INSTITUTE OF DEVELOPING ECONOMIES

IDE Discussion Papers are preliminary materials circulated
to stimulate discussions and critical comments

IDE DISCUSSION PAPER No. 943

Are spatial differentiation and product differentiation substitutes?*

Toshitaka Gokan† and Jacques-François Thisse‡

October 2024

Abstract

We revisit the Hotelling model in the case of firms selling differentiated varieties. When product differentiation outweighs travel costs, the location-then-price game has a unique subgame perfect Nash equilibrium in which the two firms located at the market center.

Keywords: location; product differentiation; agglomeration.

JEL classification: D43; L13; R12.

* We thank N. Baum-Snow for comments on an earlier version. This work was supported from JSPS KAKENHI Number JP23K01348.

† Institute of Developing Economies, JETRO (Japan). E-mail: Toshitaka_Gokan@ide.go.jp

‡ UCLouvain (Belgium), Institute of Developing Economies, JETRO (Japan) and CEPR. E-mail: jacques.thisse@uclouvain.be.

The Institute of Developing Economies (IDE) is a semigovernmental, nonpartisan, nonprofit research institute, founded in 1958. The Institute merged with the Japan External Trade Organization (JETRO) on July 1, 1998. The Institute conducts basic and comprehensive studies on economic and related affairs in all developing countries and regions, including Asia, the Middle East, Africa, Latin America, Oceania, and Eastern Europe.

INSTITUTE OF DEVELOPING ECONOMIES (IDE), JETRO
3-2-2, WAKABA, MIHAMA-KU, CHIBA-SHI
CHIBA 261-8545, JAPAN

1 Introduction

A few empirical papers suggest a positive answer to the question that serves as the title to this paper. First, until 2005, the City of Milan (Italy) imposed a minimum distance requirement between any two restaurants. Starting in 2005, a new restaurant could open anywhere it could find a profitable location. Seven years after the deregulation, restaurants were significantly more spatially concentrated than before 2005, but they were more differentiated according to different attributes such as consumer quality ratings and the type of cuisine (Leonardi and Moretti, 2023). Likewise, studying $30,000$ restaurants rated by TripAdvisor across large cities in the United Kingdom, Mossay *et al.* (2022) highlight the existence of a quality effect: the top-rated restaurants tend to be more concentrated around the city center than the low-rated restaurants. Compared to low-quality restaurants, high-quality restaurants have more opportunities to differentiate themselves along non-spatial characteristics such as the type of food and cuisine they offer. Nilsson *et al.* (2018) document the emergence of microbrewery clusters in the United States. They note that competition within clusters is eased through product differentiation caused by the artisan nature of craft beer. Last, when zoning regulation prevents spatial differentiation, Datta and Sudhir (2013) find that firms differentiate more on retail formats.

One of the cornerstones of retailing location theory is the spatial competition model developed by Hotelling (1929). According to Brown (1989, 1993), Hotelling's paper has attracted an enormous amount of academic research, especially what became to be known as the *Principle of Minimum Differentiation* (PMD). This principle states that two retailers selling the same good and competing to attract customers spatially distributed along Main Street choose to locate back-to-back at the market center. The PMD seems appealing because it suggests that competition among retailers would lead to the formation of a shopping street. Unfortunately, things are not so simple.

The PMD used in retailing models rules out price competition between sellers (see Biscaia and Mota, 2013, and Drezner and Eiselt, 2024 for surveys). When firms are located together and free to choose their prices, each retailer has an incentive to undercut its rival because doing so allows it to capture the whole market. Indeed, nothing differentiates retailers so that buying from the cheaper one seems a sensible strategy from consumers' viewpoint. In other words, retailers fall into Bertrand's trap where they make no profits. By moving away from the cluster, any firm can build on the distance that separates it from its competitor to earn positive profits. Hence, there is no PMD when retailers sell the same homogeneous good because *price competition works as a strong dispersion force.* We show below how and why product differentiation may restore the PMD.

Hotelling describes the market outcome by means of what is known as a subgame perfect Nash equilibrium of a two-stage game with two players. In the first stage, anticipating the consequences of their location choices in the ensuing price subgame, firms choose where to set up. In the second stage, locations being observed, firms compete in producer prices. Here, the issue is that a price equilibrium in pure strategies does not exist for a large domain of location pairs (d'Aspremont *et al.*, 1979). For example, when firms are symmetrically located, there exists no price equilibrium when firms set up between the first and third quartiles. Using quadratic travel costs, d'Aspremont *et al.* show that a price equilibrium exists for all location pairs and that firms choose to locate at the two endpoints of Main Street. This demonstrates that *price competition is a strong dispersion force.* When consumers' tastes are heterogeneous and described by the multinomial logit, de Palma *et al.* (1985) show that firms choose simultaneously to locate at the market center and to price above marginal cost when they sell sufficiently differentiated products.

However, this approach does not allow solving the sequential game and remains silent about what happens in the price subgames. This is where we hope to contribute.

In this paper, we assume that firms sell differentiated products while consumers have a preference for variety. Rather than the logit, we use the linear probability model that allows us to provide an analytical solution to the Hotelling model and to show that *the PMD holds true when product differentiation overcomes spatial differentiation.* Here, the geographical dispersion of consumers along Main Street is the dispersion force. What is the agglomeration force in such a setting? The answer is nonstandard. Indeed, the two firms locate back-to-back at the market center when varieties are sufficiently differentiated, travel costs are sufficiently low, or both. In other words, agglomeration emerges when transportation costs are low enough, like in Krugman (1991). However, here *agglomeration is strategic*: it is caused by the spatial interdependence between retailers selling sufficiently differentiated varieties, which compete for the location that provides the best access to potential customers. Hence, the agglomeration force used here differs from the agglomeration economies highlighted in new economic geography and urban economics (Duranton and Puga, 2004). In the CES model of monopolistic competition in which varieties are differentiated and markups are constant, retailers always choose to be agglomerated, but they need not locate at the market center (Tabuchi, 2009). This difference in results highlights the role of strategic interactions between firms.

Our model is presented in the next section. It involves only two parameters, i.e., the transportation rate (t), which measures the degree of geographical differentiation, and the size of the support of individual tastes (Δ), which gauges the degree of product differentiation. The price competition stage is studied in Section 3. Two cases may arise according to the relative values of t and Δ.

First, there is a unique price equilibrium for all location pairs when product differentiation is stronger than geographical differentiation ($\Delta > t$) and market areas fully overlap. Second, when $\Delta \leq t$, market areas partially overlap but prices may be sufficiently high to incentivize at least one firm to be the only supplier. However, a price equilibrium exists for specific domains of location pairs. When a price equilibrium exists, it displays the following features: each firm has a hinterland and consumers who buy both varieties are situated between the two firms.

In Section 4, we study the location game and show that *firms always choose to locate at the market center* when the preference for variety is strong enough for all consumers to patronize the two firms ($\Delta > t$). Hence, the answer to our title is: *yes*. Section 5 concludes.

Related literature. Our paper is related to earlier works in spatial competition theory where it is shown that firms choose to maximize their differentiation along the main characteristics while minimizing differentiation along the others (Tabuchi, 1994; Irmen and Thisse, 1998). For example, firms locate together at the market center when they sell products that display very different qualities. Unlike us, these authors work with quadratic travel costs, which implies the existence of a unique price equilibrium for any location pair (d'Aspremont *et al.*, 1979).

There are at least two reasons for consumers to visit conventional retailers. First, some services are not tradable and must be consumed on site, e.g. dining in a restaurant. Second, marketing researchers recognize that conventional retailers provide specific advantages to consumers, such as touching, tasting, or directly seeing (Balasubramanian, 1998; Guo and Lai, 2017). Further, even when consumers know the set of varieties provided by the market, they may be unsure about which variety is offered where (and at what price), thus implying that consumers must visit brick-and-mortar stores to find out their most-preferred variety. In this case, a consumer compares the cost of visiting an additional retailer with the expected gain in terms of surplus.

When a consumer wants to buy several (differentiated) goods, she can buy them in a single trip, stopping at different places or, better yet, buying goods at the same marketplace. In other words, there are economies of scope in shopping (Eaton and Lipsey, 1982; Schulz and Stahl, 1996). Trip-chaining increases competition and reduces firms' benefits from being local monopolies. When retailers are located together, the travel cost to the cluster is sunk and a consumer can visit any retailer at a very low cost. Thus, a consumer is more likely to visit a cluster than an isolated firm because of the higher probability she has of finding there a good match and a good price. Spatial clustering of firms is, therefore, a particular means by which firms can facilitate consumer search. When firms realize this, each of them understands that it might be in their own interest to form a marketplace with their competitors. For this to happen, however, competition among retailers cannot be harsh and the cluster must be sufficiently attractive (Wolinsky, 1983; Dudey, 1990; Konishi, 2005). Agglomeration economies arise here because households bear lump-sum travel costs.

The importance of trip-chaining has been largely confirmed empirically (Koster *et al.*, 2019; Miyauchi *et al.*, 2021). Such travels imply that a utility-maximizing consumer must determine a purchasing plan that satisfies the demand for a set of goods while minimizing her total expenditure. This optimization problem, known as the traveling purchaser problem, is NP-hard (Manerba *et al.*, 2017). It is associated with a particular structure of substitution between outlets, which introduces non-convexities in budget constraints and discontinuities in demands that are difficult to deal with (Thill, 1986). In such a context, the benefits of a shopping street are likely to be high because this one allows consumers to save a substantial transaction costs. And indeed, for Dutch shopping streets, Koster *et al.* (2019) find an elasticity of property rents with respect to the number of stores of at least 0.25, which is considerably higher than the agglomeration elasticities found on the production side.

In what follows, we assume that consumers make single-purpose trips, which is reasonable for some activities, such as visiting restaurants or groceries, as well as other facilities providing services that must be consumed on-site.

2 Model set-up

The market features a unit measure of consumers and two brick-and-mortar retailers, denoted 1 and 2, which produce a differentiated good at a marginal cost normalized to 0. The good sold by the two firms is horizontally differentiated along Lancasterian characteristics and consumers are heterogeneous in tastes. Consumers are uniformly distributed over the interval $[0, 1]$ and each consumer buys one unit of the good. The indirect utility of a consumer who chooses variety i is given by $V_i(x) = -p_i + \varepsilon_i$ where ε_i measures the quality of the match between the consumer and variety i. In this paper, we assume that $\varepsilon \equiv \varepsilon_2 - \varepsilon_1$ is uniformly distributed over $[-\Delta, \Delta]$ with a density $1/2\Delta$, where $\Delta > 0$. In other words, heterogeneity is modeled by the *linear probability model* where Δ measures the degree of heterogeneity across consumers.

The probabilities to buy from firms 1 and 2 are, respectively, given by

$$\begin{aligned}\delta_1 &= \Pr(V_1 \geq V_2) = \begin{cases} 0 & \text{if } p_1 \geq p_2 + \Delta, \\ \frac{-p_1+p_2+\Delta}{2\Delta}, & \text{if } -\Delta < p_1 - p_2 < \Delta, \\ 1 & \text{if } \ p_2 - \Delta \geq p_1, \end{cases} \\ \delta_2 &= 1 - \delta_1(p_1, p_2).\end{aligned}$$

Clearly, $\delta_1 > \delta_2$ if and only if $p_1 < p_2$, while $\delta_1 + \delta_2 = 1$. Profits are given by $\Pi_i(p_1, p_2) = p_i\delta_i$ and concave in firms' own prices when $\delta_i > 0$. Applying the first order conditions yields the equilibrium prices $p_1^* = p_2^* = \Delta$. Hence, product differentiation and consumer heterogeneity relaxes price competition. At the limit, when $\Delta \to 0$, we fall back on the Bertrand outcome, that is, firms price at marginal cost, here zero.

We now add geographical differentiation by assuming that firm i is located at $x_i \in [0, 1]$ with $0 \leq x_1 \leq x_2 \leq 1$, while consumers are uniformly distributed along a linear city $[0, 1]$ with a unit density. Traveling to firms is costly; let $t > 0$ be the unit travel cost. This parameter measures the degree of geographical differentiation, very much like Δ measures taste heterogeneity. Consequently, consumers are heterogeneous along *two* different dimensions, that is, their location in the geographical space and their idiosyncratic preferences. As a result, the full price $p_i(x)$ paid by a consumer located at x is equal to the producer price p_i set by the retailer plus the cost of travelling to firm i, i.e., $t\,|x - x_i|$. The indirect utility of a consumer located at x who chooses variety i is thus given by $V_i(x) = -p_i - t\,|x - x_i| + \varepsilon_i$.

Like in Hotelling's original model, the market is described by a two-stage game in which firms choose simultaneously their locations and, then, their prices. The outcome is given by a subgame perfect Nash equilibrium. As usual, the game is solved by backward induction.

3 Price competition

We consider two locations $x_1 \leq x_2$ and study the associated price subgame. The probabilities that the consumer located at x buys from firm 1 or firm 2 are, respectively, given by

$$\begin{aligned}\delta_1(x) &= \Pr(V_1(x) \geq V_2(x)) = \begin{cases} 0 & \text{if } x \geq x_{\max}, \\ \frac{\Delta - p_1 - t|x-x_1| + p_2 + t|x-x_2|}{2\Delta} & \text{if } x_{\min} < x < x_{\max}, \\ 1 & \text{if } \ x \leq x_{\min}, \end{cases}, \\ \delta_2(x) &= 1 - \delta_1(x),\end{aligned} \tag{1}$$

where $x_{\min} \geq 0$ is the marginal consumer indifferent between buying from firm 1 only or from both firms and $x_{\max} \leq 1$ the marginal consumer indifferent between buying from firm 2 only or from both firms. Therefore, $x_{\min}$ must solve the equation

$$\delta_1(x) = \frac{\Delta - p_1 - t\,|x - x_1| + p_2 + t\,|x - x_2|}{2\Delta} = 1, \tag{2}$$

while $x_{\max}$ solves the equation

$$\delta_1(x) = \frac{\Delta - p_1 - t\,|x - x_1| + p_2 + t\,|x - x_2|}{2\Delta} = 0. \tag{3}$$

Thus, unlike the standard model where there is one marginal consumer, the model with heterogeneity allows for *two marginal consumers* because market areas overlap. Furthermore, since $\delta_1(x) + \delta_2(x) = 1$, (2) and (3) have the nature of *gravity equations*: $\delta_1(x)$ increases (decreases) with p_2 and the distance to firm 2 (p_1 and the distance to firm 1) where a firm's producer price is an inverse measure of its attractiveness.

Two cases may arise. In the first one, the two market areas fully overlap, while the overlapping is partial in the second one.

3.1 Full overlapping of market areas

Assume first that $x_{\min} = 0$ and $x_{\max} = 1$. This means $\delta_{1c}(1) > 0$ and $\delta_{2a}(0) > 0$, which hold if and only if $\Delta > t$. Then, the probabilities that the consumer located at x buys from firm 1 or firm 2 are, respectively, given by

$$\begin{aligned} \delta_1(x) &= \Pr(V_1(x) \geq V_2(x)) = \begin{cases} \delta_{1a}(x) = \frac{\Delta - p_1 + p_2 + t(x_2 - x_1)}{2\Delta} & \text{if } x \leq x_1, \\ \delta_{1b}(x) = \frac{\Delta - p_1 + p_2 + t(x_1 + x_2) - 2tx}{2\Delta}, & \text{if } x_1 < x < x_2, \\ \delta_{1c}(x) = \frac{\Delta - p_1 + p_2 - t(x_2 - x_1)}{2\Delta} & \text{if } \quad x \geq x_2, \end{cases} , \qquad (4) \\ \delta_2(x) &= 1 - \delta_1(x). \end{aligned}$$

Firms' profits are then defined as follows:

$$\begin{aligned} \pi_1(p_1, p_2) &= p_1 \left(\int_0^{x_1} \delta_{1a}(x)dx + \int_{x_1}^{x_2} \delta_{1b}(x)dx + \int_{x_2}^{1} \delta_{1c}(x)dx \right) \qquad (5) \\ &= \frac{p_1}{2\Delta}(\Delta - p_1 + p_2 + t(x_2 - x_1)(x_1 + x_2 - 1)), \end{aligned}$$

$$\begin{aligned} \pi_2(p_1, p_2) &= p_2 \left(\int_0^{x_1} \delta_{2a}(x)dx + \int_{x_1}^{x_2} \delta_{2b}(x)dx + \int_{x_2}^{1} \delta_{2c}(x)dx \right) \qquad (6) \\ &= \frac{p_2}{2\Delta}(\Delta + p_1 - p_2 - t(x_2 - x_1)(x_1 + x_2 - 1)). \end{aligned}$$

Applying the first-order condition for profit maximization yields the prices:

$$p_1^*(x_1, x_2) = \Delta + \frac{t}{3}(x_2 - x_1)(x_1 + x_2 - 1), \quad p_2^*(x_1, x_2) = \Delta - \frac{t}{3}(x_2 - x_1)(x_1 + x_2 - 1). \qquad (7)$$

For the profit functions to be defined by (5) and (6), both demands must be positive at (7) over the whole interval $[0, 1]$. That is, the most distant consumer from firm 2, who is located at $x = 0$, has a positive demand for variety 2 ($\delta_{2a}(0) > 0$)), while the most distant consumer from firm 1 located at $x = 1$ has a positive demand for variety 1 ($\delta_{1c}(1) > 0$)).

Plugging (7) in (4) yields:

$$\begin{aligned} \delta_{2a}(0) &> 0 \Leftrightarrow \Delta > t(x_2 - x_1) \left(\frac{1 + 2(x_1 + x_2)}{3} \right), \qquad (8) \\ \delta_{1c}(1) &> 0 \Leftrightarrow \Delta > t(x_2 - x_1) \left(\frac{5 - 2(x_1 + x_2)}{3} \right). \qquad (9) \end{aligned}$$

Differentiating each right-hand side of these inequalities with respect to x_1 and x_2 shows that both inequalities hold for all x_1 and x_2 in $[0, 1]$ if and only if $\Delta > t$. Under this condition, as $\Pi_i(p_1, p_2)$ is strictly concave in p_i. Therefore, the prices $p_1^*(x_1, x_2)$ and $p_2^*(x_1, x_2)$ are the unique candidate interior equilibrium of the price subgame associated with locations x_1 and x_2.

Can firm 1, say, capture the whole market? For this, firm 1 must choose a price that solves

$$\delta_{1c}(1) = \frac{\Delta - p_1 + p_2^* - t(x_2 - x_1)}{2\Delta} = 1.$$

that is,

$$\tilde{p}_1 = p_2^* - t(x_2 - x_1) - \Delta = -\frac{1}{3}t\,(x_2 - x_1)\,(x_1 + x_2 + 2) \leq 0.$$

As the same argument holds for firm 2, no firm finds it profitable to undercut its competitor.

The next proposition summarizes our main result.

Proposition 1. *Assume $\Delta/t > 1$. Then, for all location pairs, there exists a unique price equilibrium given by* (7).

In other words, the existence of a price equilibrium is restored for all location pairs when product differentiation, measured by Δ, dominates geographical differentiation, i.e., t. Furthermore, the equilibrium prices increase with Δ but vary in opposite directions with t. In particular, when firms are symmetrically located, we have $p_1^*(x_1, x_2) = p_2^*(x_1, x_2) = \Delta > t$. Note that the equilibrium prices are independent of firms' locations because the heterogeneity overcomes the travel cost effect.

Clearly, $p_1^* > p_2^*$ if and only if $x_1 > 1 - x_2$, that is, firm 1 is closer to the market center $x = 1/2$ than firm 2. In other words, the firm located near the market center is able to charge a higher price than its competitor because the distance to its furthest customer is shorter. How does price p_i^* vary with x_i? Differentiating p_i^* shows that it increases when firm i locates closer to its competitor and is maximized at $x_i = 1/2$. This is because product differentiation weakens the procompetitive effect associated with a shorter distance between firms, thus allowing each firm to benefit from a more central location.

3.2 Partial overlapping of market areas

Under partial overlapping, at least one firm supplies its own hinterland, either $[0, x_{\min})$ or $(x_{\max}, 1]$, and competes with its rival over $[x_{\min}, x_{\max}]$. Different cases may arise according to the position of the marginal consumers $x_{\min}$ and $x_{\max}$ relative the locations of firms. Hence, a full analysis requires cumbersome analytical developments. To keep things simple, we now assume that firms are symmetric, that is, $x_1 + x_2 = 1$. We show in Appendix that $0 \leq x_1 \leq x_{\min} < x_{\max} \leq x_2 \leq 1$ is the only configuration that can be sustained at a price equilibrium, that is, the two marginal consumers are inside the two firms. In this case, we have:

$$x_{\min} = \frac{-\Delta - p_1 + p_2 + t}{2t} \geq x_1, \qquad x_{\max} = \frac{\Delta - p_1 + p_2 + t}{2t} \leq 1 - x_1,$$

which are symmetric about 1/2. Observe that $x_{\min}$ increases with x_1 because firm 1 gets closer to the consumers located just to the right of the marginal consumer $x_{\min}$. It also increases with x_2 because these consumers no longer buy from firm 2 which is now located further away from them. Since $x_{\max} - x_{\min} \leq 1$ implies $\Delta \leq t$, partial overlapping of market areas emerges if and only travel costs dominates the best matching. Note that $x_{\max} - x_{\min} = \Delta/t$ implies that the two marginal consumers move apart in response to a rise in Δ/t. By contrast, $x_{\min} = x_{\max}$ when $\Delta = 0$, like in Hotelling where there is a single marginal consumer.

Using (1), firms' profits are defined as follows:

$$\pi_1 = p_1 \left(\int_0^{x_{\min}} dx + \int_{x_{\min}}^{x_{\max}} \delta_{1b}(x)dx \right) = \frac{1}{2t} p_1 (p_2 - p_1 + t),$$

$$\pi_2 = p_2 \left(\int_{x_{\max}}^{1} dx + \int_{x_{\min}}^{x_{\max}} \delta_{2b}(x)dx \right) = \frac{1}{2t} p_2(t + p_1 - p_2).$$

Differentiating π_i with respect to p_i and solving the resulting system of equations yield the prices

$$p_1^*(x_1, x_2) = p_2^*(x_1, x_2) = t, \tag{10}$$

which are identical to the prices obtained by Hotelling (1929).

It is readily verified that the corresponding profits are given by

$$\pi_1^* = \pi_2^* = \frac{t}{2}. \tag{11}$$

Evaluating $x_{\min}$ at prices (10), it must be that

$$x_{\min} = \frac{-\Delta + t}{2t} \geq x_1 \Leftrightarrow 1 - 2x_1 \geq \frac{\Delta}{t}, \tag{12}$$

which requires $\Delta/t \leq 1$, for otherwise (12) would imply $x_1 < 0$. As t rises, p_i^* increases but is independent of Δ because there is more geographical differentiation and little product differentiation.

It remains to verify whether firm 1 finds it profitable to deviate from $p_1^*(x_1, x_2)$ by setting a price low enough for all consumers to buy from it only when firm 2 sets $p_2^*(x_1, x_2)$. For firm 1 to supply the whole market, it must choose a price such that the consumer at $x = 1$ prefers buying from 1 than from 2. This price is obtained by solving with respect to p_1 the indifference condition $\delta_1(p_1, p_2^*; 1) = 1$, that is,

$$t + tx_1 - \Delta = p_1 + t(1 - x_1),$$

which yields

$$\hat{p}_1 = 2tx_1 - \Delta.$$

Note that $\hat{p}_1$ decreases with Δ because consumers value more the consumption of differentiated varieties, which makes it harder for firm 1 to become the single provider.

When firm 1 chooses the price $\hat{p}_1$, its profit is equal $\hat{p}_1$ because all consumers buy only from this firm. Therefore, firm 1 does not want to deviate if and only if

$$\pi_1(p_1^*, p_2^*) \geq \pi_1(\hat{p}_1, p_2^*) \Leftrightarrow \frac{t}{2} \geq 2tx_1 - \Delta,$$

which is equivalent to

$$t(1 - 4x_1) + 2\Delta \geq 0. \tag{13}$$

Using (12) and (13), we have the following proposition.

Proposition 2. *Assume $\Delta/t \leq 1$. If firms are symmetrically located at $x_1 \in [0, 1]$ and $1 - x_1$, then there exists a unique price equilibrium with partial overlapping if and only if*

$$1 - 2x_1 \geq \frac{\Delta}{t} \geq 2x_1 - \frac{1}{2}. \tag{14}$$

Furthermore, the equilibrium prices are given by (10).

When $\Delta = 0$, the condition (13) boils down to $x_1 \leq 1/4$, which is identical to that obtained by d'Aspremont *et al.* (1979). When x_1 increases beyond 1/4, a price equilibrium exists if (14) is satisfied. In other words, a price equilibrium may exist for $x_1 > 1/4$ when $\Delta > 0$, meaning that *product differentiation allows sustaining a price equilibrium for a wider interval of locations.* Note also that $1-2x_1 > 2x_1 - 1/2$ in (14) holds if and only if $x_1 < 3/8$. That is, over the location interval $[3/8, 1/2]$, heterogeneity cannot guarantee the existence of a price equilibrium with partial overlapping. This is because firms are sufficiently close to 1/2 and products insufficiently differentiated for price competition to be relaxed. As seen in Proposition 1, Δ/t must exceed 1 for a price equilibrium to exist for such location pairs.

4 Location competition

We now turn our attention to the location game, which we can be studied only in the case where there is full overlapping ($\Delta > t$) because a unique price equilibrium exists for all location pairs. To do this, we plug the equilibrium prices (7) into the profit functions (5)-(6) and obtain:

$$\pi_1^* = \frac{[\Delta + t(x_2 - x_1)(x_1 + x_2 - 1)/3]^2}{2\Delta}, \qquad \pi_2^* = \frac{[\Delta - t(x_2 - x_1)(x_1 + x_2 - 1)/3]^2}{2\Delta},$$

which both increase with Δ while the impact of t depends on firms' locations. Hence, both firms benefit from a stronger preference for variety.

Differentiating π_i^* with respect to x_i shows that

$$\frac{d\pi_1^*}{dx_1} > 0 \Leftrightarrow x_1 < 1/2, \qquad \frac{d\pi_2^*}{dx_2} < 0 \Leftrightarrow x_2 > 1/2.$$

In other words, when $x_1 < x_2$, as conjectured by Hotelling, it is profitable for each firm to move toward the market center. Further, when $x_1 \leq x_2 < 1/2$, then each firm wants to move rightward.

Consequently, we may conclude as follows.

Proposition 3. *If $\Delta/t > 1$, the equilibrium of the location game is unique and given by $x_1^* = x_2^* = 1/2$, while the corresponding equilibrium prices are $p_1^* = p_2^* = \Delta$. Furthermore, equilibrium profits are given by $\Pi_1^* = \Pi_2^* = \Delta/2$.*

This proposition shows that the Hotelling's conjecture about firms' location choices holds when consumers are heterogeneous enough, when travelling is inexpensive, or both. Although firms are located together, it never pays for one firm to undercut its rival because such a price drop would only increase its demand marginally as the quality of the match between consumers and firms matters a lot to consumers. The end result is neat: product differentiation renders prices sufficiently sticky for firms to choose the best location.

5 Concluding remarks

We have shown that enough heterogeneity in consumers' attitude toward differentiated varieties is sufficient restore the existence of a price equilibrium for all location pairs and to prove that the PMD always holds in Hotelling's location-then-price game. An alternative approach is to appeal to mixed strategies. Using this approach, Xefteris (2013) shows that the PMD also holds in the sequential game. This similarity in results endows the PMD with some unsuspected robustness.

A final remark is in order. Our purpose being to study Hotelling's model per se, we have considered the linear probability model which is consistent with linear demands in a duopoly. Unfortunately, this model cannot be extended to oligopolistic settings because globally linear demand systems are not consistent with discrete choice theory when there are more than two varieties (Jaffe and Weyl, 2010). However, introducing an outside option through finite reservation prices leads to piecewise linear demands that can be rationalized by discrete choice under some conditions (Armstrong and Vickers, 2015). More research is called for here.

Appendix

By symmetry, it is sufficient to rule out the following three configurations: (i) $x_{\min} < x_1$, (ii) $x_1 < x_{\min} < x_2$ and (iii) $x_2 < x_{\min}$.

(i) Assume that $0 < x_{\min} < x_1 < x_2 < x_{\max} < 1$. Since the conditions (2) and (3) may be rewritten as follows:

$$\frac{\Delta - p_1 + p_2 + t(x_2 - x_1)}{2\Delta} = 1, \tag{A}$$

$$\frac{\Delta - p_1 + p_2 - t(x_2 - x_1)}{2\Delta} = 0, \tag{B}$$

we obtain:

$$x_2 - x_1 = \Delta/t.$$

Substituting this expression in (B) yields $p_1 = p_2$. Hence, firm 1's demand at any $x \in (x_{\min}, x_1)$ is given by

$$\frac{\Delta - p_1 + p_2 + t(x_2 - x_1)}{2\Delta}.$$

However, it follows from (A) that this expression is equal to 1, a contradiction.

(ii) Consider now the configuration $0 \leq x_1 < x_{\min} < x_2 < x_{\max} < 1$. In this case, (2) and (3) may be rewritten as follows:

$$\frac{\Delta - p_1 + p_2 + t(x_1 + x_2) - 2tx_{\min}}{2\Delta} = 1, \tag{C}$$

$$\frac{\Delta - p_1 + p_2 + t(x_1 - x_2)}{2\Delta} = 0, \tag{D}$$

which yield $x_{\min} = x_2 - \Delta/t$. Substituting $x_{\min} = x_2 - \Delta/t$ into profit functions, differentiating each profit function with respect to the firm's own price and solving the resulting system of equations yield the candidate equilibrium prices. Substituting these prices into (D) to obtain the value of $x_{\max}$. Substituting this value of $x_{\max}$ into prices under $x_1 + x_2 = 1$ yields the following candidate equilibrium values:

$$x_{\max} = x_2 - 3\Delta/t, \qquad p_1 = \frac{1}{2}\Delta - tx_2, \qquad p_2 = -\frac{1}{2}\Delta - tx_1 < 0,$$

which means that the configuration $0 \leq x_1 < x_{\min} < x_2 < x_{\max} < 1$ cannot be sustained as a price equilibrium.

Furthermore, the configuration $0 \leq x_1 < x_{\min} < x_2 < x_{\max} = 1$ leads to (C) and (D), which yield $x_{\min} = x_2 - \Delta/t$. Substituting $x_{\min} = x_2 - \Delta/t$ and $x_{\max} = 1$ into profit functions, differentiating each profit function with respect to the firm's own price and solving the resulting system of equations yield the candidate equilibrium prices. Substituting these prices into (D) under $x_1 + x_2 = 1$ yields

$$\frac{[t(1 - 2x_2) + \Delta]\,[t(1 - x_2) + 3\Delta]}{6t\Delta(1 - x_2) + 6\Delta^2} = 0.$$

Since $t(1-x_2)+3\Delta > 0$ and $6t\Delta(1-x_2)+6\Delta^2 > 0$, we have $t(1-2x_2)+\Delta = 0$. Combining $x_1 < x_{\min} = x_2-\Delta/t$ and $x_1 + x_2 = 1$ yields $t(1-2x_2)+\Delta < 0$, a contradiction.

(iii) Finally, the configuration $0 \leq x_1 < x_2 < x_{\min} < x_{\max} \leq 1$ implies that (2) and (3) may be rewritten as follows:

$$\begin{aligned} \frac{\Delta - p_1 - t(x_{\min} - x_1) + p_2 + t(x_{\min} - x_2)}{2\Delta} &= 1, \\ \frac{\Delta - p_1 - t(x_{\max} - x_1) + p_2 + t(x_{\max} - x_2)}{2\Delta} &= 0, \end{aligned}$$

which is equivalent to

$$\frac{\Delta - p_1 + p_2 - t(x_2 - x_1)}{2\Delta} = 1, \qquad \frac{\Delta - p_1 + p_2 - t(x_2 - x_1)}{2\Delta} = 0,$$

a contradiction. Q.E.D.

References

[1] Armstrong, M., Vickers, J. (2015) Which demand systems can be generated by discrete choice?. *Journal of Economic Theory* 158: 293-307.

[2] Balasubramanian, S. (1998) Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers. *Marketing Science* 17: 181-95.

[3] Biscaia, R., Mota, I. (2013) Models of spatial competition: A critical review. *Papers in Regional Science* 92: 851-72.

[4] Brown, S. (1989) Retail location theory: The legacy of Harold Hotelling. *Journal of Retailing* 65: 450-70.

[5] Brown, S. (1993) Retail location theory: Evolution and evaluation. *International Review of Retail, Distribution and Consumer Research* 3: 185-229.

[6] Datta, S., Sudhir, K. (2013) Does reducing spatial differentiation increase product differentiation? Effects of zoning on retail entry and format variety. *Quantitative Marketing and Economics* 11: 83-116.

[7] d'Aspremont, C., Gabszewicz, J.J., Thisse, J.-F. (1979) On Hotelling's "Stability in competition". *Econometrica* 47: 1145-50.

[8] de Palma, A., Ginsburgh, V., Papageorgiou,Y.Y., Thisse, J.-F. (1985) The principle of minimum differentiation holds under sufficient heterogeneity. *Econometrica* 53: 767-81.

[9] Drezner, Z., Eiselt, H.A. (2024) Competitive location models: A review. *European Journal of Operational Research* 316: 5-18.

[10] Dudey, M. (1990) Competition by choice: The effect of consumer search on firm location decisions. *American Economic Review* 80: 1092-104.

[11] Duranton, G., Puga, D. (2004) Micro-foundations of urban increasing returns: Theory. In: *Handbook of Regional and Urban Economics. Cities and Geography*, edited by J.V. Henderson and J.-F. Thisse, 2063–117. Amsterdam: North-Holland.

[12] Eaton, B.C., Lipsey, R.G. (1982) An economic theory of central places. *Economic Journal* 92: 56-72.

[13] Guo, W.C., Lai, F.C. (2017) Prices, locations and welfare when an online retailer competes with heterogeneous brick-and-mortar retailers. *Journal of Industrial Economics* 65: 439-68.

[14] Hotelling, H. (1929) Stability in competition. *Economic Journal* 39: 41-57.

[15] Irmen, A., Thisse, J.-F. (1998) Competition in multi-characteristics spaces: Hotelling was almost right. *Journal of Economic Theory* 78: 76-102.

[16] Jaffe, S., Weyl, E.G. (2010) Linear demand systems are inconsistent with discrete choice. *The BE Journal of Theoretical Economics* 10: 000010220219351704 1729.

[17] Konishi, H. (2005) Concentration of competing retail stores. *Journal of Urban Economics* 58: 488-512.

[18] Koster, H.R.A., Pasidis, I., van Ommeren, J. (2019) Shopping externalities and retail concentration: Evidence from Dutch shopping streets. *Journal of Urban Economics* 114: 103194.

[19] Krugman, P.R. (1991) Increasing returns and economic geography. *Journal of Political Economy* 99: 483-99.

[20] Leonardi, M., Moretti, E. (2023) The agglomeration of urban amenities: Evidence from Milan restaurants. *American Econonomic Review: Insight* 5: 141-57.

[21] Manerba, D., Mansini, R., Riera-Ledesma, J. (2017) The traveling purchaser problem and its variants. *European Journal of Operational Research* 259: 1-18.

[22] Miyauchi, Y., Nakajima, K., Redding, S.J. (2021) The economics of spatial mobility: Theory and evidence using smartphone data (No. w28497). National Bureau of Economic Research.

[23] Mossay, P., Shin, J.K., Smrkolj, G. (2022) Quality differentiation and spatial clustering among restaurants. *International Journal of Industrial Organization* 80: 102799.

[24] Nilsson, I., Reid, N., Lehnert, M. (2018) Geographic patterns of craft breweries at the intraurban scale. *The Professional Geographer* 70: 114-25.

[25] Schulz, N., Stahl, K. (1996) Do consumers search for the highest price? Equilibrium and monopolistic optimum in differentiated products markets. *RAND Journal of Economics* 27: 542-62.

[26] Tabuchi, T. (1994) Two-stage two-dimensional spatial competition between two firms. *Regional Science and Urban Economics* 24: 207-27.

[27] Tabuchi, T. (2009) Self-organizing marketplaces. *Journal of Urban Economics* 66: 179-85.

[28] Thill, J.C. (1986) A note on multipurpose and multistop shopping, sales, and market areas of firms. *Journal of Regional Science* 26: 775-84.

[29] Xefteris, D. (2013) Hotelling was right. *Journal of Economics and Management Strategy* 22: 706-12.