution by an AGE model with symmetric preference for varieties among exporters in the replication test. This approach requires re-estimation of transportation margins based on a certain assumption. Another way is to include preference weights to capture differentiation among regions, such as home bias, just like the cases of Armington-type specifications.

In the previous studies, Zhai (2008) and Balistreri, *et al*. (2011) have taken the former approach. Zhai (2008) derived unobserved transportation margins on the international trade flows assuming that the domestic trade incurs no iceberg trade costs.[11] Balistreri, *et al*. (2011) took a strategy to econometrically estimate the whole set of parameters using a nonlinear structural estimation procedure. On the other hand, Balistreri and Rutherford

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[9] For more issues related to parameterization, see Zhai (2008), Balistreri, *et al*. (2011), and Balistreri and Rutherford (2012).

[10] There has been a discussion on the relationship between the number of export varieties, volume of export quantities, and total value of exports. For instance, Hummels and Klenow (2005) found that the number of export varieties explains only 60 percent of the difference in export values across regions.

[11] Careful consideration is required to apply this assumption when one is going to handle regions instead of countries. Assuming that intra-regional trade does not incur iceberg costs, no matter how long the distances of countries grouped in the same region are, might be unrealistic in some cases.

(2012) and Dixon, *et al*. (2013) have referred to possibilities of the latter approach.[12] While Dixon, *et al*. (2013) emphasized the importance of relaxing theoretical restrictions, they have not yet shown any concrete process of calibration. Balistreri and Rutherford (2012) have explained a part of calibration procedures in both approaches. To pursue more labor-saving and simpler way, making full use of information such that we are familiar with or relatively easy to have access to, we take the latter approach assuming non-existence of unobserved trade costs. In consequence, we include and calibrate preference weights δ_{rs}^{T} in Equation (19). In addition, we assume $\sum_r \delta_{rs}^{T} = 1$ since the volumes of preference weights are adjusted in the calibration process by the scaling factor θ_s^T to pass the replication test.

Note that the CES weights $\delta_{rs}^{T}\widetilde{N}_{rs}$ in Equation (19) are now endogenous in Melitz- and Krugman-types. One of the problems of Armington-type specifications pointed out in previous studies is that the CES weights are fixed and do not change in the long-run. Contrary, Krugman- and Melitz-types can manage the case an importer endogenously changes his/her valuation of the commodity based on certain changes in the economic environment.

Another important decision has to be made. It is the choice between the "macro" and "micro" approaches that Melitz and Redding (2013) have referred to. The "macro" approach has been taken by Arkolakis, *et al*. (2012) to show a class of heterogeneous and homogeneous firm models may yield the same level of welfare gains from trade, if those models have the same domestic trade share. In the "macro" approach, the elasticity of substitution (σ^T in this paper) is assumed to have different values in different specifications.[13] On the other hand, in the "micro" approach, which has been taken by Melitz and Rodding (2013), models retain the same values for behavioral parameters. We take the "micro" approach in calibrating an AKME module, assuming that the same value applies to σ^T in every specification.

3.2 Calibration of the AKME Module with a Melitz-type Specification

To parameterize an Armington-type model by calibration, it is well known that the following kinds of information are required in advance: $p_r^M X_r$ (intermediate input at

[12] Although the discussion is limited to a Krugman-type, Francois and Roland-Holst (1997) and Francois (1998) took the latter approach.

[13] The "macro" approach is followed by Dixon and his colleagues' latest research that verifies whether the Melitz model can be regarded as an Armington-type with high substitution elasticity. Their preliminary answer is "Yes."

market price inclusive of transportation cost and import tariff); $p_r^M C_r$ (final demand at market price inclusive of transportation cost and import tariff); σ^T (elasticity of substitution across exporters); τ_{rs} (rate of transportation margin and import tariff); and trade flows at free-on-board (FOB) prices or producer prices, such as "*VXWD*" or "*VXMD*" as presented in the Global Trade Analysis Project (GTAP) database.[14] In the present framework, these two types of trade flows at the different price levels become identical.[15] Let us refer to the data related to the trade flow values as "TF_{rs}" here. TF_{rs} can be regarded as

$$TF_{rs} = p_{rs}(1 - G_{rs})N_r Q_{rs}. \tag{26}$$

In addition to the information listed above, information on γ (shape parameter related to productivity) as well as on F_{rs} (fixed cost necessary to make sales on the r-s link), G_{rs} (proportion of registered but inactive firms in region r), H_r (fixed cost necessary to establish a firm in region r), and N_r (the number of firms registered in region r) basically is necessary to include Melitz-type monopolistic competition and heterogeneous firms. However, we do not need all kinds of information on these items, since two of the latter four pieces, F_{rs} or G_{rs}, and H_r or N_r, can be derived and calibrated. In this process, initial values of other endogenous variables, which cannot be observed directly from the given data, p_{rs} (the differentiated sales price by firm in region r exclusive of transportation margin and import tariff), Q_{rs} (average distribution of the commodity to region s by active firm in region r), and φ_{rs} (average productivity of active firms in region r) also are derived by setting p_r^W (wholesale price of commodity produced in region r) to unity following the usual custom of AGE modeling.[16] After that, initial values of p_r^M (market price of the commodity inclusive of transportation margin and import tariff), and parameters, δ_{rs}^T (the demand share parameter) and θ_s^T (the scaling factor of measuring units), are derived and calibrated.

Suppose information on γ and G_{rs} is available at this moment. Then, initial values of φ_{rs} can be derived using (23):

$$\varphi_{rs} = (1 - G_{rs})^{-1/\gamma} \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T - 1)}. \tag{27}$$

Based on the values of φ_{rs} obtained by (27), initial values of p_{rs} also are derived from (21) by setting p_r^W to unity:

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14 For details, see Hertel (1997).

15 More precisely, trade flows that are dealt with here include both domestic goods ("*VDM*" in the GTAP database) and intra-regional trade in the part $r = s$.

16 Since p_r^W is given and set to unity, Equation (22) is not used for calibration.

$$p_{rs} = \left(\frac{1}{1+\eta}\right)\frac{1}{\varphi_{rs}}, \tag{28}$$

where $\eta \equiv -1/\sigma^T$.

Using (17), (20), and (24), and setting $\pi_r = 0$, as well as $\widetilde{N}_{rs} = (1 - G_{rs})N_r$, we obtain

$$H_r = \left(\frac{\sigma^T - 1}{\gamma - \sigma^T + 1}\right)\sum_s (1 - G_{rs})\, F_{rs}. \tag{29}$$

Hence, we find that H_r is a function of F_{rs}: $H_r(F_{rs})$.

Next, we can derive the following relation using (25) and (26):

$$p_r^w N_r \{\sum_s (1 - G_{rs})\, F_{rs} + H_r\} = -\eta \sum_s TF_{rs}.$$

Therefore, we obtain

$$N_r = -\frac{\eta \sum_s TF_{rs}}{\sum_s (1 - G_{rs})F_{rs} + H_r}, \tag{30}$$

setting p_r^w to unity. From (30), we find that N_r is a function of F_{rs} and H_r: $N_r(F_{rs}, H_r)$.

Substituting (21) into (26), we get

$$TF_{rs} = (1 - G_{rs})N_r \left(\frac{1}{1+\eta}\right)\frac{p_r^w}{\varphi_{rs}} Q_{rs}.$$

Therefore,

$$Q_{rs} = \frac{(1+\eta)\varphi_{rs} TF_{rs}}{(1 - G_{rs})N_r}, \tag{31}$$

since $p_r^w = 1$. Hence, we find that Q_{rs} is a function of N_r with the previously calibrated values of φ_{rs}: $Q_{rs}(N_r)$.

Plugging (24) into (23), we can derive

$$F_{rs} = -\eta \left(\frac{1}{1+\eta}\right)^{1-\sigma^T} (1 - G_{rs})^{(1-\sigma^T)/\gamma} \left(\frac{p_{rs}}{p_r^w}\right)^{\sigma^T} Q_{rs}. \tag{32}$$

From (32), we find that F_{rs} is a function of Q_{rs} with the previously calibrated values of p_{rs} and $p_r^w = 1$: $F_{rs}(Q_{rs})$.

Basically, F_{rs}, H_r, N_r, and Q_{rs} can be calibrated by solving the system of simultaneous equations (29) through (32), based on the values of φ_{rs} and p_{rs} derived by (27) and (28), when γ and G_{rs} are given. Then, let us explore more deeply into these equations.

Using (27) and (31) with $\eta \equiv -1/\sigma^T$, we obtain

$$(1 - G_{rs})^{1/\gamma} Q_{rs} = \left(\frac{\sigma^T - 1}{\sigma^T}\right)\frac{TF_{rs}}{(1 - G_{rs})N_r}\left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T - 1)}. \tag{33}$$

Using (27), (28), and (32), we get

$$F_{rs} = \frac{1}{\sigma^T - 1} \left(\frac{\gamma}{\gamma - \sigma^T + 1} \right)^{\sigma^T/(1-\sigma^T)} (1 - G_{rs})^{1/\gamma} Q_{rs}. \tag{34}$$

Using (29) and (30), we get

$$N_r \sum_s (1 - G_{rs}) F_{rs} = \left(\frac{\gamma - \sigma^T + 1}{\gamma \sigma^T} \right) \sum_s TF_{rs}. \tag{35}$$

Then, the following equation derives plugging (33) into (34):

$$(1 - G_{rs}) N_r F_{rs} = \left(\frac{\gamma - \sigma^T + 1}{\gamma \sigma^T} \right) TF_{rs}. \tag{36}$$

Notice that summing up both sides of (36) with respect to s yields exactly the same relation as the one given by (35). This implies that the system consists of Equations (27), (28), (31), and (32), and the other system of (29) and (30) independently define the same relation.[17] Therefore, one equation has to be dropped from either of those two systems. This time, we drop (30) to use (29) and (36).

Using (29) and (36), we obtain

$$N_r H_r = \left(\frac{\sigma^T - 1}{\gamma \sigma^T} \right) \sum_s TF_{rs}. \tag{37}$$

Equation (37) is the final essence of the calibration equations showing that this parameterization process is not able to go further. Thus, the mass in the left-hand side of (37) cannot be split by calibration, without making use of information on either of N_r (the number of registered firms) or H_r (fixed cost required to establish a firm). It is not affected by the existence or non-existence of Equation (30), which we dropped previously. Suppose the level of N_r is given. Then, H_r is calibrated as a parameter accordingly to scale the chosen level of N_r making the left-hand side of Equation (37) to meet the fixed proportion of $\sum_s TF_{rs}$, and *vice versa*, in the following manner:

Proposition 1 *The choice of an initial level for N_r (the number of registered firms) or a parameter value for H_r (fixed cost required to establish a firm) in the calibration process is perfectly neutral that will not affect initial levels of endogenous variables and parameter settings outside the AKME module.*

Proof. See the Appendix. ■

[17] An interesting point is that the former system corresponds to the variables specific to a Melitz-type specification, while the latter has a Krugman-based nature. It implies that Melitz- and Krugman-type specifications are clearly separated and making independent blocks in an AKME module.

By Proposition 1, an initial level of N_r (or a parameter values for H_r) can be set freely to any preferred value, such as $N_r = 1$, just as the cases we derive quantities from given value data assuming some of the initial prices to be unity.

Equation (36) also is showing that we still may not split G_{rs} (proportion of registered but inactive firms) from F_{rs} (fixed cost required to make sales on the r-s link), even if we have information on the level of N_r. In consequence, information on either of G_{rs} or F_{rs} is necessary as well. Suppose the level of G_{rs} is given. Then, F_{rs} is calibrated as a parameter accordingly to scale the chosen level of G_{rs} making the left-hand side of (36) to meet the fixed proportion of TF_{rs}, and *vice versa*, in the following manner:

Proposition 2 *The choice of an initial level for G_{rs} (proportion of registered but inactive firms) or a parameter value for F_{rs} (fixed cost required to make sales on the r-s link) in the calibration process will not affect responses of endogenous variables included in an AGE model with the AKME module to an exogenous shock given in a counterfactual simulation.*

Proof. See the Appendix. ■

Unlike the case of N_r and H_r, the choice of an initial level for G_{rs} or a parameter value for F_{rs} indeed spillovers outside the AKME module. On the other hand, deviations of endogenous variables from the base case brought by a certain shock given in a counterfactual simulation are never affected by the choice. In the ordinary AGE analysis, effects are measured and evaluated by initial volumes of endogenous variables in the base case. It implies that just changes of endogenous variables from the base case are important and essential. As far as one stays within this ordinary usage of an AGE model, choosing an initial level for G_{rs} or a parameter value for F_{rs} will not affect simulation results.[18]

Thus, the additional information required for extending an Armington-type model to be a Melitz-type is reduced to be just one kind, the shape parameter related to productivity (γ). Estimates for γ can be found in several empirical studies, such as Melitz and Redding (2013), Balistreri, *et al.* (2011), and Bernard, *et al.* (2007). At this stage, the time and efforts devoted to data collection comes to be dramatically saved.

Given initial levels of G_{rs} and N_r, H_r and φ_{rs} are calculated first using

[18] Since G_{rs} must be within the range between 0 and 1, we recommend G_{rs} not to be calibrated.

Equations (37) and (27), respectively. Then, F_{rs}, p_{rs}, and Q_{rs} are respectively obtained by Equations (36), (28), and (31). Once F_{rs}, H_r, p_{rs}, Q_{rs}, and φ_{rs} are calibrated choosing certain values whatever one likes for G_{rs} and N_r, we can derive the initial value of p_s^M and parameters δ_{rs}^T and θ_s^T as follows:

$$p_s^M = \frac{\sum_r (1+\tau_{rs}) T F_{rs}}{\sum_r (1-G_{rs}) N_r Q_{rs}}; \tag{38}$$

$$\delta_{rs}^T = \frac{(1+\tau_{rs}) p_{rs} Q_{rs}{}^{1/\sigma^T}}{\sum_{r'} (1+\tau_{r's}) p_{r's} Q_{r's}{}^{1/\sigma^T}}; \tag{39}$$

and

$$\theta_s^T = \frac{X_s + C_s}{\left[\sum_r \delta_{rs}^T (1-G_{rs}) N_r Q_{rs}^{(\sigma^T-1)/\sigma^T}\right]^{\sigma^T/(\sigma^T-1)}}, \tag{40}$$

where X_s and C_s are respectively calculated from the information on intermediate input and final demand at market price inclusive of transportation cost and import tariff, using the calibrated value of p_s^M.

Hitherto, we have demonstrated that just one kind of additional information is required to extend a standard trade model to include Melitz-type monopolistic competition and heterogeneous firms. It is information on the shape parameter related to productivity (γ). In the procedure presented above, all of the parameter values are just determined, without making any changes in the data set, at the levels that ensure the model to generate an equilibrium solution with values that reproduce the benchmark data in the reference run. For instance, there is no re-estimation of the trade costs. Our approach is on the same basis as the one taken by Zhai (2008), whereas he re-estimates unobserved transportation margins based on the assumption that domestic trade incurs no iceberg trade costs.

Different from our approach, Balistreri, *et al*. (2011) took a comprehensive strategy to estimate a whole set of core parameters and unobserved trade frictions at once based on an econometric technic. One of the reasons that motivate them to take such approach might be because they gave the top priority to the measurement of unobserved trade costs that fit to the geographic pattern of trade. Since econometric estimation requires a certain amount of data collected from several sources, we pursued a more labor-saving and simpler way, making full use of information such that we are familiar with or relatively easy to have access to.

3.3 Calibration of the AKME Module with a Krugman-type Specification

With a Krugman-type specification, H_r (fixed cost necessary to establish a firm in region

r) can be calibrated choosing a certain value whatever one likes for N_r (the number of firms registered in region r). This implies that no additional information is required in order to extend a standard model with an Armington-type trade specification to be a Krugman-type.

As in the case of a Melitz-type specification, the initial value of p_{rs} can be derived from (28) setting p_r^W and φ_{rs} to unity. Then, we can obtain H_r using (30) as follows:

$$H_r = -\frac{\eta \sum_s TF_{rs}}{N_r}. \tag{41}$$

Similarly, we obtain Q_{rs} from (31):

$$Q_{rs} = \frac{(1+\eta)TF_{rs}}{N_r}, \tag{42}$$

since $\varphi_{rs} = 1$ and $G_{rs} = 0$.

Initial value of p_s^M and parameters δ_{rs}^T and θ_s^T can be derived as follows:

$$p_s^M = \frac{\sum_r (1+\tau_{rs})TF_{rs}}{\sum_r N_r Q_{rs}}; \tag{43}$$

$$\delta_{rs}^T = \frac{(1+\tau_{rs})p_{rs}{Q_{rs}}^{1/\sigma^T}}{\sum_{r'}(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}; \tag{44}$$

and

$$\theta_s^T = \frac{X_s + C_s}{\left[\sum_r \delta_{rs}^T N_r Q_{rs}^{(\sigma^T-1)/\sigma^T}\right]^{\sigma^T/(\sigma^T-1)}}. \tag{45}$$

4. Concluding Remarks

Comparing simulation results obtained by AGE models based on the intra-industry trade specifications presented by Armington (1969), Krugman (1980), and Melitz (2003) may have considerable importance in evaluating trade-related economic policies today. This paper explained how an AKME module based on Dixon and Rimmer (2012) can be calibrated, and clarified that basically just one kind of additional information is required in order to extend a standard AGE model with an Armington-type trade specification to include Melitz-type monopolistic competition and heterogeneous firms, as far as a calibration procedure is utilized to parameterize a model. The necessary information is the one on the shape parameter related to productivity (γ) only. To include Krugman-type monopolistic competition, no additional information is required.

One of the most important findings related to calibrating an AGE model with an

AKME module is that the choice of an initial level for the number of registered firms (N_r) or a parameter value for the fixed cost necessary to establish a firm in region r (H_r) is perfectly neutral so that initial levels of endogenous variables and parameter settings outside the module will never be affected. Furthermore, the choice of an initial level for the proportion of inactive firms (G_{rs}) or a parameter value for the fixed cost necessary to make sales on the r-s link (F_{rs}) will not affect responses of endogenous variables included in an AGE model with an AKME module to exogenous shocks given in counterfactual simulations. Then, initial levels of G_{rs} and N_r (or parameter values for F_{rs} and H_r) can be set freely to any preferred value, just as the cases we derive quantities from given value data assuming some of the initial prices to be unity.

At this stage, the cost-performance of introducing an AKME module in a multi-regional AGE model comes to be extremely high. Model builders in applied economics may now be released from the time-consuming burden of data collection and reconciliation, and have chance to fully enjoy the featured properties of the theoretical models invented by Krugman (1980) and Melitz (2003) in practical policy evaluations at low cost.

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Appendix

When a Melitz-type specification applies, an AKME module consists of the following seven equations:

$$X_s + C_s = \theta_s^T \left\{\sum_r \delta_{rs}^T (1 - G_{rs}) N_r Q_{rs}^{(\sigma^T - 1)/\sigma^T}\right\}^{\sigma^T/(\sigma^T - 1)}; \tag{A1}$$

$$(1 + \tau_{rs}) p_{rs} = \delta_{rs}^T p_s^M (\theta_s^T)^{(\sigma^T - 1)/\sigma^T} \left(\frac{X_s + C_s}{Q_{rs}}\right)^{1/\sigma^T}; \tag{A2}$$

$$p_{rs} = \left(\frac{\sigma^T}{\sigma^T - 1}\right) \frac{p_r^w}{\varphi_{rs}}; \tag{A3}$$

$$\sum_s (1 - G_{rs}) N_r \frac{Q_{rs}}{\varphi_{rs}} = Z_r - \sum_s (1 - G_{rs}) N_r F_{rs} - N_r H_r; \tag{A4}$$

$$G_{rs} = 1 - \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{\gamma/(\sigma^T - 1)} \varphi_{rs}{}^{-\gamma}; \tag{A5}$$

$$\varphi_{rs} = \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T - 1)} \frac{(\sigma^T)^{\sigma^T/(\sigma^T - 1)}}{\sigma^T - 1} \left(\frac{p_r^w}{p_{rs}}\right)^{\sigma^T/(\sigma^T - 1)} \left(\frac{F_{rs}}{Q_{rs}}\right)^{1/(\sigma^T - 1)}; \tag{A6}$$

and

$$\sigma^T p_r^w \{\sum_s (1 - G_{rs}) F_{rs} + H_r\} = \sum_s p_{rs} (1 - G_{rs}) Q_{rs}. \tag{A7}$$

Equations (A1) through (A7) respectively correspond to Equations (19) through (25) in Section 2, while $\eta = -1/\sigma^T$ and $\widetilde{N}_{rs} = (1 - G_{rs}) N_r$ are substituted into some of them.

Rearranging (A2), we obtain

$$Q_{rs} = (X_s + C_s) \left\{\frac{\delta_{rs}^T p_s^M (\theta_s^T)^{(\sigma^T - 1)/\sigma^T}}{(1 + \tau_{rs}) p_{rs}}\right\}^{\sigma^T}. \tag{A8}$$

Totally differentiating (A8), we get

$$\mathrm{d}Q_{rs} = \left\{\frac{\delta_{rs}^T p_s^M (\theta_s^T)^{(\sigma^T - 1)/\sigma^T}}{(1 + \tau_{rs}) p_{rs}}\right\}^{\sigma^T} \left[\begin{array}{c} \mathrm{d}(X_s + C_s) \\ + \sigma^T (X_s + C_s) \left\{\begin{array}{c} \frac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T} + \frac{\mathrm{d}p_s^M}{p_s^M} - \frac{\mathrm{d}p_{rs}}{p_{rs}} \\ + \left(\frac{\sigma^T - 1}{\sigma^T}\right) \frac{\mathrm{d}\theta_s^T}{\theta_s^T} \end{array}\right\} \end{array}\right]. \tag{A9}$$

Dividing (A9) by (A8), the following relation derives:

$$\frac{\mathrm{d}Q_{rs}}{Q_{rs}} = \frac{\mathrm{d}(X_s + C_s)}{X_s + C_s} + \sigma^T \left\{\frac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T} + \frac{\mathrm{d}p_s^M}{p_s^M} - \frac{\mathrm{d}p_{rs}}{p_{rs}} + \left(\frac{\sigma^T - 1}{\sigma^T}\right) \frac{\mathrm{d}\theta_s^T}{\theta_s^T}\right\}. \tag{A10}$$

Totally differentiating (A3), we get

$$\mathrm{d}p_{rs} = \left(\frac{\sigma^T}{\sigma^T - 1}\right) \left\{\frac{\mathrm{d}p_r^w}{\varphi_{rs}} - \left(\frac{p_r^w}{\varphi_{rs}}\right) \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}}\right\}. \tag{A11}$$

Dividing (A11) by (A3), the following relation derives:

$$\frac{\mathrm{d}p_{rs}}{p_{rs}} = \frac{\mathrm{d}p_r^w}{p_r^w} - \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}}. \tag{A12}$$

Rearranging (A4), we obtain

$$Z_r = \sum_s (1 - G_{rs}) N_r \frac{Q_{rs}}{\varphi_{rs}} + \sum_s (1 - G_{rs}) N_r F_{rs} + N_r H_r. \tag{A13}$$

Totally differentiating (A13), we get

$$\begin{aligned} \mathrm{d}Z_r = &\sum_s \left[(1 - G_{rs}) N_r \frac{Q_{rs}}{\varphi_{rs}} \left\{ \frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}Q_{rs}}{Q_{rs}} - \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}} \right\} \right] \\ &+ \sum_s \left[(1 - G_{rs}) N_r F_{rs} \left\{ \frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}F_{rs}}{F_{rs}} \right\} \right] \\ &+ N_r H_r \left(\frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}H_r}{H_r} \right). \end{aligned} \tag{A14}$$

Dividing (A14) by (A13), the following relation derives:

$$\begin{aligned} \frac{\mathrm{d}Z_r}{Z_r} = &\sum_s \left[\frac{(1-G_{rs}) N_r \frac{Q_{rs}}{\varphi_{rs}}}{Z_r} \left\{ \frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}Q_{rs}}{Q_{rs}} - \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}} \right\} \right] \\ &+ \sum_s \left[\frac{(1-G_{rs}) N_r F_{rs}}{Z_r} \left\{ \frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}F_{rs}}{F_{rs}} \right\} \right] \\ &+ \frac{N_r H_r}{Z_r} \left(\frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}H_r}{H_r} \right). \end{aligned} \tag{A15}$$

Rearranging (A5), we obtain

$$1 - G_{rs} = \left(\frac{\gamma}{\gamma - \sigma^T + 1} \right)^{\gamma/(\sigma^T - 1)} {\varphi_{rs}}^{-\gamma}. \tag{A16}$$

Totally differentiating (A16), we get

$$\mathrm{d}(1 - G_{rs}) = -\gamma \left(\frac{\gamma}{\gamma - \sigma^T + 1} \right)^{\gamma/(\sigma^T - 1)} {\varphi_{rs}}^{-\gamma} \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}}. \tag{A17}$$

Dividing (A17) by (A16), the following relation derives:

$$\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} = -\gamma \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}}. \tag{A18}$$

Substituting (A3) into (A6) and after some manipulation, we obtain

$$\varphi_{rs} = \left\{ \frac{\gamma - \sigma^T + 1}{\gamma(\sigma^T - 1)} \right\} \frac{Q_{rs}}{F_{rs}}. \tag{A19}$$

Totally differentiating (A19), we get

$$\mathrm{d}\varphi_{rs} = \frac{\gamma - \sigma^T + 1}{\gamma(\sigma^T - 1)} \left\{ \frac{\mathrm{d}Q_{rs}}{F_{rs}} - \left(\frac{Q_{rs}}{F_{rs}} \right) \frac{\mathrm{d}F_{rs}}{F_{rs}} \right\}. \tag{A20}$$

Dividing (A20) by (A19), the following relation derives:

$$\frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}} = \frac{\mathrm{d}Q_{rs}}{Q_{rs}} - \frac{\mathrm{d}F_{rs}}{F_{rs}}. \tag{A21}$$

The equations used in the calibration process of an AKME module are as follows:

$$\varphi_{rs} = (1 - G_{rs})^{-1/\gamma} \left(\frac{\gamma}{\gamma - \sigma^T + 1}\right)^{1/(\sigma^T - 1)}; \tag{A22}$$

$$p_r^w N_r H_r = \left(\frac{\sigma^T - 1}{\gamma \sigma^T}\right) \sum_s TF_{rs}; \tag{A23}$$

$$p_r^w (1 - G_{rs}) N_r F_{rs} = \left(\frac{\gamma - \sigma^T + 1}{\gamma \sigma^T}\right) TF_{rs}; \tag{A24}$$

$$p_{rs} = \left(\frac{\sigma^T}{\sigma^T - 1}\right) \frac{p_r^w}{\varphi_{rs}}; \tag{A25}$$

$$Q_{rs} = \left(\frac{\sigma^T - 1}{\sigma^T}\right) \frac{\varphi_{rs} TF_{rs}}{p_r^w (1 - G_{rs}) N_r}; \tag{A26}$$

$$p_s^M = \frac{\sum_r (1 + \tau_{rs}) TF_{rs}}{\sum_r (1 - G_{rs}) N_r Q_{rs}}; \tag{A27}$$

$$\delta_{rs}^T = \frac{(1 + \tau_{rs}) p_{rs} Q_{rs}{}^{1/\sigma^T}}{\sum_{r'} (1 + \tau_{r's}) p_{r's} Q_{r's}{}^{1/\sigma^T}}; \tag{A28}$$

and

$$\theta_s^T = (X_s + C_s) \left\{\sum_r \delta_{rs}^T (1 - G_{rs}) N_r \, Q_{rs}^{(\sigma^T - 1)/\sigma^T}\right\}^{\sigma^T/(1 - \sigma^T)}, \tag{A29}$$

where TF_{rs} is given data on trade flow values at FOB prices when there is no export duty/subsidy, which can be regarded as

$$TF_{rs} = p_{rs}(1 - G_{rs}) N_r Q_{rs}. \tag{A30}$$

Equations (A22) through (A30) correspond to Equations (27), (37), (36), (28), (31), (38), (39), and (19) in Section 3, respectively, while $\eta = -1/\sigma^T$ and $\tilde{N}_{rs} = (1 - G_{rs}) N_r$ are substituted into some of them. X_s and C_s also are given data calculated from the information on intermediate input and final demand at market price inclusive of transportation cost and import tariff, using the calibrated value of p_s^M. In addition, we follow the usual procedure to set p_r^w to unity. Hence, TF_{rs}, X_s, C_s, and p_r^w are constant and excluded from the variables to be differentiated.

There are several equations that can be laid aside at this moment. Equations (A22) and (A25) are the same as (A5) and (A3), respectively. Equation (A26) can be derived from plugging (A25) into (A30).

Then, setting $p_r^w = 1$ and using (A23), we obtain

$$H_r = \left(\frac{\sigma^T - 1}{\gamma \sigma^T}\right) \frac{\sum_s TF_{rs}}{N_r}. \tag{A31}$$

Totally differentiating (A31), we get

$$\mathrm{d}H_r = -\left(\frac{\sigma^T-1}{\gamma\sigma^T}\right)\left(\frac{\sum_s TF_{rs}}{N_r}\right)\frac{\mathrm{d}N_r}{N_r}. \tag{A32}$$

Dividing (A32) by (A31), the following relation derives:

$$\frac{\mathrm{d}H_r}{H_r} = -\frac{\mathrm{d}N_r}{N_r}. \tag{A33}$$

In a similar manner, we obtain

$$F_{rs} = \left(\frac{\gamma-\sigma^T+1}{\gamma\sigma^T}\right)\frac{TF_{rs}}{(1-G_{rs})N_r}. \tag{A34}$$

Totally differentiating (A34), we get

$$\mathrm{d}F_{rs} = -\left(\frac{\gamma-\sigma^T+1}{\gamma\sigma^T}\right)\frac{TF_{rs}}{(1-G_{rs})N_r}\left\{\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r}\right\}. \tag{A35}$$

Dividing (A35) by (A34), the following relation derives:

$$\frac{\mathrm{d}F_{rs}}{F_{rs}} = -\left\{\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r}\right\}. \tag{A36}$$

Totally differentiating (A27), we obtain

$$\mathrm{d}p_s^M = -\frac{\sum_r(1+\tau_{rs})TF_{rs}}{\sum_r(1-G_{rs})N_rQ_{rs}}\sum_r\left[\frac{(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1-G_{r's})N_{r'}Q_{r's}}\left\{\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\}\right]. \tag{A37}$$

Dividing (A37) by (A27), we get

$$\frac{\mathrm{d}p_s^M}{p_s^M} = -\sum_r\left[\frac{(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1-G_{r's})N_{r'}Q_{r's}}\left\{\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r} + \frac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\}\right]. \tag{A38}$$

Totally differentiating (A28), we obtain

$$\begin{aligned}\mathrm{d}\delta_{rs}^T = {}& \frac{(1+\tau_{rs})p_{rs}{Q_{rs}}^{1/\sigma^T}}{\sum_{r'}(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}\left\{\frac{\mathrm{d}p_{rs}}{p_{rs}} + \left(\frac{1}{\sigma^T}\right)\frac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\} \\ & -\frac{(1+\tau_{rs})p_{rs}{Q_{rs}}^{1/\sigma^T}}{\sum_{r'}(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}\sum_{r'}\left[\begin{array}{c}\dfrac{(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}{Q_{r''s}}^{1/\sigma^T}} \\ \times\left\{\dfrac{\mathrm{d}p_{r's}}{p_{r's}} + \left(\dfrac{1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{r's}}{Q_{r's}}\right\}\end{array}\right].\end{aligned} \tag{A39}$$

Dividing (A39) by (A28), we get

$$\frac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T} = \left\{\frac{\mathrm{d}p_{rs}}{p_{rs}} + \left(\frac{1}{\sigma^T}\right)\frac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\} - \sum_{r'}\left[\begin{array}{c}\dfrac{(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}{Q_{r''s}}^{1/\sigma^T}} \\ \times\left\{\dfrac{\mathrm{d}p_{r's}}{p_{r's}} + \left(\dfrac{1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{r's}}{Q_{r's}}\right\}\end{array}\right]. \tag{A40}$$

Totally differentiating (A29), we obtain

$$\mathrm{d}\theta_s^T = (X_s + C_s)\left\{\sum_r \delta_{rs}^T(1-G_{rs})N_r\, Q_{rs}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(1-\sigma^T)}$$

$$\times\left(\frac{\sigma^T}{1-\sigma^T}\right)\sum_r\left[\begin{array}{c}\dfrac{\delta_{rs}^T(1-G_{rs})N_rQ_{rs}^{(\sigma^T-1)/\sigma^T}}{\sum_{r'}\delta_{r's}^T(1-G_{r's})N_{r'}Q_{r's}^{(\sigma^T-1)/\sigma^T}}\\ \times\left\{\dfrac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T}+\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}+\dfrac{\mathrm{d}N_r}{N_r}+\left(\dfrac{\sigma^T-1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\}\end{array}\right]. \tag{A41}$$

Dividing (A41) by (A29), we get

$$\frac{\mathrm{d}\theta_s^T}{\theta_s^T}=\left(\frac{\sigma^T}{1-\sigma^T}\right)\sum_r\left[\begin{array}{c}\dfrac{\delta_{rs}^T(1-G_{rs})N_rQ_{rs}^{(\sigma^T-1)/\sigma^T}}{\sum_{r'}\delta_{r's}^T(1-G_{r's})N_{r'}Q_{r's}^{(\sigma^T-1)/\sigma^T}}\\ \times\left\{\dfrac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T}+\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}+\dfrac{\mathrm{d}N_r}{N_r}+\left(\dfrac{\sigma^T-1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\}\end{array}\right]. \tag{A42}$$

Then, the following relation can be derived using (A28):

$$\begin{aligned}\frac{\delta_{rs}^T(1-G_{rs})N_rQ_{rs}^{(\sigma^T-1)/\sigma^T}}{\sum_{r'}\delta_{r's}^T(1-G_{r's})N_{r'}Q_{r's}^{(\sigma^T-1)/\sigma^T}}&=\frac{\dfrac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}Q_{r's}{}^{1/\sigma^T}}}{\dfrac{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}Q_{r''s}{}^{1/\sigma^T}}}\\&=\frac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}.\end{aligned} \tag{A43}$$

Substituting (A43) into (A42), the following relation derives:

$$\frac{\mathrm{d}\theta_s^T}{\theta_s^T}=\left(\frac{\sigma^T}{1-\sigma^T}\right)\sum_r\left[\begin{array}{c}\dfrac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}\\ \times\left\{\dfrac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T}+\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}+\dfrac{\mathrm{d}N_r}{N_r}+\left(\dfrac{\sigma^T-1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\}\end{array}\right]. \tag{A44}$$

A.1 Effects of Changing the Level of N_r in the Calibration Process

Since the values of p_r^W and $(1-G_{rs})$ are given exogenously in the calibration process, we obtain

$$\frac{\mathrm{d}p_r^W}{p_r^W}=\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}=0. \tag{A45}$$

Substituting (A45) into (A12) and (A18), we get

$$\frac{\mathrm{d}p_{rs}}{p_{rs}}=\frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}}=0. \tag{A46}$$

Then, the following derives from (A21), (A33), and (A36):

$$\frac{\mathrm{d}Q_{rs}}{Q_{rs}}=\frac{\mathrm{d}F_{rs}}{F_{rs}}=\frac{\mathrm{d}H_r}{H_r}=-\frac{\mathrm{d}N_r}{N_r}. \tag{A47}$$

Substituting (A45) through (A47) into (A15) and (A38), we get

$$\frac{\mathrm{d}Z_r}{Z_r}=\frac{\mathrm{d}p_s^M}{p_s^M}=0. \tag{A48}$$

Using (A46) and (A47), (A40) can be rewritten to

$$\frac{\mathrm{d}\delta_{rs}^{T}}{\delta_{rs}^{T}} = -\left(\frac{1}{\sigma^{T}}\right)\left[\frac{\mathrm{d}N_r}{N_r} - \sum_{r'}\left\{\frac{(1+\tau_{r's})p_{r's}Q_{r's}{}^{1/\sigma^{T}}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}Q_{r''s}{}^{1/\sigma^{T}}}\cdot\frac{\mathrm{d}N_{r'}}{N_{r'}}\right\}\right]. \tag{A49}$$

Substituting (A45), (A47), and (A49) into (A44), and after some manipulation, we obtain the following:

$$\frac{\mathrm{d}\theta_{s}^{T}}{\theta_{s}^{T}} = \left(\frac{\sigma^{T}}{1-\sigma^{T}}\right)\sum_{r}\left[\begin{array}{c}\dfrac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}\\ \times\left(\dfrac{1}{\sigma^{T}}\right)\sum_{r'}\left\{\dfrac{(1+\tau_{r's})p_{r's}Q_{r's}{}^{1/\sigma^{T}}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}Q_{r''s}{}^{1/\sigma^{T}}}\cdot\dfrac{\mathrm{d}N_{r'}}{N_{r'}}\right\}\end{array}\right]$$

$$= \left(\frac{1}{1-\sigma^{T}}\right)\sum_{r'}\left\{\frac{(1+\tau_{r's})p_{r's}Q_{r's}{}^{1/\sigma^{T}}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}Q_{r''s}{}^{1/\sigma^{T}}}\cdot\frac{\mathrm{d}N_{r'}}{N_{r'}}\right\}\frac{\sum_{r}(1+\tau_{rs})p_{rs}(1-G_{rs})N_rQ_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}$$

$$= \left(\frac{1}{1-\sigma^{T}}\right)\sum_{r}\left\{\frac{(1+\tau_{rs})p_{rs}Q_{rs}{}^{1/\sigma^{T}}}{\sum_{r'}(1+\tau_{r's})p_{r's}Q_{r's}{}^{1/\sigma^{T}}}\cdot\frac{\mathrm{d}N_{r}}{N_{r}}\right\}. \tag{A50}$$

Finally, we get the following relation plugging (A45), (A46), and (A48) through (A50) into (A10):

$$\frac{\mathrm{d}(X_s+C_s)}{X_s+C_s} = 0. \tag{A51}$$

Proof of Proposition 1. Equations (A48) and (A51) show that the choice of N_r in the calibration process is perfectly neutral and will not affect initial values outside the AKME module. ■

A.2 Effects of Changing the Level of G_{rs} in the Calibration Process

Since the values of p_r^W and N_r are given exogenously in the calibration process, we obtain

$$\frac{\mathrm{d}p_r^W}{p_r^W} = \frac{\mathrm{d}N_r}{N_r} = 0. \tag{A52}$$

Substituting (A52) into (A33), we get

$$\frac{\mathrm{d}H_r}{H_r} = 0. \tag{A53}$$

Then, the following derives from (A12), (A18), (A21), and (A36):

$$\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} = -\frac{\mathrm{d}F_{rs}}{F_{rs}} = -\gamma\frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}} = \gamma\frac{\mathrm{d}p_{rs}}{p_{rs}} = -\left(\frac{\gamma}{1+\gamma}\right)\frac{\mathrm{d}Q_{rs}}{Q_{rs}}. \tag{A54}$$

Substituting (A52) through (A54) into (A15) and (A38), we get

$$\frac{\mathrm{d}Z_r}{Z_r} = 0, \tag{A55}$$

and

$$\frac{\mathrm{d}p_s^M}{p_s^M} = \left(\frac{1}{\gamma}\right)\sum_r \left\{\frac{(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1-G_{r's})N_{r'}Q_{r's}} \cdot \frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\right\}. \tag{A56}$$

Using (A54), (A40) can be rewritten to

$$\frac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T} = -\left(\frac{\gamma-\sigma^T+1}{\gamma\sigma^T}\right)\left[\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} - \sum_{r'}\left\{\begin{array}{c}\dfrac{(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}{Q_{r''s}}^{1/\sigma^T}} \\ \times\dfrac{\mathrm{d}(1-G_{r's})}{1-G_{r's}}\end{array}\right\}\right]. \tag{A57}$$

Substituting (A54) and (A57) into (A44), and after some manipulation, we obtain the following:

$$\begin{aligned}
\frac{\mathrm{d}\theta_s^T}{\theta_s^T} &= \left(\frac{\sigma^T}{1-\sigma^T}\right)\sum_r\left(\begin{array}{c}\dfrac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}} \\ \times\left[\begin{array}{c}\left(\dfrac{\gamma-\sigma^T+1}{\gamma\sigma^T}\right)\sum_{r'}\left\{\dfrac{(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}{Q_{r''s}}^{1/\sigma^T}}\cdot\dfrac{\mathrm{d}(1-G_{r's})}{1-G_{r's}}\right\} \\ -\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\end{array}\right]\end{array}\right) \\
&= \left(\frac{\gamma-\sigma^T+1}{\gamma-\gamma\sigma^T}\right)\sum_{r'}\left\{\begin{array}{c}\dfrac{(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}{\sum_{r''}(1+\tau_{r''s})p_{r''s}{Q_{r''s}}^{1/\sigma^T}} \\ \times\dfrac{\mathrm{d}(1-G_{r's})}{1-G_{r's}}\end{array}\right\}\frac{\sum_r(1+\tau_{rs})p_{rs}(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}} \\
&\quad + \left(\frac{\sigma^T}{\sigma^T-1}\right)\sum_r\left\{\frac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}\cdot\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\right\} \\
&= \left(\frac{\gamma-\sigma^T+1}{\gamma-\gamma\sigma^T}\right)\sum_r\left\{\frac{(1+\tau_{rs})p_{rs}{Q_{rs}}^{1/\sigma^T}}{\sum_{r'}(1+\tau_{r's})p_{r's}{Q_{r's}}^{1/\sigma^T}}\cdot\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\right\} \\
&\quad + \left(\frac{\sigma^T}{\sigma^T-1}\right)\sum_r\left\{\frac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}\cdot\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\right\}.
\end{aligned} \tag{A58}$$

Finally, we get the following relation plugging (A54) and (A56) through (A58) into (A10):

$$\begin{aligned}
\frac{\mathrm{d}Q_{rs}}{Q_{rs}} - \frac{\mathrm{d}(X_s+C_s)}{X_s+C_s} &= -\left(\frac{\gamma+1}{\gamma}\right)\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \left(\frac{\sigma^T}{\gamma}\right)\sum_r\left\{\frac{(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1-G_{r's})N_{r'}Q_{r's}}\cdot\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\right\} \\
&\quad +\sigma^T\sum_r\left\{\frac{(1+\tau_{rs})p_{rs}(1-G_{rs})N_r Q_{rs}}{\sum_{r'}(1+\tau_{r's})p_{r's}(1-G_{r's})N_{r'}Q_{r's}}\cdot\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}\right\}.
\end{aligned} \tag{A59}$$

Equations (A55), (A56), and (A59) show that the choice of $(1-G_{rs})$ in the calibration process spillovers outside the AKME module, through p_s^M and (X_s+C_s).

A.3 Effects of Changes in N_r and G_{rs} in Counterfactual Simulations

At this stage, we will not use Equations (A33), (A36), (A38), and (A42), because these are derived in relation with the information given in the calibration process such as TF_{rs} and $(X_s + C_s)$. Instead, Equation (A1) is utilized. Since δ_{rs}^T and θ_s^T are constant at this moment, totally differentiating (A1) yields

$$\mathrm{d}(X_s + C_s) = \theta_s^T \left\{\sum_r \delta_{rs}^T (1 - G_{rs}) N_r\, Q_{rs}^{(\sigma^T-1)/\sigma^T}\right\}^{\sigma^T/(1-\sigma^T)}$$

$$\times \left(\frac{\sigma^T}{\sigma^T-1}\right) \sum_r \left[\begin{array}{c} \dfrac{\delta_{rs}^T (1-G_{rs}) N_r Q_{rs}^{(\sigma^T-1)/\sigma^T}}{\sum_{r'} \delta_{r's}^T (1-G_{r's}) N_{r'} Q_{r's}^{(\sigma^T-1)/\sigma^T}} \\ \times \left\{\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \dfrac{\mathrm{d}N_r}{N_r} + \left(\dfrac{\sigma^T-1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\} \end{array}\right]. \quad \text{(A61)}$$

Dividing (A61) by (A1), the following relation derives:

$$\frac{\mathrm{d}(X_s+C_s)}{X_s+C_s} = \left(\frac{\sigma^T}{\sigma^T-1}\right) \sum_r \left[\begin{array}{c} \dfrac{\delta_{rs}^T (1-G_{rs}) N_r Q_{rs}^{(\sigma^T-1)/\sigma^T}}{\sum_{r'} \delta_{r's}^T (1-G_{r's}) N_{r'} Q_{r's}^{(\sigma^T-1)/\sigma^T}} \\ \times \left\{\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \dfrac{\mathrm{d}N_r}{N_r} + \left(\dfrac{\sigma^T-1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\} \end{array}\right]. \quad \text{(A62)}$$

Substituting (A43) into (A62), we obtain

$$\frac{\mathrm{d}(X_s+C_s)}{X_s+C_s} = \left(\frac{\sigma^T}{\sigma^T-1}\right) \sum_r \left[\begin{array}{c} \dfrac{(1+\tau_{rs}) p_{rs} (1-G_{rs}) N_r Q_{rs}}{\sum_{r'} (1+\tau_{r's}) p_{r's} (1-G_{r's}) N_{r'} Q_{r's}} \\ \times \left\{\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \dfrac{\mathrm{d}N_r}{N_r} + \left(\dfrac{\sigma^T-1}{\sigma^T}\right)\dfrac{\mathrm{d}Q_{rs}}{Q_{rs}}\right\} \end{array}\right]. \quad \text{(A63)}$$

F_{rs}, H_r, δ_{rs}^T, and θ_s^T are parameters that stay constant in counterfactual simulations,. Therefore, we obtain

$$\frac{\mathrm{d}F_{rs}}{F_{rs}} = \frac{\mathrm{d}H_r}{H_r} = \frac{\mathrm{d}\delta_{rs}^T}{\delta_{rs}^T} = \frac{\mathrm{d}\theta_s^T}{\theta_s^T} = 0. \quad \text{(A64)}$$

Substituting (A60) into (A18), (A21), and (A40), the following relation derives:

$$\frac{\mathrm{d}Q_{rs}}{Q_{rs}} = \frac{\mathrm{d}\varphi_{rs}}{\varphi_{rs}} = -\sigma^T \frac{\mathrm{d}p_{rs}}{p_{rs}} = -\left(\frac{1}{\gamma}\right)\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}. \quad \text{(A65)}$$

Using (A64), (A63) can be rewritten to

$$\frac{\mathrm{d}(X_s+C_s)}{X_s+C_s} = \sum_r \left[\begin{array}{c} \dfrac{(1+\tau_{rs}) p_{rs} (1-G_{rs}) N_r Q_{rs}}{\sum_{r'} (1+\tau_{r's}) p_{r's} (1-G_{r's}) N_{r'} Q_{r's}} \\ \times \left\{\left(\dfrac{\gamma\sigma^T-\sigma^T+1}{\gamma\sigma^T-\gamma}\right)\dfrac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \left(\dfrac{\sigma^T}{\sigma^T-1}\right)\dfrac{\mathrm{d}N_r}{N_r}\right\} \end{array}\right]. \quad \text{(A66)}$$

Plugging (A64) and (A65) into (A15), we get

$$\frac{\mathrm{d}Z_r}{Z_r} = \sum_s \left[\frac{(1-G_{rs}) N_r \frac{Q_{rs}}{\varphi_{rs}}}{Z_r}\left\{\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r}\right\}\right] + \sum_s \left[\frac{(1-G_{rs}) N_r F_{rs}}{Z_r}\left\{\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}} + \frac{\mathrm{d}N_r}{N_r}\right\}\right]$$

$$+\left(\frac{N_r H_r}{Z_r}\right)\frac{\mathrm{d}N_r}{N_r}. \tag{A67}$$

Using (A65), (A12) can be modified to

$$\frac{\mathrm{d}p_r^W}{p_r^W} = \left(\frac{1-\sigma^T}{\gamma\sigma^T}\right)\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}. \tag{A68}$$

Finally, we obtain the following relation substituting (A64) and (A65) into (A10):

$$\frac{\mathrm{d}p_s^M}{p_s^M} = -\left(\frac{1}{\sigma^T}\right)\frac{\mathrm{d}(X_s+C_s)}{X_s+C_s}. \tag{A69}$$

Proof of Proposition 2. Equations (A66) through (A69) show that the absolute levels of N_r and $(1-G_{rs})$ will not affect variables outside the AKME module in counterfactual simulations, while changes of N_r and $(1-G_{rs})$ from the base case affect through (X_s+C_s), Z_r, p_r^W, and p_s^M. Since the values of the variables in terms of absolute level that enter Equations (A66) and (A67), which are listed below, are all given in the calibration process based on (A30) to absorb individual levels of N_r and $(1-G_{rs})$ adjusting calibrated parameter values of H_r and F_{rs}:

$p_{rs}(1-G_{rs})N_r Q_{rs} = TF_{rs}$;

$(1-G_{rs})N_r \frac{Q_{rs}}{\varphi_{rs}} = \left(\frac{\sigma^T-1}{\sigma^T}\right)TF_{rs}$;

$(1-G_{rs})N_r F_{rs} = \left(\frac{\gamma-\sigma^T+1}{\gamma\sigma^T}\right)TF_{rs}$; and

$N_r H_r = \left(\frac{\sigma^T-1}{\gamma\sigma^T}\right)\sum_s TF_{rs}$.

Thus, not the absolute levels of N_r and $(1-G_{rs})$, but the changing rates from the base case, $\frac{\mathrm{d}N_r}{N_r}$ and $\frac{\mathrm{d}(1-G_{rs})}{1-G_{rs}}$, matter in simulation analyses. Suppose there are two models calibrated to different choices of N_r, 1 and 100 for instance ,while those are built on an identical data set. Then, a shock given in a simulation experiment, which may change the value of N_r in one model from 1 to 1.2, will change the value in another model from 100 to 120, and the percentage changes in all of the endogenous variables become identical in both models. In this meaning, it may be said that the choice of N_r will not affect "reactions" of endogenous variables in counterfactual simulations. ■