Chapter 14
Framework of International Input-Output Model

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Input-Output Models for Two Different Economies

For simplifying a discussion, let's assume that there are tow different economies, one is a primarily industrialized economy (Economy A) and another is a industrialized economy (Economy B). Economy A is an agro-based economy, which produces agricultural products and imports manufactured products. Economy B is a comprehensive economy, which produces a part of agricultural products and manufactured products, and imports a part of agricultural products. These two economic accounts will be illustrated as Table 1.a and 1.b in the form of input-output table.

Table 1.a Table 1.b

Primarily Industrialized Economy (A)							
	1	2	3	FD	EX	IM	GO
1	5			80	15		100
2	10		8	42		-60	0
3	3		1	16			20
VA	82		11				93
GO	100	0	20	138	15	-60	213
Industrialized Economy (B)							
	1	2	3	FD	EX	IM	GO
1	5	35	10	80		-15	115
2	20	150	50	70	60		350
3	5	30	20	65			120
VA	85	135	40				260
GO	115	350	120	215	60	-15	845

(Notes) 1 = Agriculture, 2 = Manufacturing, 3 = Others, VA = Value added, GO = Gross outputs, FD = Final demands, EX = Exports, and IM = Imports

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Economy A produces 100 units of agricultural products and 20 units of goods and services in other sectors, and exports 15 units of agricultural products and imports 60 units of manufactured products. In this economy, since it is assumed that no manufactured goods are produced, these goods are imported for intermediate uses and final demands.

Economy B produces 115 units of agricultural products, 250 units of manufactured products, and 120 units of goods and services in other sectors, and exports 15 units of agricultural products and imports 60 units of manufactured products.

Input-Output Models for an Economy

Closed economy

If an economy would not import and export goods and services, an input-output model will be quite simple as follows:

$$(1) \quad \mathbf{X} = \mathbf{AX} + \mathbf{F}$$

$$\text{while } \mathbf{A} = \begin{bmatrix} a_{11} & a_{1j} & a_{1n} \\ a_{i1} & a_{1j} & a_{in} \\ a_{n1} & a_{nj} & a_{nn} \end{bmatrix}, a_{ij} = \frac{x_{ij}}{X_j}$$

$$(2) \quad \mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{F}$$

In the above equation, $(\mathbf{I} - \mathbf{A})^{-1}$ is so called as Leontief's inverse matrix.

The following Table 2 is a result of Leontief's inverse matrix obtained from the tables shown above in Table 1.

Table 2:

Country A

	$(I-A)^{-1}$			
	1	2	3	FL
1	1.05	0.00	0.00	1.05
2	0.12	1.00	0.42	1.54
3	0.03	0.00	1.05	1.09
BL	1.20	1.00	1.47	3.68

Country B

	$(I-A)^{-1}$			
	1	2	3	FL
1	1.10	0.23	0.22	1.54
2	0.41	1.98	1.03	3.41
3	0.10	0.21	1.32	1.63
BL	1.60	2.42	2.57	6.58

The primitive economy A has less industrial linkages than the comprehensive economy B. Total repercussion effect is 3.68 for Economy A, and 6.58 for Economy B, respectively.

Open economy

- ***Competitive import model***

As an economy has imports and exports with a partner economy, in fact, an input-output model should be modified as follows:

$$\mathbf{X} = \mathbf{AX} + \mathbf{F} + \mathbf{E} - \mathbf{M} \tag{3}$$

Then, the induced production will be calculated as follows:

$$\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{F} + \mathbf{E} - \mathbf{M}) \tag{4}$$

In the above model, imports are treated as exogenous variable. If imports and domestic products would be competitive for consumers or users, such imports will be treated as indigenous variable, so that the following equation can be realistic.

$$\mathbf{M} = \hat{\mathbf{M}}(\mathbf{AX} + \mathbf{F}), \text{ where } \hat{m}_{ij} = \frac{m_i}{\sum_j x_{ij} + f_j} \tag{5}$$

Then,

$$\mathbf{X} = \mathbf{AX} + \mathbf{F} + \mathbf{E} - \hat{\mathbf{M}}(\mathbf{AX} + \mathbf{F}) \tag{6}$$

$$\mathbf{X} = \{\mathbf{I} - (\mathbf{I} - \hat{\mathbf{M}})\mathbf{A}\}^{-1}\{(\mathbf{I} - \hat{\mathbf{M}})\mathbf{F} + \mathbf{E}\} \tag{7}$$

(8) $\mathbf{M} = \hat{\mathbf{M}}(\mathbf{AX} + \mathbf{F})$

Table 3 shows results of inter-industrial repercussion in non-competitive import type for Economy A and B.

Table 3:

Economy A: Economy B:

$\{I-(I-M)A\}^{-1}$

	1	2	3	FL
1	1.05	0.00	0.00	1.05
2	0.00	1.00	0.00	1.00
3	0.03	0.00	1.05	1.09
BL	1.09	1.00	1.05	3.14

$\{I-(I-M)A\}^{-1}$

	1	2	3	FL
1	1.08	0.20	0.19	1.48
2	0.40	1.96	1.02	3.38
3	0.10	0.21	1.31	1.63
BL	1.58	2.37	2.53	6.49

- ***Non-competitive import model***

In case that imported goods were non-competitive to domestic products, demand-supply equation should be divided into domestic products and imported goods as follows:

(8) $\mathbf{X} = \mathbf{A}^d\mathbf{X} + \mathbf{F}^d + \mathbf{E}$

(9) $\mathbf{M} = \mathbf{A}^m\mathbf{X}^d + \mathbf{F}^m$

Where $\mathbf{A}^d$ and $\mathbf{A}^m$ is defined as follows:

(9.1) $\mathbf{A} = \begin{bmatrix} a^d_{11} & a^d_{1j} & a^d_{1n} \\ a^d_{i1} & a^d_{1j} & a^d_{in} \\ a^d_{n1} & a^d_{nj} & a^d_{nn} \end{bmatrix}, a^d_{ij} = \dfrac{x^d_{ij}}{X_j}$

(9.1) $\mathbf{A} = \begin{bmatrix} a^m_{11} & a^m_{1j} & a^m_{1n} \\ a^m_{i1} & a^m_{1j} & a^m_{in} \\ a^m_{n1} & a^m_{nj} & a^m_{nn} \end{bmatrix}, a^m_{ij} = \dfrac{x^m_{ij}}{X_j}$

Then, domestic production and import requirements will be obtained from the following equations.

(10) $\mathbf{X} = (\mathbf{I} - \mathbf{A}^d)^{-1}(\mathbf{F}^d + \mathbf{E})$

(11) $\mathbf{M} = \mathbf{A}^m(\mathbf{I} - \mathbf{A}^d)^{-1}(\mathbf{F}^d + \mathbf{E}) + \mathbf{F}^m$

Input-output tables for two countries will be combined for complete analysis for international inter-industrial interdependency.

Consolidation of Two Economies' Input-Output Tables

For international input-output analysis, two tables should be consolidated into one comprehensive input-output table. Table 4 shows a flow of consolidation of these two tables into an international input-output table.

Table 4: A Flow of Consolidation of Two I-O Tables into One International I-O

Primarily Industrialized Economy (A)

	1	2	3	FD	EX	IM	GO
1	5			80	15		100
2	10		8	42		-60	0
3	3		1	16			20
VA	82		11				93
GO	100	0	20	138	15	-60	213

Industrialized Economy (B)

	1	2	3	FD	EX	IM	GO
1	5	35	10	80		-15	115
2	20	150	50	70	60		350
3	5	30	20	65			120
VA	85	135	40				260
GO	115	350	120	215	60	-15	845

Modified I-O Table of Primarily Industrialized Economy (A) for International I-O

		A			B			A	B	W	
		1	2	3	1	2	3	FD	FD	XM	GO
A	1	5						80		15	100
	2	10		8				42		-60	0
	3	3		1				16			20
B	1										
	2										
	3										
VA		82		11							93
GO		100	0	20				138		-45	213

Modified I-O Table of Industrialized Economy (B) for International I-O

		A			B			A	B	W	
		1	2	3	1	2	3	FD	FD	XM	GO
A	1										
	2										
	3										
B	1				5	35	10		80	-15	115
	2				20	150	50		70	60	350
	3				5	30	20		65		120
VA					85	135	40				260
GO					115	350	120		215	60	845

Simplly Combined I-O Table for Primarily Industrialized Economy (A) and Industrialized Economy (S)

		A			B			A	B	W	
		1	2	3	1	2	3	FD	FD	XM	GO
A	1	5	0	0	0	0	0	80	0	15	100
	2	10	0	8	0	0	0	42	0	-60	0
	3	3	0	1	0	0	0	16	0	0	20
B	1	0	0	0	5	35	10	0	80	-15	115
	2	0	0	0	20	150	50	0	70	60	350
	3	0	0	0	5	30	20	0	65	0	120
VA		82	0	11	85	135	40	0	0	0	353
GO		100	0	20	115	350	120	138	215	15	1058

Simplly Combined I-O Table for Primarily Industrialized Economy (A) and Industrialized Economy (S)

		A			B			A	B	W	
		1	2	3	1	2	3	FD	FD	XM	GO
A	1	5						80		15	100
	2	10		8				42		-60	0
	3	3		1				16			20
B	1				5	35	10		80	-15	115
	2				20	150	50		70	60	350
	3				5	30	20		65		120
VA		82		11	85	135	40				260
GO		100	0	20	115	350	120	138	215	60	845

Non-competitive-type International I-O Table for Primarily Industrialized Economy (A) and Industrialized Economy (S)

		A			B			A	B	XM	GO	TS
		1	2	3	1	2	3	FD	FD			
A	1	5			1	4	1	80	9	0	100	100
	2	0	0	0	0	0	0	0	0	0	0	0
	3	3		1				16			20	20
B	1				4	31	9	0	71	0	115	115
	2	10	0	8	20	150	50	42	70	0	350	350
	3				5	30	20		65		120	120
VA		82		11	85	135	40				260	353
GO		100	0	20	115	350	120	138	215	60	845	1118
TI		100	0	20	115	350	120	138	215	0	965	1058

Table 5 is a result of Leontief's inverse matrix obtained from the international input-output table for Economy A and B, which is shown in Table 4. We can find a total requirement in this analytical table is the highest among the analytical tables mentioned above. Total of backward or forward linkage is 11.01 (see Table 6). This is a complete evaluation of international input-output linkage effects for two economies. For an analysis for a single country, the results based on one economy's input-output table will be undervalued since the international effects on imports and exports were not completely covered in a single economy's input-output table. Therefore, we are planning to construct the international input-output table for

Thailand and Indochina economies.

Table 5:

$(I-A)^{-1}$

		A			B			FW
		1	2	3	1	2	3	
A	1	1.06	0.00	0.01	0.01	0.03	0.03	1.13
	2	0.00	1.00	0.00	0.00	0.00	0.00	1.00
	3	0.03	0.00	1.05	0.00	0.00	0.00	1.09
B	1	0.02	0.00	0.08	1.08	0.20	0.20	1.58
	2	0.23	0.00	0.83	0.40	1.97	1.02	4.46
	3	0.03	0.00	0.09	0.10	0.21	1.32	1.74
BL		1.37	1.00	2.07	1.60	2.41	2.56	11.01

Table 6: Comparison of Backward and Forward Linkage

		IIO: $(I-A)^{-1}$		NIO: $(I-A)^{-1}$		$\{I-(I-M)A\}^{-1}$	
		BL	FW	FL	BL	FL	BL
A	1	1.37	1.13	1.05	1.20	1.05	1.09
	2	1.00	1.00	1.54	1.00	1.00	1.00
	3	2.07	1.09	1.09	1.47	1.09	1.05
B	1	1.60	1.58	1.54	1.60	1.48	1.58
	2	2.41	4.46	3.41	2.42	3.38	2.37
	3	2.56	1.74	1.63	2.57	1.63	2.53
BL		11.01	11.01	10.26	10.26	9.62	9.62

Table 7 shows induced gross outputs by final demands of A and B. For example, 86% of total gross output of sector 1 of A (100 units) are induced by final demands of A, residual 14% by B. 33% and 67% of industrial products (sector 2) of B are induced by A and B, respectively. This means that the industrial sector of B is highly depended upon final demands of A.

Table 7:

Induced GO by Final Demand of A and B

		$(I-A)^{-1}F$		GO	C% $(I-A)^{-1}F$		R% $(I-A)^{-1}F$		GO
		A	B		A	B	A	B	
A	1	86	14	100	0.35	0.03	0.86	0.14	1.00
	2	0	0	0	0.00	0.00	...	...	0.00
	3	20	0	20	0.08	0.00	0.98	0.02	1.00
B	1	12	103	115	0.05	0.22	0.10	0.90	1.00
	2	115	235	350	0.47	0.51	0.33	0.67	1.00
	3	12	108	120	0.05	0.23	0.10	0.90	1.00
X^*		244	461	705	1.00	1.00			

Estimation of Input-Output Tables for Indochina Countries

Input-output tables for Indochina countries are not available except Vietnam[1]. In this project, we have to estimate the tables from quite limited statistical information and published data. This section will discuss how to estimate the tables for our analysis.

Case 1: Only gross outputs, value added, final demand, exports and imports are available from published data

In case those only gross outputs, value added, final demand, exports and imports are available from published data, an incomplete input-output table is available as shown in Table 4.1. For the estimation of intermediate transactions, we can employ some other input-output coefficients from the economy, which has similar industrial structures with the country concerned. Let us say such input coefficients as the preliminary input coefficients (see Table 4.1).

[1] Input-output tables for Vietnam: There are several information of Vietnamese input-output tables. The latest input-output table is for 1987. However, no table is published and available from official sources.

Table 4.1

Case 1: GO, VA, FD, EX and IM available

	1	2	3	FD	EX	IM	GO
1				70	15		100
2				56		-60	40
3				51			60
VA	82	10	40				132
GO	100	40	60	177	15	-60	332

Preliminary Input Coefficients

	1	2	3
1	0.10	0.20	0.01
2	0.20	0.30	0.25
3	0.05	0.10	0.10

Then, we can estimate a preliminary input-output table, shown in Table 4.2, based on two tables mentioned above. Table 4.2 has not yet been balanced both in input-side and output-side. Therefore, it is necessary to make a balance for the table. For this purpose, RAS method is usually employed. Table 4.3 shows the balancing steps by RAS method. Step 2 shows the table, which is balanced in column-wise only. Step 2 is the table balanced in row-wise. In this example, Step 2 and Step 3 balanced the table for intermediate transactions. If the size of table were larger than this, it would be repeated these balancing procedures several times until the statistical discrepancies were eliminated. Table 4.4 is a final estimated table and an estimated input coefficient matrix in this procedure.

Table 4.2: Preliminary Input-Output Table

Estimated I-O Table from Available Information

	1	2	3	FD	EX	IM	GO	TO	RD
1	10	8	1	70	15		100	104	-4
2	20	12	15	56		-60	40	43	-3
3	5	4	6	51			60	66	-6
VA	82	10	40	0			132	132	0
GO	100	40	60	177	15	-60	332	332	0
TI	117	34	62	177	15	-60	332	345	-13
CD	-17	6	-2	0	0	0	0	-13	13

Table 4.3

Step 1:

	1	2	3	TIS	TID	RD
1	10	8	1	15	19	-4
2	20	12	15	44	47	-3
3	5	4	6	9	15	-6
TIC	18	30	20	68	68	0
TII	35	24	22	68	81	-13
CD	-17	6	-2	0	-13	13

Step 2:

	1	2	3	TIS	TID	RD
1	5	10	1	15	16	-1
2	10	15	14	44	39	5
3	3	5	6	9	13	-4
TIC	18	30	20	68	68	0
TII	18	30	20	68	68	0
CD	0	0	0	0	0	0

Step 3:

	1	2	3	TIS	TID	RD
1	5	10	1	15	15	0
2	12	17	16	44	44	0
3	2	3	4	9	9	0
TIC	18	30	20	68	68	0
TII	18	30	20	68	68	0
CD	-0	0	0	0	0	0

Table 4.4:

Estimated I-O Table from Availbale Information

	1	2	3	FD	EX	IM	GO	TO	RD
1	5	10	1	70	15		100	100	0
2	12	17	16	56		-60	40	40	0
3	2	3	4	51			60	60	0
VA	82	10	40	0			132	132	0
GO	100	40	60	177	15	-60	332	332	0
TI	100	40	60	177	15	-60	332	332	0
CD	0	0	0	0	0	0	0	0	0

Estimated Input Coefficients

	1	2	3
1	0.05	0.24	0.01
2	0.12	0.42	0.26
3	0.02	0.09	0.06

Case 2: Only gross outputs, value added, exports and imports are available from published data

In this case, the similar balancing procedure can be applied after adding the final demand vector in calculation. Table 4.5 and 4.6 shows the steps and the result. In this procedure, the step 3 still remain the unbalanced value in the column sector 3 and the final demand, but in repeating this iteration until the table was balanced, we can obtained the final balanced input-output table shown in Table 4.6.

Table 4.5

Step 1:

	1	2	3	FD	TIS	TID	RD
1	10	8	1	80	85	98	-13
2	20	12	15	53	100	100	0
3	5	4	6	44	60	59	1
TIC	18	30	20	177	245	258	-13
TII	35	24	22	177	245	258	-13
CD	-17	6	-2	0	0	0	0

Step 2:

	1	2	3	FD	TIS	TID	RD
1	5	10	1	80	85	95	-10
2	10	15	14	53	100	92	8
3	3	5	6	44	60	57	3
TIC	18	30	20	177	245	245	0
TII	18	30	20	177	245	245	0
CD	0	0	0	0	0	0	0

Step 3:

	1	2	3	FD	TIS	TID	RD
1	5	9	0	71	85	85	0
2	11	16	15	58	100	100	0
3	3	5	6	46	60	60	0
TIC	18	30	20	177	245	245	0
TII	18	30	21	175	245	245	0
CD	0	0	-1	2	0	0	0

Table 4.6

	1	2	3	FD	EX	IM	GO	TO	RD
1	5	9	0	71	15		100	100	0
2	11	16	14	59		-60	40	40	0
3	3	5	5	47			60	60	0
VA	82	10	40	0			132	132	0
GO	100	40	60	177	15	-60	332	332	0
TI	100	40	60	177	15	-60	332	332	0
CD	0	-0	-0	0	0	0	0	0	0

	1	2	3
1	0.05	0.22	0.01
2	0.11	0.40	0.24
3	0.03	0.13	0.09

Actually, there will be more complicated cases for Indochina countries due to lack of statistical information and data published officially. However, it will be significant to try such estimation and to analyze international interindustrial interdependence among Thai economy and Indochina economies based on this method.