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IDE DISCUSSION PAPER No. 605

A note on three factor model of discounting

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May 2016

Abstract

In Montiel Olea and Strzalecki (2014), authors have axiomatically developed an algorithm to infer the parameters of beta-delta model of cognitive bias (present and future biases). While this is extremely useful, it allows the implied beta to become very large when the response is impatient in the future choices relative to present choices, i.e., when there is a strong future bias. I modify the model to further exponentiate the functional form to get more reasonable beta values.

Keywords: discount factor, experimental method

JEL classification: C91, D99

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A note on three factor model of discounting

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I Introduction: Motivation

When asked about the consumption choices in the future, a respondent may discount the future beyond what is suggested by standard exponential discounting. For example, an individual may feel tiring to consider about the choices of distant future. Tired of waiting and thinking about the future, one may show little tolerance to wait further and choose the larger consumption once it arrives. Alternatively, a terminally ill patient may discount the utility gain/loss heavily if one is asked about 10 years from now. If the question is posed against the concurrent consumption, there shall not be such disproportionately heavy discounting.

In Montiel Olea and Strzalecki (2014), authors have axiomatically developed an algorithm to infer the parameters of $\beta - \delta$ model of cognitive bias (present and future biases discussed in Ainslie and Haslam, 1992; Laibson, 1997; Loewenstein, 1987; Rubinstein, 2006; Sayman and Öncüler, 2009; Takeuchi, 2011). While this is extremely useful, it allows the implied β to become very large when the response is impatient in the future choices relative to present choices, i.e., when there is a strong future bias. In particular, when the accepted future waiting period is short and δ is small, β becomes astronomically large.

In the field study conducted in a township of South Africa, the responses to the questions that follow Montiel Olea and Strzalecki (2014)'s algorithm suggest the tolerable waiting period to be long in the present choices but short in the furture choices. Here are the examples.

```
           beta     delta present s  future s
1: 9.678635e+09 0.2131497        15 0.1250000
2: 1.934672e+05 0.2131497         8 0.1250000
3: 2.618018e+02 0.5783179        11 0.8333333
4: 1.089273e+12 0.3802776        29 0.3333333
5: 5.219914e+07 0.4043368        20 0.3750000
6: 1.853144e+08 0.4993843        28 0.5833333
```

Responses are shown in each row. present s shows reported tolerable waiting time (switch point) measured in days in the present choice, present s shows that of future choice. One sees a large discrepancy between the two, and it is much shorter with future s which shows only a fraction of a day that the respondents are willing to wait for a larger future consumption. Columns under delta and beta show implied parameter values. One sees that implied value of β to be too large.

II A model

Consider a plan of day w consumption x_{a1}, day $w + s + 1$ consumption x_{a2}, and day $w + s + 2$ consumption x_{a3}. Let the day w as day 0, or today. So we are considering today's consumption x_{a1}, day $s + 1$ consumption x_{a2}, and the day $s + 2$ consumption x_{a3}. Assume that a discount rate β is used for reference day and its day after, and a different discount factor δ is used for the rest of future days, as in usual $\beta - \delta$ model.

Utility under this plan, denoted as $r_a^p = (x_{a1}, x_{a2}, x_{a3}, w, s) = (x_{a1}, x_{a2}, x_{a3}, 0, s)$, is expressed as

$$u\left(r^p\right) = u(x_{a1}) + \beta\delta^s u(x_{a2}) + \beta\delta^{s+1} u(x_{a3}), \quad \beta, \delta \in \mathbb{U}^2.$$

Let us consider exactly the same consumption stream, only the day 0 is shifted into the future by w days. So day 0 is w days from today. We introduce a waiting cost $d(w)$ for waiting until day w arrives. $d(w)$ acts exactly the same as the discount factor and is assumed to take the values between

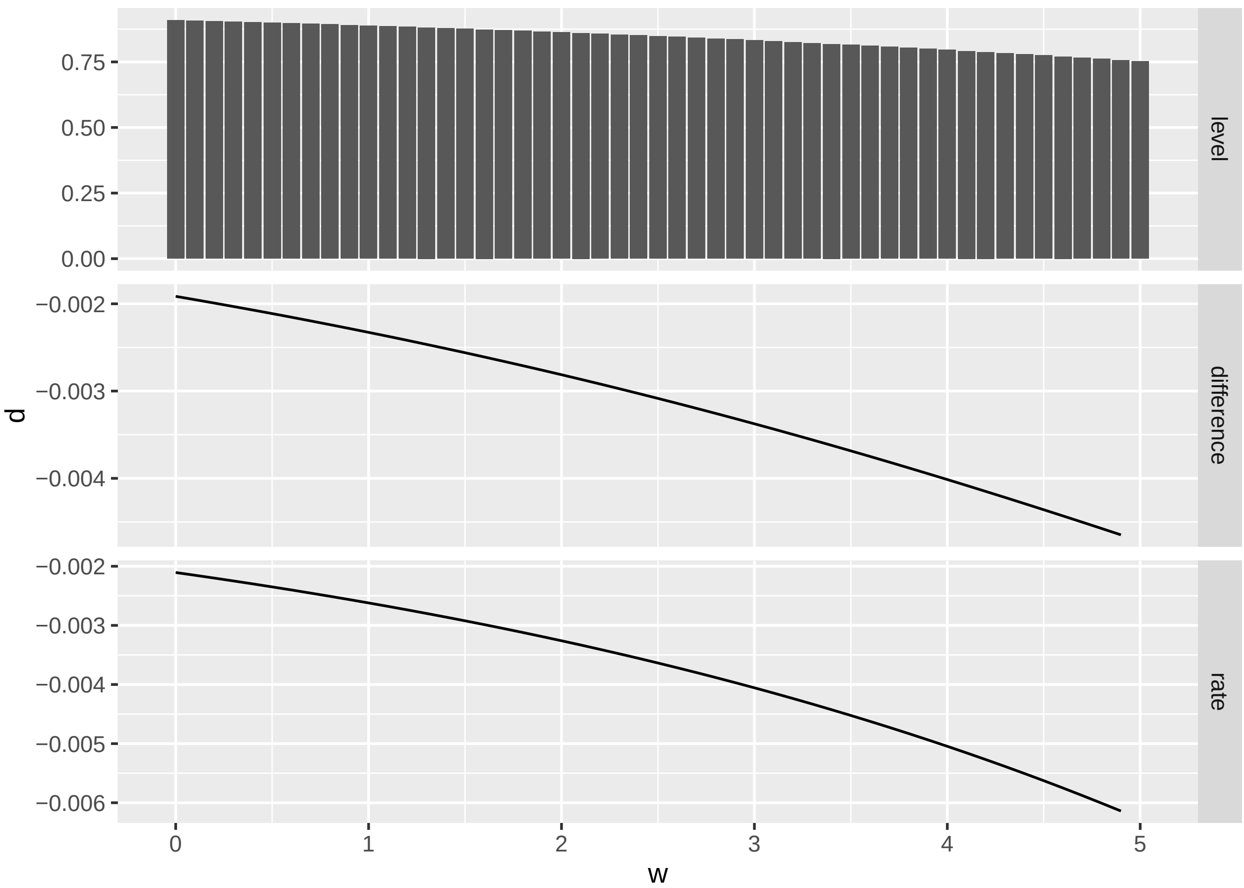

図 1　$d(w)$ with $a = 1.1, b = 1.2, k = 1.2$: levels, first differences, rates of change

0 and 1. Utility of a future plan $r_a^f = (x_{a1}, x_{a2}, x_{a3}, w, s)$ can be expressed as:

$$u\left(r_a^f\right) = \beta d(w)u(x_{a1}) + \beta d(w+s)\delta^s u(x_{a2}) + \beta d(w+s+1)\delta^{s+1}u(x_{a3}).$$

To be consistent, we will also let waiting for s days will incurr a cost in present choices.

$$u\left(r_a^p\right) = u(x_{a1}) + \beta d(s)\delta^s u(x_{a2}) + \beta d(s+1)\delta^{s+1}u(x_{a3}).$$

We parameterise $d(w)$ as

$$d(w) = a^{-b^{wk}}, \quad a, b \in [1, +\infty)^2,\ k \geqslant 1 \tag{1}$$

We choose this doubly exponentiating paramterisation as we intend to impose disproportionately heavy discounting in the future. Note

$$\frac{d(w+s)}{d(w)} = \left(a^{-b^{wk}}\right)^{b^{sk}-1} = d(w)^{b^{sk}-1}, \quad \frac{d(w+s+1)}{d(w)} = \left(a^{-b^{wk}}\right)^{b^{(s+1)k}-1} = d(w)^{b^{(s+1)k}-1}.$$

As can be seen in the figure, this doubly exponentiated discount function discounts the future at an increasing speed. This introduces the increasing costs of considering about the further future.

Let us impose sufficient restrictions on the consumption stream to set the bounds on parameters.

$$\begin{aligned} x_{a1} &= x_1, \quad x_{b1} = x_2, \\ x_{a2} &= x_2, \quad x_{b2} = x_1, \quad 0 < x_1 < x_2. \\ x_{a3} &= x_2, \quad x_{b2} = x_1, \end{aligned}$$

Assume that an individual with this utility chooses the bundle b over a: $r_a^f \leqslant r_b^f$. Then:

$$\begin{aligned}\beta d(w)u(x_2)+&\beta d(w+s)\delta^s u(x_1)+\beta d(w+s+1)\delta^{s+1}u(x_1)\\ &\leqslant \beta d(w)u(x_1)+\beta d(w+s)\delta^s u(x_2)+\beta d(w+s+1)\delta^{s+1}u(x_2).\end{aligned}$$

Then

$$\delta^s\frac{d(w+s)}{d(w)}+\delta^{s+1}\frac{d(w+s+1)}{d(w)}\leqslant 1, \tag{2}$$

Under our parametrisation, (2) becomes

$$d(w)^{b^{sk}-1}\delta^s+d(w)^{b^{(s+1)k}-1}\delta^{s+1}\leqslant 1. \tag{3}$$

Note that $d(w)^{b^{sk}-1}\leqslant 1$ for $a,b\in[1,+\infty)^2$ and $s\geqslant 0$, $k\geqslant 1$. When we specify $d(w)=c^{-w}$ with $c\geqslant 1$, δ that satisfy (2) with an equality will be smaller than that of eqrefdelta.ubound2.

Next, consider the present consumption plans. Let $r_a^p=(x_{a1},x_{a2},x_{a3},0,s)$ be plans of today, s and $s+1$ days later. Then:

$$u\left(r_a^p\right)=u(x_{a1})+\beta d(s)\delta^s u(x_{a2})+\beta d(s+1)\delta^{s+1}u(x_{a3}).$$

If $r_a^p\leqslant r_b^p$, then:

$$u(x_2)+\beta d(s)\delta^s u(x_1)+\beta d(s+1)\delta^{s+1}u(x_1)\leqslant u(x_1)+\beta d(s)\delta^s u(x_2)+\beta d(s+1)\delta^{s+1}u(x_2).$$

This suggests

$$\{d(s)+d(s+1)\delta\}^{-1}\delta^{-s}\leqslant\beta, \tag{4}$$

or

$$\left\{1+d(s)^{k-1}\delta\right\}^{-1}\delta^{-s}\leqslant d(s)\beta. \tag{5}$$

Given $d(s)^{k-1}\simeq 1$ for k close to 1, this increasing speed discounting function reduces the value of β compared to constant speed discounting function nearly by the factor of $d(s)$ even when we have the same δ. When $s>1$ and a large b, this scaling down effect is not trivial. With an increasing speed discount function, δ will be larger as we have seen in (3) which further reduces the LHS of (5).

III Conclusion: Reconciling with a too large β

For an individual with increasing speed discounting, we expect her future choice r_a^f to become extremely impatient relative to the present choice of same consumption bundle r_a^p. This reflects that such an individual is "taxed" by waiting until the future start date and would like to consume a larger bundle immediately once it arrives. Such an individual would express a very short tolerable duration s, such as a few hours, that r_a^f and r_b^f become indifferent.

If we use the standard β–δ model, or single exponential discounting, implied β becomes extremely large because δ obtained from the future choice is smaller and s is less than 1 (day). Because β is derived in (4) by taking a reciprocal, this $(1/\delta^s)$ will have a disproportionately large effect on the implied value. This is what we observe in some of responses in our field data. While it is easy to dismiss such a response as an error or inability to understand the question, the intention of this paper is to show it is possible to reconcile it with a rational choice framework.

The intuition behind assuming a doubly exponentiated discounting function is a heavy penalty of waiting until future. This can be interpreted as wait fatigue or low survival probability felt in the individual's mind.

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