INSTITUTE OF DEVELOPING ECONOMIES

IDE Discussion Papers are preliminary materials circulated
to stimulate discussions and critical comments

IDE DISCUSSION PAPER No. 890

‘Made in the World’: Measuring the Productivity of Global Value Chains

Wenyin CHENG[1*], Bo MENG[2], Yuning GAO[3]

March 2023

Abstract

As an additional approach to the traditional Jorgenson’s accounting method based on the sectoral total factor productivity (TFP), this paper aims to measure GVC-based TFP by explicitly considering intermediate inputs as an endogenous variable. Based on the theoretical derivations, simulations, and a recursive approach, we first clarify the distinction between the Domar- and Leontief-based GVC TFP. We further point out the knife-edge feature of Domar aggregation based on the sectoral TFP, as well as the “missing productivity” of the conventional approach based on the share-weighted sectoral TFP or aggregate production function. Finally, we unify the Domar- and Leontief-based GVC TFP within Jorgenson’s accounting framework and decompose it into four parts. Using the world input–output database, we show that the new GVC TFP helps better understand the nature and structure of international fragmentation production and the evolution of global resource allocations.

Keywords: Made in the world, Global value chain, Total factor productivity, Domar aggregation, Leontief inverse

JEL classification: D24, O19, O47

1: Researcher, IDE-JETRO, Japan (wenyin_cheng@ide.go.jp); 2: Senior Researcher, IDE-JETRO, Japan (bo_meng@ide.go.jp); 3: Associate Professor, School of Public Policy and Management, Tsinghua University, China.

The Institute of Developing Economies (IDE) is a semigovernmental, nonpartisan, nonprofit research institute, founded in 1958. The Institute merged with the Japan External Trade Organization (JETRO) on July 1, 1998. The Institute conducts basic and comprehensive studies on economic and related affairs in all developing countries and regions, including Asia, the Middle East, Africa, Latin America, Oceania, and Eastern Europe.

INSTITUTE OF DEVELOPING ECONOMIES (IDE), JETRO
3-2-2, WAKABA, MIHAMA-KU, CHIBA-SHI
CHIBA 261-8545, JAPAN

‘Made in the World’:
Measuring the Productivity of Global Value Chains

Wenyin CHENG[1], Bo MENG[2*], Yuning GAO[3]

Abstract: As an additional approach to the traditional Jorgenson’s accounting method based on the sectoral total factor productivity (TFP), this paper aims to measure GVC-based TFP by explicitly considering intermediate inputs as an endogenous variable. Based on the theoretical derivations, simulations, and a recursive approach, we first clarify the distinction between the Domar- and Leontief-based GVC TFP. We further point out the knife-edge feature of Domar aggregation based on the sectoral TFP, as well as the “missing productivity” of the conventional approach based on the share-weighted sectoral TFP or aggregate production function. Finally, we unify the Domar- and Leontief-based GVC TFP within Jorgenson’s accounting framework and decompose it into four parts. Using the world input–output database, we show that the new GVC TFP helps better understand the nature and structure of international fragmentation production and the evolution of global resource allocations.

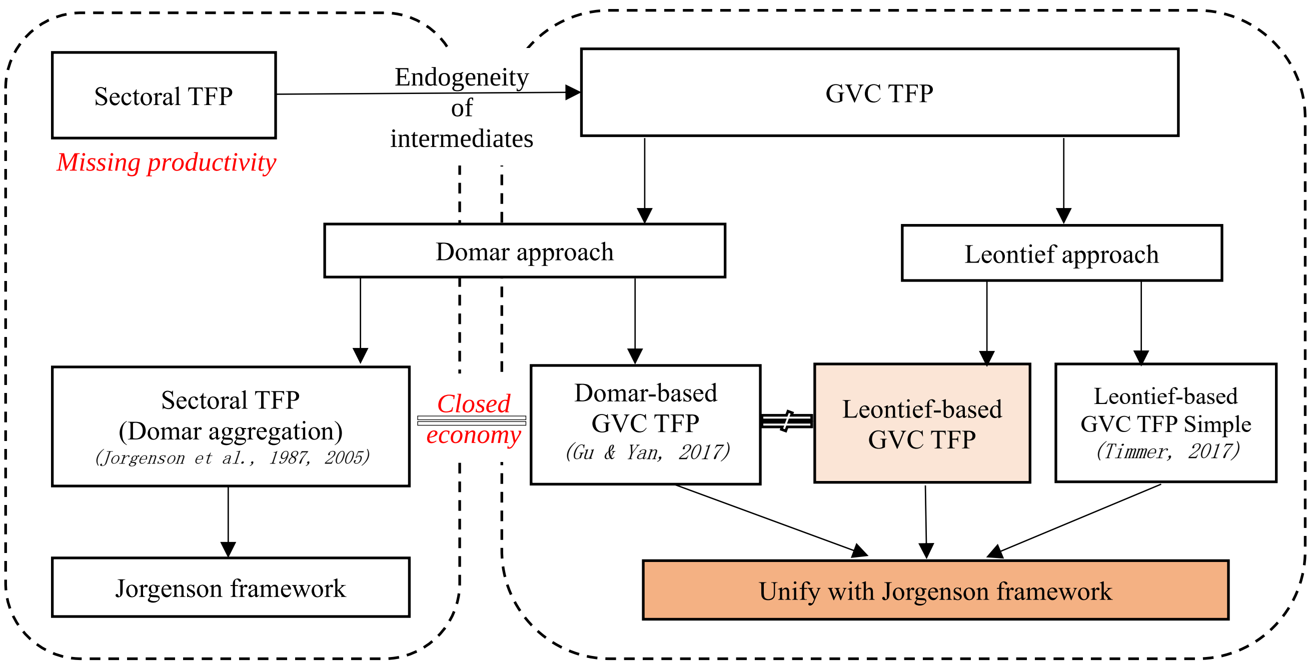
Keywords: made in the world, global value chain, total factor productivity, Domar aggregation, Leontief inverse

JEL classification: O47, D24, O47

Highlights:

1. Gap between the Domar and Leontief-based GVC TFP was clarified.
2. Domar and Leontief approaches were unified within Jorgenson's accounting framework.
3. The knife-edge feature of Domar aggregation based on sectoral TFP was pointed out.
4. The performance and evolution of sectoral TFP and those of GVC TFP differ greatly.

Graphical Abstract:



1 Introduction

The new century has witnessed rapid globalization and the rise of global value chains (GVCs), making production activities seem more like a "spider" (production network) than a "snake" (sector). The "made in" label, typical of manufactured goods, which attributes them to a specific economy, has become an archaic symbol of a bygone era as most manufactured goods are now "made in the world." Although sectoral total factor productivity (TFP) well describes the economy in which products are made, it is insufficient in describing and accounting for the globalized manufacturing process, especially the activities of multinational enterprises. Therefore, new theories are needed to provide appropriate explanations of new phenomena and constructive suggestions for policymakers in this new era.

"The correct measurement of the rate of growth in economy-wide productivity is no less important today than it has ever been" (Gollop, 1979). The aggregate production function (APF) approach, proposed by the seminal work of Solow (1956, 1957), has been criticized for its stringent assumptions and lack of micro-foundations (Jorgenson et al., 2005). The conventional aggregated approach in calculating aggregate TFP with micro-foundations uses output or value-added shares as weights. However, these approaches neglect the endogeneity of intermediate inputs and are likely to underestimate the TFP. Assuming A and B as two sectors, that provide intermediate goods for one another, their TFPs are likely to be affected by one another. This confers a kind of mutual causality that is similar to the endogeneity issue in econometric models.

Notably, Domar aggregation provides a progressive intermediate endogenizing capability. Domar (1961) worked out a method of integrating and aggregating with and without input–output relations, respectively, and the method has been widely cited in productivity literature, especially in the canonical framework of Jorgenson's productivity accounting. Hulten (1978) proved that the Domar-weighted sectoral TFP change equals the share-weighted effective rate of productivity change, which is derived from recent GVC insights into the propagation of intermediate goods in production networks; the result is the Domar-based GVC TFP. Although the effective rate of productivity change is expressed as a complex function across a series of partial elasticities, which is difficult for empirical analysis, later scholars (Peterson, 1979; Durand, 1996; Aulin-Ahmavaara, 1999; Basu et al., 2013) have calculated the Domar-based GVC TFP based on sectoral TFP with the help of the Leontief inverse. However, the early research highlighted by those works focuses on closed economies. Only recently has the approach been extended to open economies (Gu & Yan, 2017).

Another branch of literature that endogenizes intermediates is that of GVC TFP, which is directly based on the Leontief inverse without relying on sectoral TFP (Timmer, 2017; Timmer & Ye, 2020; Turégano, 2021). We call it Leontief-based GVC TFP. However, the Domar- and Leontief-based GVC TFPs have mistakenly been considered duplicative (Gu & Yan, 2017; Timmer, 2017). Hence, illuminating the differences between them is crucial for understanding GVC TFP in general, which remains nascent in the literature. If we succeed in this, we will gain a better understanding of the micro-foundations of aggregate TFP.

This paper measures GVC TFP by explicitly considering intermediate inputs as an endogenous variable. We aim to contribute to productivity accounting theory in the following three ways: (1)

We clarify the gap between the Domar- and Leontief-based GVC TFPs, which remains ambiguous in the literature; however, understanding the differences is critical to comprehending the relationship between sectoral TFP and GVC TFP, and also the micro-foundations of aggregate TFP; (2) We integrate GVC TFP into the accounting framework of Jorgenson and provide a symmetrical interpretation of GVC TFP as the traditional Jorgenson's accounting method based on sectoral TFP; and (3) We highlight the knife-edge feature of Domar aggregation based on sectoral TFP and the "missing productivity" of the conventional approaches based on share-weighted sectoral TFP. Furthermore, we obtain many novel empirical findings in terms of GVC TFP and its gap with sectoral TFP.

The structure of the remaining article is as follows: Section 2 briefly introduces the TFP measures at disaggregate and aggregate levels respectively, points out the difference between sectoral TFP and GVC TFP, and also the properties of accurate aggregate TFP measures. Section 3 provides top-down and bottom-up derivations of the Leontief-based GVC TFP; Section 4 compares the Domar and Leontief-based GVC TFP models, points out the potential issues with Domar aggregation, and integrates GVC TFP into Jorgenson's framework; In Section 5, we investigate the missing productivity of conventional approaches based on the share-weighted sectoral TFP; In Section 6, we apply our model to the World Input–Output Database (WIOD) and provide empirical evidence at the world, country, and country-sector levels while decomposing GVC TFP growth; Finally, Section 7 concludes the paper.

2 Definition of different types of GVC TFP

Sectoral TFP measures the residual growth of sectoral gross output not accounted for by the growth of intermediate and primary inputs of the sector.

$$\pi_j = \frac{dx_j}{x_j} - \sum_i \frac{p_i x_{ij}}{p_j x_j}\frac{dx_{ij}}{x_{ij}} - \frac{r_j k_j}{p_j x_j}\frac{dk_j}{k_j} - \frac{w_j l_j}{p_j x_j}\frac{dl_j}{l_j} = -\left(\sum_i p_i da_{ij} + r_j d\kappa_j + w_j d\ell_j\right)/p_j$$

$$\Rightarrow \pi = -\left(pd\boldsymbol{A} + rd\hat{\kappa} + wd\hat{\ell}\right)\hat{p}^{-1} \qquad \text{(II-1)}$$

where $\kappa_j = k_j/x_j$ and $\ell_j = l_j/x_j$ represent the capital and labor service of sector j directly required to produce one unit of output j. r, w, κ, ℓ, p and π refers to the row vector of r_j, w_j, κ_j, ℓ_j, p_j and sectoral TFP growth π_j. $\boldsymbol{A}$ is the direct input coefficient matrix, where the element a_{ij} refers to the output of sector i delivered as intermediate inputs to sector j. The hat notation over a vector denotes a diagonal matrix with the diagonal filled with the elements of the vector. The letter in bold black refers to a matrix.

However, GVC TFP measures the residual growth of sectoral final products not accounted for by the growth of primary inputs of various sectors within the GVC. There are two different ways to calculate GVC TFP, as indicated in Wolf (1994). The first one can be derived using the recursive approach indicated in Domar (1961), and thus we call it Domar-based GVC TFP growth π^D.

$$\pi_j^D = \sum_i \pi_i b_{ij}\frac{p_i}{p_j}$$

$$\Rightarrow \pi^D = \pi\boldsymbol{S} \qquad \text{(II-2)}$$

where π^D is the row vector of Domar-based GVC TFP growth. $\boldsymbol{S} = \hat{p}\boldsymbol{B}\hat{p}^{-1}$. $\boldsymbol{B} = (\boldsymbol{I}-\boldsymbol{A})^{-\boldsymbol{1}}$ is the Leontief inverse.

The aggregate TFP growth rate based on Domar approach can be obtained from the following equation:

$$\vartheta^D = \pi^D \frac{\hat{p}y}{py} = \pi \frac{\hat{p}x}{py} \tag{II-3}$$

where $\frac{\hat{p}x}{py}$ is the Domar weight. y and x denotes the column vector of final output and gross output respectively. Equation (II-3) means that Domar approach provides a succinct way to aggregate both sectoral TFP and GVC TFP.

The second one can be derived directly from Solow residual, and thus we call it Solow-based GVC TFP growth π^S. The Solow-based GVC TFP growth of each commodity can be expressed as:

$$\pi_j^S = \frac{dy_j}{y_j} - \left(\sum_i \frac{r_i\left(y_j\gamma_{ij}\right)}{p_jy_j} * \frac{d\left(y_j\gamma_{ij}\right)}{y_j\gamma_{ij}} + \sum_i \frac{w_i\left(y_j\lambda_{ij}\right)}{p_jy_j} * \frac{d\left(y_j\lambda_{ij}\right)}{y_j\lambda_{ij}}\right)$$

$$\Rightarrow \pi_j^S = -\left(\sum_i r_i d\gamma_{ij} + \sum_i w_i d\lambda_{ij}\right)/p_j$$

$$\Rightarrow \pi^S = -(rd\boldsymbol{\gamma} + wd\boldsymbol{\lambda})\hat{p}^{-1} \tag{II-4}$$

where $\boldsymbol{\gamma} = \hat{\kappa}\boldsymbol{B}$ and $\boldsymbol{\lambda} = \hat{\ell}\boldsymbol{B}$ denotes the matrix with element γ_{ij} and λ_{ij} representing the total capital and labor service of sector i directly and indirectly resulting from one unit of final demand j.

Both Domar-based GVC TFP and Solow-based GVC TFP relies on Leontief inverse, but in different ways. There might be some relationships between them. Wolf (1994) proved that the two types of GVC TFP growth are equal to each other if the prices of capital and labor across sectors to are assumed to be constant: $\bar{r}$ and $\bar{w}$. In this case, the Solow-based GVC TFP growth of each commodity can be expressed as:

$$\pi_j^{SS} = -\left(\bar{r}\, d\gamma_j + \bar{w} d\lambda_j\right)/p_j$$

$$\Rightarrow \pi^{SS} = -(\bar{r}d\gamma + \bar{w}d\lambda)\hat{p}^{-1} \tag{II-5}$$

where $\gamma = \kappa\boldsymbol{B}$ and $\lambda = \ell\boldsymbol{B}$ denotes the row vector of γ_j and λ_j, respectively, and $\gamma_j = \sum_i \gamma_{ij}$, $\lambda_j = \sum_i \lambda_{ij}$. The difference between π^S and π^{SS} can be boiled down simply to whether the heterogeneity across sectors and thus the reallocation effect between sectors has been taken into account.

With the help of the following two equations.

$$p(\boldsymbol{I}-\boldsymbol{A}) = \bar{r}\kappa + \bar{w}\ell \tag{II-6}$$

$$d\boldsymbol{I} \equiv (\boldsymbol{I}-\boldsymbol{A})\boldsymbol{d}(\boldsymbol{I}-\boldsymbol{A})^{-\boldsymbol{1}} + [\boldsymbol{d}(\boldsymbol{I}-\boldsymbol{A})](\boldsymbol{I}-\boldsymbol{A})^{-\boldsymbol{1}} \tag{II-7}$$

We can prove that

$$\pi^{SS} = \pi^S = \pi^D \tag{II-8}$$

However, the equation will not hold if we consider the heterogeneity among sectors.

$$\pi^S = -(rd\boldsymbol{\gamma} + wd\boldsymbol{\lambda})\hat{p}^{-1} = -\left(rd\hat{\kappa}\boldsymbol{B} + wd\hat{\ell}\boldsymbol{B}\right)\hat{p}^{-1} = -\left[\left(r\hat{\kappa} + w\hat{\ell}\right)d\boldsymbol{B} + \left(rd\hat{\kappa} + wd\hat{\ell}\right)\boldsymbol{B}\right]\hat{p}^{-1}$$
$$= -\left[p(\boldsymbol{I} - \boldsymbol{A})d\boldsymbol{B} + \left(rd\hat{\kappa} + wd\hat{\ell}\right)\boldsymbol{B}\right]\hat{p}^{-1} = -\left[pd\boldsymbol{A} + rd\hat{\kappa} + wd\hat{\ell}\right]\boldsymbol{B}\hat{p}^{-1}$$
$$= \pi\hat{p}\boldsymbol{B}\hat{p}^{-1} = \pi^D$$

$$p(\boldsymbol{I} - \boldsymbol{A}) = r\hat{\kappa} + w\hat{\ell} \qquad \text{(II-9)}$$

In general, the prices of capital and labor are not the same in different sectors. Therefore, we should find new ways to the investigate the difference and relationship between Domar-based GVC TFP and Solow-based GVC TFP.

$$\Rightarrow \pi = -\left(pd\boldsymbol{A} + rd\hat{\kappa} + wd\hat{\ell}\right)\hat{p}^{-1} \qquad \text{(II-1)}$$
$$p(\boldsymbol{I} - \boldsymbol{A}) = r\hat{\kappa} + w\hat{\ell} \qquad \text{(II-6)}$$
$$d\boldsymbol{I} \equiv (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{d}(\boldsymbol{I} - \boldsymbol{A})^{-\mathbf{1}} + [\boldsymbol{d}(\boldsymbol{I} - \boldsymbol{A})](\boldsymbol{I} - \boldsymbol{A})^{-\mathbf{1}} \qquad \text{(II-7)}$$

3 Difference between two types of GVC TFP

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}}\,;\, x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} x_{12}^{\gamma_{12}} \qquad \text{(II-1)}$$

By substituting x_1 and x_2 reciprocally into the two equations and using a recursive process, we can derive $\acute{\pi}$, *i.e.* GVC TFP 1.

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}} = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} (\delta_{21} A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} x_{12}^{\gamma_{12}})^{\gamma_{21}} \qquad \text{(II-2)}$$

$$\text{where } \delta_{ij} = \frac{x_{ij}}{x_i}, \gamma_{ij} = \frac{p_i}{p_j} a_{ij} = \frac{p_i x_{ij}}{p_j x_j}$$

$$\Rightarrow \dot{x}_1 = \dot{A}_1 + \alpha_1 \dot{l}_1 + \beta_1 \dot{k}_1 + a_{11}\dot{x}_1 + \frac{p_2}{p_1} a_{21}\dot{A}_2 + \frac{p_2}{p_1} a_{21}\alpha_2 \dot{l}_2 + \frac{p_2}{p_1} a_{21}\beta_2 \dot{k}_2 + \frac{p_2}{p_1} a_{21} \frac{p_1}{p_2} a_{12}\dot{x}_1 + \frac{p_2}{p_1} a_{21} a_{22}\dot{x}_2$$

$$\Rightarrow \acute{\pi}_1 = \dot{A}_1 + a_{11}\acute{\pi}_1 + \frac{p_2}{p_1} a_{21}\dot{A}_2 + \frac{p_2}{p_1} a_{21} \frac{p_1}{p_2} a_{12}\acute{\pi}_1 + \frac{p_2}{p_1} a_{21} a_{22}\acute{\pi}_2 \qquad \text{(II-3)}$$

$$x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} x_{12}^{\gamma_{12}} = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} (\delta_{12} A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}})^{\gamma_{12}} \qquad \text{(II-4)}$$

$$\Rightarrow \dot{x}_2 = \dot{A}_2 + \alpha_2 \dot{l}_2 + \beta_2 \dot{k}_2 + a_{22}\dot{x}_2 + \frac{p_1}{p_2} a_{12}\dot{A}_1 + \frac{p_1}{p_2} a_{12}\alpha_1 \dot{l}_1 + \frac{p_1}{p_2} a_{12}\beta_1 \dot{k}_1 + a_{11} \frac{p_1}{p_2} a_{12}\dot{x}_1 + \frac{p_1}{p_2} a_{12} \frac{p_2}{p_1} a_{21}\dot{x}_2$$

$$\Rightarrow \acute{\pi}_2 = \pi_2 + a_{22}\acute{\pi}_2 + \frac{p_1}{p_2}a_{12}\pi_1 + a_{11}\frac{p_1}{p_2}a_{12}\acute{\pi}_1 + \frac{p_1}{p_2}a_{12}\frac{p_2}{p_1}a_{21}\acute{\pi}_2 \tag{II-5}$$

Therefore, we have: $\boldsymbol{\acute{\pi}} = \boldsymbol{\pi S}$, (II-6)

where $\acute{\pi} = (\acute{\pi}_1, \acute{\pi}_2) = \left(\dot{A}_1^{GVC1}, \dot{A}_2^{GVC1}\right), \pi = (\pi_1, \pi_2) = \left(\dot{A}_1, \dot{A}_2\right)$

The recursive process of $\acute{\pi}_1$ is as follows. For simplicity, we assume $a_{11} = a_{22} = 0$.

$$\dot{A}_1 + \gamma_{21}(\dot{A}_2 + \gamma_{12}\dot{A}_1)$$

$$\dot{A}_1 + \gamma_{21}\dot{A}_2 + \gamma_{21}\gamma_{12}\left(\dot{A}_1 + \gamma_{21}\dot{A}_2\right)$$

$$\dot{A}_1 + \gamma_{21}\dot{A}_2 + \gamma_{21}\gamma_{12}\dot{A}_1 + \gamma_{21}\gamma_{12}\gamma_{21}\dot{A}_2 + \gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12}\dot{A}_1 + \cdots$$

$$(1 + \gamma_{21}\gamma_{12} + \gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12} + \cdots)\dot{A}_1 + (\gamma_{21} + \gamma_{21}\gamma_{12}\gamma_{21} + \gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12}\gamma_{21} + \cdots)lnA_2 = \frac{lnA_1 + \gamma_{21}lnA_2}{1 - \gamma_{21}\gamma_{12}}$$

The TFP contribution of sector 2 to sector 1 includes different channels:

- $\gamma_{21}\dot{A}_2$: the intermediate inputs directly delivered to sector 1. 2-1
- $\gamma_{21}\gamma_{12}\gamma_{21}\dot{A}_2$: indirect channel, 2-1-2-1
- $\gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12}\gamma_{21}$: indirect channel, 2-1-2-1-2-1

3 Leontief-Based GVC TFP

3.1 Leontief inverse: final demand and inputs in GVCs

The computation of the GVC TFP requires information about the final demand and inputs within the GVC. Although the final demand can be obtained directly from the input–output matrix, the intermediate and primary inputs to each GVC must be derived indirectly from the Leontief inverse. Prior to explaining the GVC TFP model, some matrixes related to input–output tables must be defined. For notational convenience, we consider only the dimension of the sector and suppress the country dimensions. The sector-country setting can be derived in the same way.

For expositional convenience, we employ capital letters to represent the nominal variables and lowercase letters to represent variables in physical units. The prime symbol in the upper-right corner refers to the transpose of a matrix or vector. The E in the upper-right corner refers to the sum of all matrix or vector elements, and the D in the upper-right corner refers to the sum of the diagonal elements in a matrix. The dot over variables indicates their growth rate. The hat notation over a vector denotes a diagonal matrix with the diagonal filled with the elements of the vector, and the hat over a matrix denotes a diagonal matrix with the diagonal filled with the diagonal elements of the matrix. The letter in bold black refers to a matrix or vector. Table 3-2 provides the definition of symbols.

Table 3-2: Definition of Symbols

Symbols	Definitions
	Elements
Z_{ij}	Value of output i directly used as intermediates of sector j.
a_{ij}	Value of output i directly used as intermediates by one unit value of output j.
b_{ij}	Total value of output i directly and indirectly used as intermediates by one unit value of output j.
c_{ij}	Total value of output i directly and indirectly resulting from one unit value of final demand j.
l_{ij} (k_{ij})	Total labor (capital) service of sector i directly and indirectly resulting from final demand j.
	Column vectors
$\boldsymbol{X}$	Column vector of output value X_i with $n \times 1$ dimension.
$\boldsymbol{V}$	Column vector of value added V_i with $n \times 1$ dimension.
$\boldsymbol{Y}$	Column vector of final demand Y_i with $n \times 1$ dimension.
$\boldsymbol{L}$	Column vector of labor compensation L_i with $n \times 1$ dimension.
$\boldsymbol{K}$	Column vector of capital compensation K_i with $n \times 1$ dimension.
$\boldsymbol{y}$	Column vector of real final demand y_i with $n \times 1$ dimension.
$\boldsymbol{l}$	Column vector of labor service l_i with $n \times 1$ dimension.
$\boldsymbol{k}$	Column vector of real capital service k_i with $n \times 1$ dimension.
$\boldsymbol{i}$	Column vector with ones as the elements
	Matrices
$\boldsymbol{Z}$	Intermediate use matrix (x_{ij}) with $n \times n$ dimension.
$\boldsymbol{I}$	Identity matrix with $n \times n$ dimension.
$\mathbf{A}$	Direct input coefficient matrix (a_{ij}) with $n \times n$ dimension.
$\boldsymbol{C}$	Leontief inverse matrix (c_{ij}) with $n \times n$ dimension.
$\boldsymbol{A_l}$ ($\boldsymbol{A_k}$)	Direct labor (capital) service coefficient vector.
$\boldsymbol{C_l}$ ($\boldsymbol{C_k}$)	Labor (capital) service Leontief inverse matrix.

(1) Intermediate inputs in GVCs

$$\boldsymbol{X} = \boldsymbol{Zi} + \boldsymbol{Y}, or X_i = \sum_{j=1}^{n} X_{ij} + Y_i \quad (i = 1,2,\dots,n) \qquad (3\text{-}1)$$

Then we have

$$\boldsymbol{X} = \boldsymbol{AX} + \boldsymbol{Y}, or X_i = \sum_{j=1}^{n} a_{ij} X_j + Y_i \quad (i = 1,2,\dots,n) \qquad (3\text{-}2)$$

We can then rewrite equations (3-2) as

$$\boldsymbol{X} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{Y} = \boldsymbol{CY}, or X_i = \sum_{j=1}^{n} c_{ij} Y_j \quad (i = 1,2,\dots,n) \qquad (3\text{-}3)$$

Where $\boldsymbol{C} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots + \boldsymbol{A}^{n-1}$ is Leontief inverse matrix. The element c_{ij} of Leontief inverse matrix represents the total value of output i resulting from one unit value of final demand j.

(2) Primary inputs in GVCs

Like the direct input coefficient, we can also obtain **direct labor service coefficient vector** and **direct capital service coefficient vector**: $\boldsymbol{A_l} = \left(\frac{l_1}{X_1}, \frac{l_2}{X_2}, \ldots, \frac{l_n}{X_n}\right)$ and $\boldsymbol{A_k} = \left(\frac{k_1}{X_1}, \frac{k_2}{X_2}, \ldots, \frac{k_n}{X_n}\right)$, respectively. Note that $\frac{l_j}{X_j}$ $(\frac{k_j}{X_j})$ means the labor (capital) service of sector j directly required to produce one unit value of output j.

Equation (3-3) indicates that the Leontief inverse matrix serves as an amplifier to transform the final demand matrix to gross output matrix. Therefore, we could obtain the **total labor service coefficient matrix $\boldsymbol{C_l}$** as shown in Equation (3-4).

$$\boldsymbol{C_l} = \widehat{\boldsymbol{A_l}}\boldsymbol{C}, \text{ or } l_{ij} = \frac{l_i}{X_i} c_{ij} \ (i = 1,2, \ldots, \mathrm{n}; j = 1,2, \ldots, \mathrm{n}) \qquad (3\text{-}4)$$

Where element l_{ij} represents the total labor service of sector i directly and indirectly resulting from final demand j.

Similarly, we have the **total capital service coefficient matrix, $\boldsymbol{C_k}$,** as shown in Equation (3-5).

$$\boldsymbol{C_k} = \widehat{\boldsymbol{A_k}}\boldsymbol{C}, \text{ or } k_{ij} = \frac{k_i}{X_i} c_{ij} \ (i = 1,2, \ldots, \mathrm{n}; j = 1,2, \ldots, \mathrm{n}) \qquad (3\text{-}5)$$

Where element k_{ij} represents the total labor service of sector i directly and indirectly resulting from final demand j.

Let $\boldsymbol{W} = (W_1, W_2, \ldots, W_n)'$, and $\boldsymbol{R} = (R_1, R_2, \ldots, R_n)'$; hence, we can obtain the following **total labor compensation coefficient matrix**

$$\boldsymbol{C_L} = \widehat{\boldsymbol{W}}\boldsymbol{C_l} \quad \text{or} \quad L_{ij} = \frac{W_i l_i}{X_i} c_{ij} \ (i = 1,2, \ldots, \mathrm{n}; j = 1,2, \ldots, \mathrm{n}) \qquad (3\text{-}6)$$

Likewise, **total capital compensation coefficient matrix** can be given as

$$\boldsymbol{C_K} = \widehat{\boldsymbol{R}}\boldsymbol{C_k} \quad \text{or} \quad K_{ij} = \frac{R_i k_i}{X_i} c_{ij} \ (i = 1,2, \ldots, \mathrm{n}; j = 1,2, \ldots, \mathrm{n}) \qquad (3\text{-}7)$$

(3) Intermediate inputs canceled out

The **total value-added coefficient matrix** can be given as

$$\boldsymbol{C_V} = (\widehat{\boldsymbol{W}}\widehat{\boldsymbol{A_l}} + \widehat{\boldsymbol{R}}\widehat{\boldsymbol{A_k}})\boldsymbol{C} \quad \text{or} \quad V_{ij} = \frac{W_i l_i + R_i k_i}{X_i} c_{ij} \ (i = 1,2, \ldots, \mathrm{n}; j = 1,2, \ldots, \mathrm{n}) \qquad (3\text{-}8)$$

Where element V_{ij} represents the total value added of sector i directly and indirectly resulting from the final demand of sector j. Because the sum of intermediate input, labor compensation, and capital compensation equals the value of the output, we have

$$\boldsymbol{i}'(\boldsymbol{I}-\boldsymbol{A}) = \boldsymbol{W}\widehat{\boldsymbol{A}_l} + \boldsymbol{R}\widehat{\boldsymbol{A}_k}, \text{ or } 1 - \sum_{i=1}^{n} a_{ij} = \frac{W_j l_j}{X_j} + \frac{R_j k_j}{X_j} \; (i = 1,2,\dots,\text{n}; j = 1,2,\dots,\text{n}) \quad (3\text{-}9)$$

Next, we right-multiply both sides by $\boldsymbol{C}$ to obtain

$$\boldsymbol{i}' = \boldsymbol{W}\boldsymbol{C}_{\boldsymbol{l}} + \boldsymbol{R}\boldsymbol{C}_{\boldsymbol{k}} = \boldsymbol{i}'\boldsymbol{C}_{\boldsymbol{L}} + \boldsymbol{i}'\boldsymbol{C}_{\boldsymbol{K}} \quad (3\text{-}10)$$

The j^{th} element of vector $\boldsymbol{W}\boldsymbol{C}_{\boldsymbol{l}}$ represents the total labor compensation of all sectors directly and indirectly resulting from final product j. Equation (3-10) indicates that the total labor and capital compensation coefficients embodied in each final product sum to unity. Thus, all intermediates are canceled out in the total coefficient setting.

3.2 Leontief-based GVC TFP: Top-down

As mentioned above, GVC TFP measures the residual growth of sectoral final products not accounted for by the growth of labor and capital within a GVC, and it focuses on all production stages, regardless of sector. We assume that the price of primary inputs is homogeneous for a sector, regardless of the value chain in which they are engaged. Then the growth rate of GVC TFP could be noted as:

$$\dot{A}_j^{LeontiefGVC} = \dot{y}_j - \sum_{i=1}^{n} \frac{W_i l_{ij}}{P_j y_j} \dot{l}_{ij} - \sum_{i=1}^{n} \frac{R_i k_{ij}}{P_j y_j} \dot{k}_{ij} \quad (j = 1,2, \dots \text{n}) \quad (3\text{-}11)$$

$$\frac{\left(\sum_{i=1}^{n} W_i l_{ij}\right)}{\sum_{j=1}^{n} P_j y_j} \dot{l}_j$$

To prepare the above formulas for empirical analysis, we obtain y_i and $P_j y_j$ directly from the input-output tables, and l_{ij}, k_{ij} $W_i l_{ij}$, and $R_i k_{ij}$ are obtained indirectly based on the Leontief inverse (see Equation (3-4)~ (3-7) respectively). Timmer (2017) assumes that all the labor (capital) service inputs are homogenous across sectors in a representative GVC, which is unrealistic because it treats one hour work in Thailand and that in Japan are same. Because there is no input-output relationship among GVCs, we can aggregate the TFP of each GVC with the share of each's final demand as the weight to obtain aggregate productivity growth (hereinafter "APG").

$$APG_{LeontiefGVC} = \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \dot{A}_j^{LeontiefGVC} = \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \dot{y}_j - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{W_i l_{ij}}{\sum_{j=1}^{n} P_j y_j} \dot{l}_{ij} - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{R_i k_{ij}}{\sum_{j=1}^{n} P_j y_j} \dot{k}_{ij} \quad (3\text{-}12)$$

Thus, the aggregate TFP can be expressed as the gap between the Divisia index of the final demand and that of the total primary input.

4 GVC TFP: Leontief *vs.* Domar

The endogeneity issue lies in the static input-output relationship[①]. Therefore, aggregate productivity (hereinafter " AP ") is a better starting point than the transtemporal APG for understanding the difference among various TFP aggregation approaches. After that, we will focus on the APG based on different TFP aggregation approaches.

There are two ways to calculate aggregate TFP based on Domar approach. The first is the aggregate sectoral TFP based on Domar weight ($AP_{DomarSector}$), and the second is aggregate GVC TFP (i.e. aggregate from Domar-based GVC TFP) ($AP_{DomarGVC}$). By proposing the effective rate of productivity change, which is derived from recent GVC production network insights, Hulten (1978) theoretically proved that the two methods are identical. However, the differences between the Leontief and Domar approaches remain unclear (Gu & Yan, 2017; Timmer, 2017). Recall that snake-like sectoral TFP and spider-like GVC TFP are not directly comparable. Therefore, we derive $AP_{DomarGVC}$, which serves as an excellent intermediary to help illuminate the gap between $AP_{DomarSector}$ and $AP_{LeontiefGVC}$ (i.e., aggregate TFP based on the Leontief approach).

4.1 Theoretical Derivations

In the scenario where there are no input-output relations, Domar aggregation is a very useful and concise approach in calculating aggregate TFP. However, this is only an extreme scenario. Now we provide more general scenarios, with the intermediates delivered to each other.

$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\boldsymbol{z} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$, where $\boldsymbol{x}$ refers to the vector of output, $\boldsymbol{y}$ is the vector of final demand, $\boldsymbol{z}$ is the matrix of intermediate input, $\sum_j x_{ij} + y_i = x_i$

Then we have direct input coefficient matrix

$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where $0 < a_{ij} = \frac{x_{ij}}{x_j} < 1$

$\Rightarrow \boldsymbol{I} - \boldsymbol{A} = \begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix}$

Then the Leontief inverse matrix can be expressed as:

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \frac{1}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}} \begin{bmatrix} 1 - a_{22} & a_{12} \\ a_{21} & 1 - a_{11} \end{bmatrix} \quad (4\text{-}1)$$

Here we provide a very simple scenario, with the intermediate inputs directly, but not indirectly, delivered to the sector itself to be zero. More complex scenarios are provided in Appendix III.

$$x_1 = F_1(A_1, l_1, k_1, x_{21})\ ;\ x_2 = F_2(A_2, l_2, k_2, x_{12}) \quad (4\text{-}2)$$

① It can also be extended to the dynamic scenario.

For convenience's sake, we assume that the prices of output and intermediate input are equal to one. It is worth mentioning that, in this case, the physical TFP (TFPQ) and revenue TFP (TFPR) are equal. The distinction between TFPQ and TFPR has been well-documented in the literature (e.g., Syverson, 2004; Foster et al., 2008; Hsieh & Klenow, 2009; Braguinsky et al., 2015; Haltiwanger, 2016; Li et al., 2016; Yang & Chen, 2019; Grover & Maloney, 2022). TFPR incorporates the impacts of both demand shock, which relates to output price[①], and TFPQ is measured as the quantity of output per unit input. The selection of survival firms is based on profitability, including both technical efficiency and demand shocks. Therefore, TFPR, which equals the product of TFPQ and output price, is more suitable for explaining reallocations and firm turnover than TFPQ[②].

(1) Aggregate TFP based on Leontief

Since the intermediate inputs directly delivered to the sector itself are zero. The direct input coefficient matrix can be expressed as: $\boldsymbol{A} = \begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix}$, and thus we have $y_1 = x_1 - x_{12} = x_1 - a_{12}x_2$; $y_2 = x_2 - x_{21} = x_2 - a_{21}x_1$.

Then the Leontief inverse can be expressed as: $\boldsymbol{C} = \frac{1}{1-a_{12}a_{21}}\begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix}$. To note that the diagonal elements of $\boldsymbol{C}$ are not zero, which means that the intermediate inputs indirectly delivered to the sector itself are zero, despite the zero in the diagonal elements of $\boldsymbol{A}$.

With Leontief inverse, we obtain the induced output of each sector by each final demand:

$$\boldsymbol{C}\widehat{\boldsymbol{Y}} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} y_1 & \\ & y_2 \end{bmatrix} = \begin{bmatrix} c_{11}y_1 & c_{12}y_2 \\ c_{21}y_1 & c_{22}y_2 \end{bmatrix} \tag{4-3}$$

The induced primary inputs of each sector by each final demand can be expressed as:

$$\boldsymbol{C_l}\widehat{\boldsymbol{Y}} = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} = \begin{bmatrix} \frac{l_1}{x_1}c_{11}y_1 & \frac{l_1}{x_1}c_{12}y_2 \\ \frac{l_2}{x_2}c_{21}y_1 & \frac{l_2}{x_2}c_{22}y_2 \end{bmatrix} \Rightarrow \boldsymbol{C_L}\widehat{\boldsymbol{Y}} = \begin{bmatrix} \frac{W_1l_1}{x_1}c_{11}y_1 & \frac{W_1l_1}{x_1}c_{12}y_2 \\ \frac{W_2l_2}{x_2}c_{21}y_1 & \frac{W_2l_2}{x_2}c_{22}y_2 \end{bmatrix} \tag{4-4}$$

$$\boldsymbol{C_k}\widehat{\boldsymbol{Y}} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{k_1}{x_1}c_{11}y_1 & \frac{k_1}{x_1}c_{12}y_2 \\ \frac{k_2}{x_2}c_{21}y_1 & \frac{k_2}{x_2}c_{22}y_2 \end{bmatrix} \Rightarrow \boldsymbol{C_K}\widehat{\boldsymbol{Y}} = \begin{bmatrix} \frac{R_1k_1}{x_1}c_{11}y_1 & \frac{R_1k_1}{x_1}c_{12}y_2 \\ \frac{R_2k_2}{x_2}c_{21}y_1 & \frac{R_2k_2}{x_2}c_{22}y_2 \end{bmatrix} \tag{4-5}$$

Based on the definition of Leontief-based GVC TFP, we have

$$lnA_1^{LeontiefGVC} = lny_1 - \frac{W_1l_1}{x_1}c_{11} * lnl_{11} - \frac{W_2l_2}{x_2}c_{21} * lnl_{21} - \frac{R_1k_1}{x_1}c_{11} * lnk_{11} - \frac{R_2k_2}{x_2}c_{21} * lnk_{21} \tag{4-6}$$

① It is highly possible that firms facing particularly high demand would charge higher prices.

② Entrants usually have higher technical efficiency but would set lower prices than incumbents (Foster et al., 2008).

$$lnA_2^{LeontiefGVC} = lny_2 - \frac{W_1 l_1}{x_1} c_{12} * lnl_{12} - \frac{W_2 l_2}{x_2} c_{22} * lnl_{22} - \frac{R_1 k_1}{x_1} c_{12} * lnk_{12} - \frac{R_2 k_2}{x_2} c_{22} * lnk_{22} \quad (4\text{-}7)$$

Then we obtain the aggregate Leontief-based TFP:

$$AP_{LeontiefGVC} = \frac{y_1}{y_1+y_2} lnA_1^{LeontiefGVC} + \frac{y_2}{y_1+y_2} lnA_2^{LeontiefGVC} \quad (4\text{-}8)$$

(2) Aggregate TFP based on Domar

(1) $\boldsymbol{AP_{DomarSector}}$

$$lnA_1 = lnx_1 - \frac{W_1 l_1}{x_1} * lnl_1 - \frac{R_1 k_1}{x_1} * lnk_1 - \frac{x_{21}}{x_1} * lnx_{21} \quad (4\text{-}9)$$

$$lnA_2 = lnx_2 - \frac{W_2 l_2}{x_2} * lnl_2 - \frac{R_2 k_2}{x_2} * lnk_2 - \frac{x_{12}}{x_2} * lnx_{12} \quad (4\text{-}10)$$

Then we obtain the aggregate TFP based on Domar weight:

$$AP_{DomarSector} = \frac{x_1}{y_1+y_2} lnA_1 + \frac{x_2}{y_1+y_2} lnA_2 \quad (4\text{-}11)$$

(2) $\boldsymbol{AP_{DomarGVC}}$

In order to demonstrate the process of calculating Domar-based GVC TFP, it is beneficial to use a Cobb–Douglas function as the starting point. However, it should never be used as an ending point since the GVC TFP model is independent of the forms of its production function.

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{21}^{\gamma_{21}};\ x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{12}^{\gamma_{12}} \quad (4\text{-}12)$$

Following Domar (1961), we define the share of gross output of sector 2 used as intermediate inputs of sector 1 to be $\delta_{ij} = \frac{x_{ij}}{x_i}$, where $0 < \delta_{ij} < 1$.

Then we have: $x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{21}^{\gamma_{21}} = A_1 l_1^{\alpha_1} k_1^{\beta_1} (\delta_{21} A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{12}^{\gamma_2})^{\gamma_{21}}$

$$\boldsymbol{lnx_1} = \boldsymbol{lnA_1} + \alpha_1 lnl_1 + \beta_1 lnk_1 + \gamma_{21} ln\delta_{21} + \boldsymbol{\gamma_{21} lnA_2} + \gamma_{21}\alpha_2 lnl_2 + \gamma_{21}\beta_2 lnk_2 + \gamma_{21}\gamma_{12} ln\delta_{12} + \boldsymbol{\gamma_{21}\gamma_{12} lnx_1} \quad (4\text{-}13)$$

$lnA_1 + \gamma_{21}(lnA_2 + \gamma_{12} lnA_1)$

$lnA_1 + \gamma_{21} lnA_2 + \gamma_{21}\gamma_{12}(lnA_1 + \gamma_{21} lnA_2)$

$lnA_1 + \gamma_{21} lnA_2 + \gamma_{21}\gamma_{12} lnA_1 + \gamma_{21}\gamma_{12}\gamma_{21} lnA_2 + \gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12} lnA_1 + \cdots$

$$(1+\gamma_{21}\gamma_{12}+\gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12}+\cdots)lnA_1+\gamma_{21}(1+\gamma_{12}\gamma_{21}+\gamma_{12}\gamma_{21}\gamma_{12}\gamma_{21}+\cdots)lnA_2=\frac{lnA_1+\gamma_{21}lnA_2}{1-\gamma_{21}\gamma_{12}} \tag{4-19}$$

$$\Rightarrow lnA_1^{DomarGVC}=lnA_1+\gamma_{21}lnA_2+\gamma_{21}\gamma_{12}lnA_1^{DomarGVC}=\frac{lnA_1+\gamma_{21}lnA_2}{1-\gamma_{21}\gamma_{12}} \tag{4-14}$$

With perfect competition assumption, we have $\gamma_{ij}=a_{ij}$.

$$\Rightarrow lnA_1^{DomarGVC}=\frac{lnA_1+a_{21}lnA_2}{1-a_{21}a_{12}}=$$
$$\frac{(lnx_1-a_{21}*lnx_{21})-\left(\frac{W_1l_1}{x_1}*lnl_1+\frac{R_1k_1}{x_1}*lnk_1\right)+a_{21}(lnx_2-a_{12}*lnx_{12})-a_{21}(\frac{W_2l_2}{x_2}*lnl_2+\frac{R_2k_2}{x_2}*lnk_2)}{1-a_{21}a_{12}}$$

$$=\frac{(lnx_1-a_{21}*lnx_{21})-(\frac{W_1l_1}{x_1}*lnl_1+\frac{R_1k_1}{x_1}*lnk_1)}{1-a_{21}a_{12}}+\frac{a_{21}(lnx_2-a_{12}*lnx_{12})-a_{21}(\frac{W_2l_2}{x_2}*lnl_2+\frac{R_2k_2}{x_2}*lnk_2)}{1-a_{21}a_{12}} \tag{4-15}$$

Similarly, we have: $lnA_2^{DomarGVC}=\frac{lnA_2+a_{12}lnA_1}{1-a_{21}a_{12}}=$
$$\frac{(lnx_2-a_{12}*lnx_{12})-\left(\frac{W_2l_2}{x_2}*lnl_2+\frac{R_2k_2}{x_2}*lnk_2\right)+a_{12}(lnx_1-a_{21}*lnx_{21})-a_{12}\left(\frac{W_1l_1}{x_1}*lnl_1+\frac{R_1k_1}{x_1}*lnk_1\right)}{1-a_{21}a_{12}}$$

Then we obtain the aggregate Domar-based TFP:

$$AP_{DomarGVC}=\frac{y_1}{y_1+y_2}lnA_1^{DomarGVC}+\frac{y_2}{y_1+y_2}\ lnA_2^{DomarGVC} \tag{4-16}$$

(3) Domar vs. Leontief

Proof 1: $AP_{DomarSector}=AP_{DomarGVC}$

$$\begin{aligned}
AP_{DomarGVC}&=\frac{y_1}{y_1+y_2}lnA_1^{DomarGVC}+\frac{y_2}{y_1+y_2}\ lnA_2^{DomarGVC}\\
&=\frac{x_1-a_{12}x_2}{y_1+y_2}lnA_1^{DomarGVC}+\frac{x_2-a_{21}x_1}{y_1+y_2}\ lnA_2^{DomarGVC}\\
&=\frac{x_1-a_{12}x_2}{y_1+y_2}\frac{lnA_1+a_{21}lnA_2}{1-a_{21}a_{12}}+\frac{x_2-a_{21}x_1}{y_1+y_2}\frac{lnA_2+a_{12}lnA_1}{1-a_{21}a_{12}}\\
&=\frac{(x_1-a_{12}x_2+a_{12}x_2-a_{12}a_{21}x_1)lnA_1}{(y_1+y_2)(1-a_{21}a_{12})}\\
&+\frac{(a_{21}x_1-a_{21}a_{12}x_2+x_2-a_{21}x_1)lnA_2}{(y_1+y_2)(1-a_{21}a_{12})}\\
&=\frac{(x_1-a_{12}a_{21}x_1)lnA_1}{(y_1+y_2)(1-a_{21}a_{12})}+\frac{(-a_{21}a_{12}x_2+x_2)lnA_2}{(y_1+y_2)(1-a_{21}a_{12})}\\
&=\frac{x_1}{y_1+y_2}lnA_1+\frac{x_2}{y_1+y_2}lnA_2=AP_{DomarSector}
\end{aligned}$$

To note that if we use value added share or output share as the weight to aggregate $lnA_j^{DomarGVC}$, then $AP_{DomarGVC}$ will not be equal to $AP_{DomarSector}$. This indicates that $lnA_j^{DomarGVC}$ captures only the GVC TFP of final goods, rather than that of gross output.

Proof 2: $AP_{DomarGVC} > AP_{LeontiefGVC}$

$$lnA_1^{LeontiefGVC} = lny_1 - [c_{11} * (\frac{W_1 l_1}{x_1} lnl_{11} + \frac{R_1 k_1}{x_1} lnk_{11}) + c_{21}(\frac{W_2 l_2}{x_2} lnl_{21} + \frac{R_2 k_2}{x_2} * lnk_{21})] = lny_1 - \frac{\left(\frac{W_1 l_1}{x_1} * lnl_{11} + \frac{R_1 k_1}{x_1} * lnk_{11}\right) + a_{21}(\frac{W_2 l_2}{x_2} * lnl_{21} + \frac{R_2 k_2}{x_2} * lnk_{21})}{1 - a_{12}a_{21}} = lny_1 - \frac{(lnx_1 - a_{21}lnx_{21} - lnA_1) + a_{21}(lnx_2 - a_{12}lnx_{12} - lnA_2)}{1 - a_{12}a_{21}} = lny_1 - \frac{(lnx_1 - a_{21}lnx_{21}) + a_{21}(lnx_2 - a_{12}lnx_{12})}{1 - a_{12}a_{21}} + \frac{lnA_1 + a_{21}lnA_2}{1 - a_{12}a_{21}} \quad (4\text{-}17)$$

$$lnA_1 = lnx_1 - a_{21}lnx_{21} - \frac{W_1 l_1}{x_1} * lnl_{11} - \frac{R_1 k_1}{x_1} * lnk_{11} \Rightarrow lnA_1^{DomarGVC} - lnA_1^{LeontiefGVC} = \frac{(lnx_1 - a_{21}lnx_{21}) + a_{21}(lnx_2 - a_{12}lnx_{12})}{1 - a_{12}a_{21}} - \ln(x_1 - x_{12}) = \frac{(lnx_1 - a_{12}a_{21}lnx_{12}) + a_{21}lnx_2 - a_{21}lnx_{21}}{1 - a_{12}a_{21}} - \ln(x_1 - x_{12}) = \frac{(lnx_1 - a_{12}a_{21}lnx_1) - a_{12}a_{21}ln\delta_{12} + a_{21}(lnx_2 - lnx_{21})}{1 - a_{12}a_{21}} - ln(1 - \delta_{12}) - \ln x_1 = \frac{-a_{12}a_{21}ln\delta_{12} - a_{21}ln\delta_{21}}{1 - a_{12}a_{21}} - ln(1 - \delta_{12}) > 0 \quad (4\text{-}18)$$

Similarly we have: $lnA_2^{DomarGVC} > lnA_2^{LeontiefGVC}$

Thus, $AP_{DomarGVC} > AP_{LeontiefGVC}$

The reason behind the gap between $AP_{DomarGVC}$ and $AP_{LeontiefGVC}$ can be identified from the recursive process of $A_1^{DomarGVC}$ as follows:

$$lnA_1 + \gamma_{21}(lnA_2 + \gamma_{12}lnA_1)$$

$$lnA_1 + \gamma_{21}lnA_2 + \gamma_{21}\gamma_{12}(lnA_1 + \gamma_{21}lnA_2)$$

$$lnA_1 + \gamma_{21}lnA_2 + \gamma_{21}\gamma_{12}lnA_1 + \gamma_{21}\gamma_{12}\gamma_{21}lnA_2 + \gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12}lnA_1 + \cdots$$

$$(1 + \gamma_{21}\gamma_{12} + \gamma_{21}\gamma_{12}\gamma_{21}\gamma_{12} + \cdots)lnA_1 + \gamma_{21}(1 + \gamma_{12}\gamma_{21} + \gamma_{12}\gamma_{21}\gamma_{12}\gamma_{21} + \cdots)lnA_2 = \frac{lnA_1 + \gamma_{21}lnA_2}{1 - \gamma_{21}\gamma_{12}} \quad (4\text{-}19)$$

In the recursive process, the TFP contributions of sector 2, as intermediate inputs, to sector 1 have been decomposed into different parts: the intermediate inputs directly delivered to sector 1 ($\gamma_{21}lnA_2$), and the intermediate inputs indirectly delivered to sector 1 through channels such as first sector 1 and then sector 2 ($\gamma_{21}\gamma_{12}\gamma_{21}lnA_2$). Of course, there are lots of different indirect channels. However, all these direct and indirect channels are considered to be the homogenous, which is shown in the element of a Leontief inverse matrix, that is, the total output of sector *i* resulting from one unit of final product j.

In other words, in the Domar-based GVC TFP, the intermediate products of a sector delivered to other sectors or the same sector through different channels are considered to be heterogenous, because they are aggregated by aggregating sectoral TFP with a weight. However, in the Leontief-based GVC TFP, these intermediate products are considered to be homogenous, because they are aggregated with simple addition.

In terms of the more general scenario, with n sectors providing intermediate goods to each other and to themselves, the Domar-based GVC TFP cannot be derived with recursive approach easily, but we have proved that the results based on recursive approach are in line with the results calculated based on sectoral TFP with the help of Leontief inverse. Please see appendix III for detailed proof. The formula can be expressed as follows.

$$\begin{bmatrix} lnA_1^{DomarGVC} \\ lnA_2^{DomarGVC} \\ \vdots \\ lnA_n^{DomarGVC} \end{bmatrix} = \boldsymbol{C'} \begin{bmatrix} lnA_1 \\ lnA_2 \\ \vdots \\ lnA_n \end{bmatrix} = \boldsymbol{C'} \begin{bmatrix} \frac{p_{v1}v_1}{p_1x_1} lnA_{v1} \\ \frac{p_{v1}v_1}{p_1x_1} lnA_{v2} \\ \vdots \\ \frac{p_{vn}v_n}{p_nx_n} lnA_{vn} \end{bmatrix},$$

lnA_j is the sectoral TFP based on gross output production function, and lnA_{vj} is the sectoral TFP based on value added production function. This means that Domar-based GVC TFP is a function of sectoral TFP based on output or value added production function.

We provide a simple case to illustrate the reason behind the difference between Domar-based GVC TFP and Leontief-based GVC TFP. Let's take the GVC of a car manufacture in the United States as an example. China provides steels, as intermediate goods, to the US car manufacturer through three channels. First, China delivers steels to the axles manufacturer in Japan, and then to the US car manufacturer (x_{C-J-U}). Second, China delivers steels to the engines manufacturer in Germany, and then to the US car manufacturer (x_{C-G-U}). Third, China delivers steels directly to US car manufacturer (x_{C--U}). These (direct and indirect) channels are treated differently in the Domar-based GVC TFP. However, all the different channels are considered to be homogeneous and aggregated together with simple addition, that is, $x_{C-U} = x_{C-J-U} + x_{C-G-U} + x_{C--U}$.

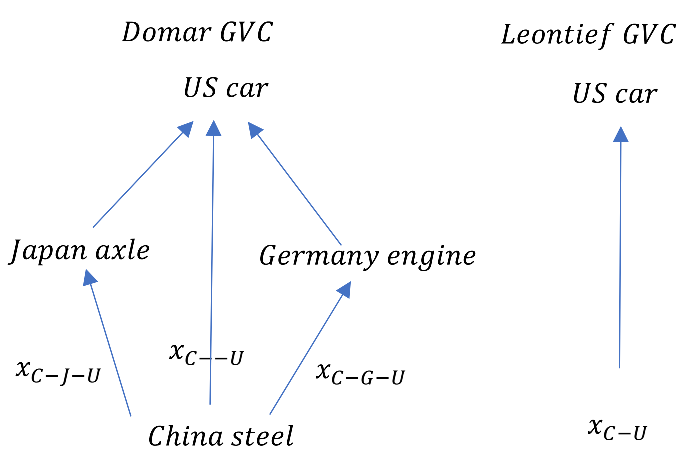


Figure 4-1: Domar-based GVC TFP v.s. Leontief-based GVC TFP

(4) Extreme Scenarios

Scenario 1: Aggregation

Assume that $a_{11} = a_{12} = a_{21} = a_{22} = \delta_{12} = \delta_{21} = 0$, then we obtain an economy with two sectors lacking input–output relations.

$$y_1 = x_1 = F_1(A_1, l_1, k_1)\ ; y_2 = x_2 = F_2(A_2, l_2, k_2) \tag{4-20}$$

In this scenario, each sector forms an independent GVC, and thus we have $A_i^{LeontiefGVC} = A_i^{DomarGVC} = A_i$. There is only aggregation process (i.e. aggregation of different GVCs), without integration (i.e. integration of different production stages of a GVC). The aggregate Leontief-based GVC TFP and aggregate Domar-based TFP can be expressed as follows, respectively:

$$AP_{LeontiefGVC} = \frac{y_1}{y_1+y_2} lnA_1^{LeontiefGVC} + \frac{y_2}{y_1+y_2} lnA_2^{LeontiefGVC} \tag{4-21}$$

$$AP_{DomarGVC} = \frac{y_1}{y_1+y_2} ln\ A_1^{DomarGVC} + \frac{y_2}{y_1+y_2} ln\ A_2^{DomarGVC} \tag{4-22}$$

$$AP_{DomarSector} = \frac{x_1}{y_1+y_2} lnA_1 + \frac{x_2}{y_1+y_2} lnA_2 \tag{4-23}$$

$\Rightarrow AP_{LeontiefGVC} = AP_{DomarGVC} = AP_{DomarSector}$

Scenario 2: Integration (different sectors)

Assume that $a_{11} = a_{12} = a_{22} = \delta_{12} = 0$ and $\delta_{21} = 1$, then we obtain an economy with two sectors, with sector two providing intermediates to sector one.

$$y_1 = x_1 = F_1(A_1, l_1, k_1, x_{21})\ ; x_2 = x_{21} = F_2(A_2, l_2, k_2) \tag{4-24}$$

In this scenario, the two sectors form an integrate GVC. There is only integration process (i.e. integration of different production stages of a GVC), without aggregation (i.e. aggregation of different GVCs). We have, $\boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ a_{21} & 1 \end{bmatrix}$; $\boldsymbol{C_l}\widehat{\boldsymbol{Y}} = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix} = \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix}$.

$$lnA_1^{LeontiefGVC} = lny_1 - \left[\left(\frac{W_1 l_1}{x_1} * lnl_{11} + \frac{R_1 k_1}{x_1} * lnk_{11}\right) + a_{21}\left(\frac{W_2 l_2}{x_2} * lnl_{21} + \frac{R_2 k_2}{x_2} * lnk_{21}\right)\right] =$$
$$lnx_1 - [(lnx_1 - a_{21}lnx_{21} - lnA_1) + a_{21}(lnx_2 - lnA_2)] = lnA_1 + a_{21}lnA_2 \tag{4-25}$$

$$lnA_1^{DomarGVC} = lnA_1 + a_{21}lnA_2 \tag{4-26}$$

$$lnA_1^{DomarSector} = \frac{x_1}{y_1+y_2} lnA_1 + \frac{x_2}{y_1+y_2} lnA_2 = \frac{x_1}{y_1} lnA_1 + \frac{x_2}{y_1} lnA_2 = lnA_1 + a_{21}lnA_2 \tag{4-27}$$

$\Rightarrow AP_{LeontiefGVC} = AP_{DomarGVC} = AP_{DomarSector}$

Scenario 3: Integration (same sector)

Now we further consider an economy with one sector, with intermediates delivered to itself. In this scenario, the intermediate products of a sector delivered to the same sector in different production stages are considered to be heterogenous.

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_1} \tag{4-28}$$

$$\boldsymbol{C} = 1/(1-\gamma_1) \Rightarrow \boldsymbol{C}\widehat{\boldsymbol{Y}} = (1-\gamma_1)x_1/(1-\gamma_1) = x_1 \Rightarrow \boldsymbol{C_l}\widehat{\boldsymbol{Y}} = l_1, \boldsymbol{C_k}\widehat{\boldsymbol{Y}} = k_1$$

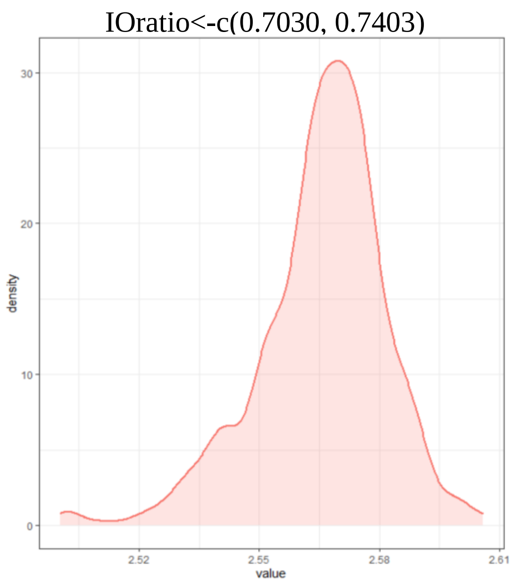
$$AP_{LeotiefGVC} = ln[(1-\gamma_1)x_1] - \frac{\alpha_1}{1-\gamma_1}lnl_1 - \frac{\beta_1}{1-\gamma_1}lnk_1 = lnx_1 - \frac{\alpha_1}{1-\gamma_1}lnl_1 - \frac{\beta_1}{1-\gamma_1}lnk_1 + ln(1-\gamma_1) \quad (4\text{-}29)$$

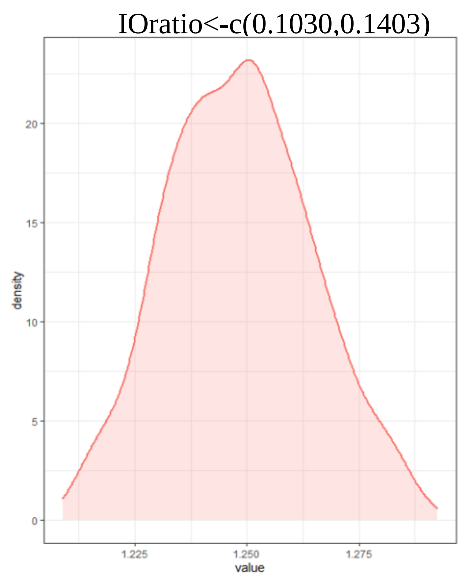
$$AP_{DomarGVC} = \frac{lnA}{1-\gamma_1} = \frac{lnx_1-\alpha_1 lnl_1-\beta_1 lnk_1-\gamma_1 ln(\gamma_1 x_1)}{1-\gamma_1} = \frac{(1-\gamma_1)lnx_1-\alpha_1 lnl_1-\beta_1 lnk_1-\gamma_1 ln\gamma_1}{1-\gamma_1} \quad (4\text{-}30)$$

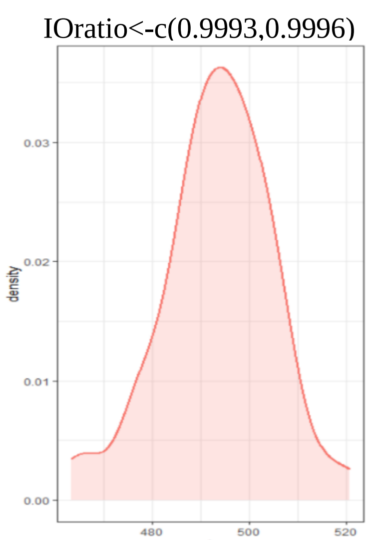
$$AP_{DomarGVC} - AP_{LeontiefGVC} = \frac{-\gamma_1 ln\gamma_1-(1-\gamma_1)ln(1-\gamma_1)}{1-\gamma_1} > 0 \quad (4\text{-}31)$$

4.2 Simulation

Figure 4-2 shows the ratio of the aggregate Domar-based to Leontief-based GVC TFP. We set different parameters for their input–output ratio (intermediate input to output). As can be seen, the ratio is greater than one in all situations, meaning that the Domar-based GVC TFP is larger than the Leontief-based GVC TFP at the aggregate level. Furthermore, we find that a larger input–output ratio brings a larger ratio, indicating that the gap between the two types of GVC TFP results from the role of intermediate inputs.







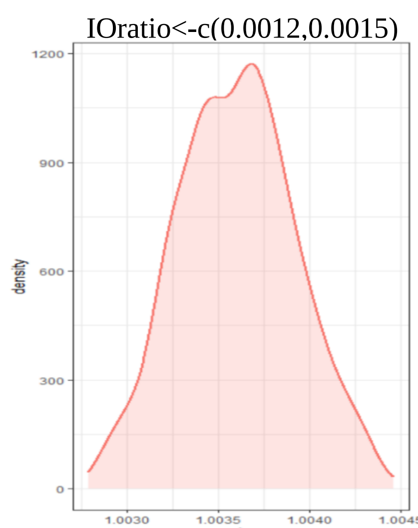


Figure 4-2: The ratio of aggregate Domar-based to Leontief-based GVC TFP

Note: Repeat generating random matrix 1,000 times.

4.3 From Jorgenson and Solow to GVC TFP

Based on the analysis above, the key difference between $AP_{DomarGVC}$ and $AP_{LeontiefGVC}$ is to treat the intermediate products of a sector delivered to other sectors or the same sector through different channels as heterogenous (weighted sum) or homogenous (simple addition). Following Jorgenson's framework, where the ratio of weighted sum to simple addition of factors is defined as the reallocation effect, the gap between $AP_{DomarGVC}$ and $AP_{LeontiefGVC}$ can be considered as the intra-sector (and intra-GVC) reallocation effect. Likewise, the gap between the weighted sum to the simple addition of factors across sectors within a GVC can be viewed as the intra-GVC (and inter-sector) reallocation effect. Furthermore, the gap between the weighted sum to the simple addition of factors across GVCs can be viewed as the inter-GVC reallocation effect.

The APG based on PPF (Jorgenson type) could be expressed as:

$$APG_{PPFSector} = \sum_{j=1}^{n} \frac{P_j^v v_j}{\sum_{j=1}^{n} P_j^v v_j} \dot{v}_j - \frac{Wl}{Py}\left(\sum_{j=1}^{n} \dot{l_j}\right) - \frac{Rk}{Py}\left(\sum_{j=1}^{n} \dot{k_J}\right) \tag{4-32}$$

Similarly, the APG based on PPF (GVC type) could be expressed as:

$$APG_{PPFGVC} = \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \dot{y}_j - \frac{Wl}{Py}\dot{l} - \frac{Rk}{Py}\dot{k} \tag{4-33}$$

Then we have the following decomposition framework, which unifies the Domar-based and Leontief-based GVC TFPs, and integrates the GVC TFPs into the framework of Jorgenson. Jorgenson's model has been widely used as an accounting framework in aggregating sectoral TFP. This paper is the first attempt to apply Jorgenson's model to GVC TFP aggregation.

$$APG_{PPFGVC} = APG_{DomarGVC} +$$

$$(APG_{LeontiefGVC} - APG_{DomarGVC})$$

$$+\left\{\sum_{j=1}^{n}\left[\sum_{i=1}^{n} \frac{W_i l_{ij}}{\sum_{j=1}^{n} P_j y_j}\dot{l}_{ij} - \frac{\left(\sum_{i=1}^{n} W_i l_{ij}\right)}{\sum_{j=1}^{n} P_j y_j}\dot{l}_j\right] + \sum_{j=1}^{n}\left[\sum_{i=1}^{n} \frac{R_i k_{ij}}{\sum_{j=1}^{n} P_j y_j}\dot{k}_{ij} - \frac{\left(\sum_{i=1}^{n} R_i k_{ij}\right)}{\sum_{j=1}^{n} P_j y_j}\dot{k}_j\right]\right\}$$

$$+\left\{\left[\sum_{j=1}^{n} \frac{\left(\sum_{i=1}^{n} W_i l_{ij}\right)}{\sum_{j=1}^{n} P_j y_j}\dot{l}_j - \frac{Wl}{Py}\dot{l}\right] + \left[\sum_{j=1}^{n} \frac{\left(\sum_{i=1}^{n} R_i k_{ij}\right)}{\sum_{j=1}^{n} P_j y_j}\dot{k}_j - \frac{Rk}{Py}\dot{k}\right]\right\} \tag{4-34}$$

In other words, PPF-based aggregate TFP growth (Jorgenson type) can be decomposed into different components with value chain connections. APG_{PPF}=$APG_{DomarGVC}$ + Intra-sector (and intra-GVC) reallocation effect + Intra-GVC (and inter-sector) reallocation effect+ Inter-GVC reallocation effect. TFP measures our ignorance, but our framework provides more knowledge to the "ignorance" than existing productivity studies. It is worth mentioning that the inter-GVC reallocation, which cannot be captured in studies based on sectoral TFP, matters a lot since job-hopping is likely to happen within the same sector but across GVCs considering the asset specificity

(Williamson, 1979), such as hopping from the upstream companies of Toyota to the upstream companies of Ford.

Since the pioneer works of Cobb & Douglas (1928), Tinbergen (1942) and Solow (1957), APF has become a widely used approach for calculating APG. The Solow-type TFP can be expressed as follows

$$APG_{APFSector} = APG_{PPFSector} - \left[\sum_{j=1}^{n} \frac{P_j^V v_j}{\sum_{j=1}^{n} P_j^V v_j} \dot{v}_j - \left(\sum_{j=1}^{n} \dot{v_j}\right)\right] \tag{4-35}$$

In Jorgenson's model, the APF-based aggregate TFP growth (Solow type) is the sum of PPF-based aggregate TFP growth and value added reallocation effect or substitution bias.

$$APG_{APFSector} = APG_{PPFSector} - \left[\sum_{j=1}^{n} \frac{P_j^V v_j}{\sum_{j=1}^{n} P_j^V v_j} \dot{v}_j - \left(\sum_{j=1}^{n} \dot{v_j}\right)\right] \tag{4-36}$$

There are some strict assumptions for the existence of APF: all sectors have a value added production function and these functions are identical; capitals (or labors) in all sectors are homogeneous (Domar, 1961; Hulten, 1978; Jorgenson *et al*., 1987; Wu, 2020). Furthermore, this approach treat value added and inputs in different ways: geometric index is used for aggregating labor and capital, while arithmetic index is employed for aggregating outputs (Domar, 1961). Despite of the possible drawbacks, it has been widely used due to the availability of country-level data.

The APF-based aggregate TFP growth (GVC type) is the sum of PPF-based aggregate TFP growth and final demand reallocation effect or substitution bias.

$$APG_{APFGVC} = \left(\sum_{j=1}^{n} \dot{y_j}\right) - \frac{\left(\sum_{j=1}^{n} W_{GVCj} l_{GVCj}\right)}{\left(\sum_{j=1}^{n} P_j y_j\right)}\left(\sum_{j=1}^{n} \dot{l}_{GVCj}\right) - \frac{\left(\sum_{j=1}^{n} R_{GVCj} k_{GVCj}\right)}{\left(\sum_{j=1}^{n} P_j y_j\right)}\left(\sum_{j=1}^{n} \dot{k}_{GVCj}\right) \tag{4-37}$$

Where l_{GVCj} means all the labor services embodied in value chain j, and l_j refers to the labor service in sector j. We can have the following equations only in a closed economy $APG_{APFGVC} = APG_{APFSector}$, and thus decompose $APG_{APFSector}$ into components with value chains connections. In this way, we can link the Solow type TFP to the GVC TFP.

5 Conventional approaches: missing productivity

5.1 Conventional approaches

The aggregation of sectoral TFP offers two conventional methods of calculating aggregate TFP. The first approach aggregates sectoral TFP based on the output production function (Watanabe, 1971), and its common expression of sectoral TFP is as follows:

$$lnA_i^{GOsim} = lnx_i - \frac{W_i l_i}{P_i x_i} lnl_i - \frac{R_i k_i}{P_i x_i} lnk_i - \frac{P_i^M m_i}{P_i x_i} lnm_i \tag{5-1}$$

If the industrial origins of intermediate inputs are considered, sectoral TFP is expressed as

$$lnA_i^{GO} = lnx_i - \frac{W_i l_i}{P_i x_i} lnl_i - \frac{R_i k_i}{P_i x_i} lnk_i - \sum_j \frac{P_{ji} x_{ji}}{P_i x_i} lnx_{ji} \tag{5-2}$$

The second approach is based on the value-added production function (Kendrick, 1961, 1973), and its common expression of sectoral TFP is as follows:

$$lnA_i^V = lnv_i - \frac{W_i l_i}{P_i^V v_i} lnl_i - \frac{R_i k_i}{P_i^V v_i} lnk_i \tag{5-3}$$

Using output or value-added shares as the weight, Equations (5-1)–(5-3) can be used to aggregate TFP. The aggregate TFP corresponding to Equation (5-1) is expressed as follows:

$$lnA^{GOsim} = \sum_{i=1}^{n} \frac{P_i x_i}{\sum_{=1}^{n} P_i x_i} lnA_i^{GOsim} \tag{5-4}$$

The aggregate TFP corresponding to Equation (5-2) is expressed as follows:

$$lnA^{GO} = \sum_{i=1}^{n} \frac{P_i x_i}{\sum_{=1}^{n} P_i x_i} lnA_i^{GO} \tag{5-5}$$

The aggregate TFP corresponding to Equation (5-3) is expressed as follows:

$$lnA^{V} = \sum_{i=1}^{n} \frac{P_i^V v_i}{\sum_{=1}^{n} P_i^V v_i} lnA_i^{V} \tag{5-6}$$

These conventional approaches are widely used because the sectoral inputs and output are often easily obtained. However, intermediates are considered exogenous in sectoral TFPs. In fact, the TFP growth of Sector A might benefit from the TFP growth of upstream sectors through intermediates, which could further benefit itself. Therefore, the intermediate inputs are actually endogenous.

5.2 Scenarios

Scenario 1: one sector, with all upstream stages coming from the same sector

It is highly likely that upstream production stages come from the same sector, which is also covered in our model above. We consider an extreme scenario, in which all upstream stages come from the same sector.

$$lnA_1^{LeontiefGVC} = lnA_1 + \gamma_1 lnA_2 + \gamma_1\gamma_2 lnA_3 + \gamma_1\gamma_2 \ldots \gamma_{n-1} lnA_n \tag{5-7}$$

According to the Equation (5-7), we have $lnA_1^{LeontiefGVC} = \sum_{i=1}^{n} \gamma_1^{i-1} \cdot lnA_1 = \frac{lnA_1}{1-\gamma_1} > lnA_1$. In this scenario, the share-weighted method underestimates sectoral TFP owing to its ignorance of input–output relations within the sector. In the real world, these relations might not be successive in production stages; they could alternate with intermediate inputs from other sectors.

Scenario 2: Two sectors, with one final sector and one intermediate sector

Next, we again consider the extreme scenario in section 4.1.

$$y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{21}^{\gamma_{21}};\ x_{21} = A_2 l_2^{\alpha_2} k_2^{\beta_2} \tag{5-8}$$

Then we have

$$lnA_1^{LeontiefGVC} = \frac{P_1 y_1}{P_1 y_1} lnA_1 + \frac{P_2 x_{21}}{P_1 y_1} lnA_2 > \frac{P_1 y_1}{P_1 y_1 + P_2 x_{21}} lnA_1 + \frac{P_2 x_{21}}{P_1 y_1 + P_2 x_{21}} lnA_2.$$

In this scenario, the share-weighted aggregate TFP is smaller than the aggregate GVC TFP because x_{21} is part of y_1, which leads to the double-counting in the denominator of the share-weighted aggregate TFP.

Scenario 3: Two sectors, providing intermediate inputs for each other.

$$y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} (y_{21})^{\gamma_{21}};\ y_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} (y_{12})^{\gamma_{12}} \tag{5-9}$$

$$\Rightarrow y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} \left[\delta_{21} A_2 l_2^{\alpha_2} k_2^{\beta_2} (\delta_{12} y_1)^{\gamma_{12}}\right]^{\gamma_{21}} =$$

$$\left[\left(\delta_{21}^{\gamma_{21}} \delta_{12}^{\gamma_{12}\gamma_{21}}\right)\left(A_1 A_2^{\gamma_{21}}\right)\left(l_1^{\alpha_1} l_2^{\alpha_2\gamma_{21}}\right)\left(k_1^{\beta_1} k_2^{\beta_2\gamma_{21}}\right)\right]^{\frac{1}{1-\gamma_{12}\gamma_{21}}} \tag{5-10}$$

Then the Leontief-based GVC TFP can be expressed as

$$lnA_1^{LeontiefGVC} = \frac{(\gamma_{21} ln\delta_{21} + \gamma_{12}\gamma_{21} ln\delta_{12}) + (lnA_1 + \gamma_{21} lnA_2)}{1-\gamma_{12}\gamma_{21}} \tag{5-11}$$

Since $0 \leq \delta_{12}, \delta_{21} \leq 1 \Rightarrow \gamma_{21} ln\delta_{21} + \gamma_{12}\gamma_{21} ln\delta_{12} \leq 0$, $\gamma_{21} lnA_2 \geq 0$, and $1 - \gamma_{12}\gamma_{21} \leq 1$, we have $lnA_1^{LeontiefGVC} > lnA_1$

Similarly, we could also have:

$$y_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} \left[\delta_{12} A_1 l_1^{\alpha_1} k_1^{\beta_1} (\delta_{21} y_2)^{\gamma_{21}}\right]^{\gamma_{12}} =$$

$$\left[\left(\delta_{12}^{\gamma_{12}} \delta_{21}^{\gamma_{21}\gamma_{12}}\right)\left(A_2 A_1^{\gamma_{12}}\right)\left(l_2^{\alpha_2} l_1^{\alpha_1\gamma_{12}}\right)\left(k_2^{\beta_2} k_1^{\beta_1\gamma_{12}}\right)\right]^{\frac{1}{1-\gamma_{21}\gamma_{12}}} \tag{5-12}$$

$$\Rightarrow lnA_2^{LeontiefGVC} = \frac{(\gamma_{12} ln\delta_{21} + \gamma_{21}\gamma_{12} ln\delta_{12}) + (lnA_2 + \gamma_{12} lnA_1)}{1-\gamma_{12}\gamma_{21}} > lnA_2 \tag{5-13}$$

Therefore, we could obtain aggregate GVC TFP $lnA^{LeontiefGVC}$ and share-weighted aggregate TFP lnA^{GO}:

$$lnA^{LeontiefGVC} = \frac{P_1(y_1 - y_{12})}{P_1(y_1 - y_{12}) + P_2(y_2 - y_{21})} lnA_1^{LeontiefGVC} + \frac{P_2(y_2 - y_{21})}{P_1(y_1 - y_{12}) + P_2(y_2 - y_{21})} lnA_2^{LeontiefGVC} \tag{5-14}$$

$$lnA^{GO} = \frac{P_1 y_1}{P_1 y_1 + P_2 y_2} lnA_1 + \frac{P_2 y_2}{P_1 y_1 + P_2 y_2} lnA_2 < lnA^{LeontiefGVC} \tag{5-15}$$

To sum up, share-weighted aggregate sectoral TFP is smaller than aggregate GVC TFP.

5.3 Simulation

As discussed, the conventional approach fails to consider the endogeneity of intermediate inputs. Figure 5-1 illustrates the ratios of aggregate TFP based on conventional approaches and that based on Leontief approach. The conventional approaches include the aggregate TFP based on value added function (Kendrick, 1961, 1973), that based on output function (i.e., the weighted or simple sum of intermediate inputs). The ratios are all smaller than one, indicating that conventional approaches underestimate the aggregate TFP, and thus brings missing productivity.

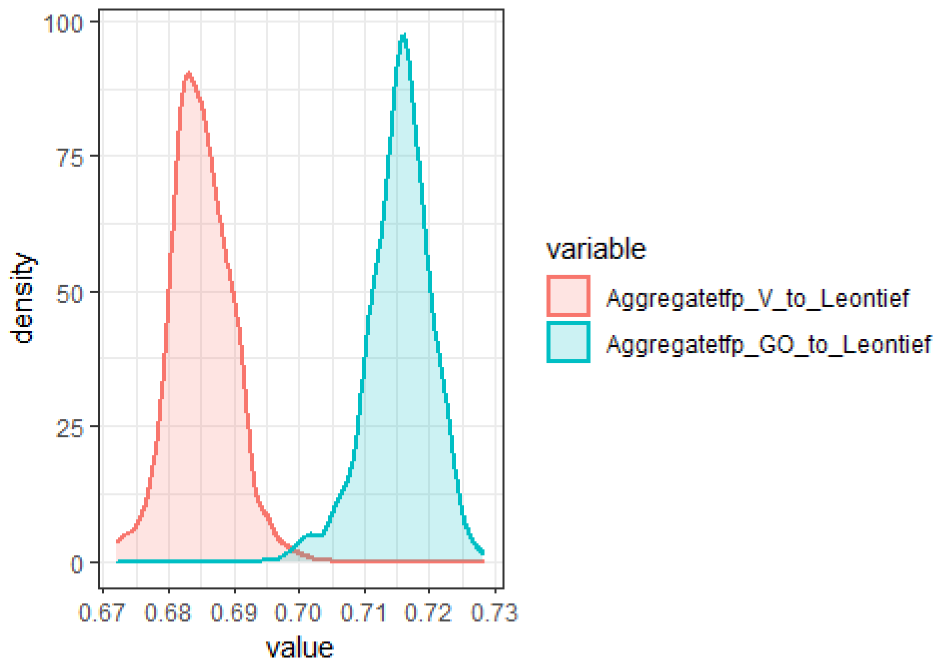


Figure 5-1: The ratio of aggregate TFP based on conventional approach and that based on Leontief approach

Note: Repeat generating random matrix 1,000 times.

6 Empirical Analysis

6.1 Data

We used the WIOD, which covers 56 sectors in 44 countries ranging from 2000 to 2014. It includes the world input–output tables (WIOTs)[①] and social economic accounts (SEAs)[②], which provide abundant information on output, value-added, intermediate input, labor input, capital input, and price indices at the country-sector level. The output, value added, and intermediate input from the WIOT and those from the SEA are basically equal.

Whereas the WIOT contains information on the output and intermediate input of the rest of the world (ROW), the SEA dataset does not. Thus, we estimate the primary inputs of ROW by assuming that the average ratio between primary inputs and output of all the middle-income countries equals to that of ROW. 10 out of 44 countries in the WIOT are middle-income countries.

We converted the local currency in the SEA to USD based on the exchange rate data provided by the WIOD and transformed all the nominal values into real values using the price index with 2010 as the basic year. The unit of all values was USD, and the unit of labor was a person.

[①] Source: http://www.wiod.org/database/wiots16
[②] Source: http://www.wiod.org/database/seas16

The number of employees (EMPE) and total hours worked by employees (H_EMPE) were missing for China; thus, we used the number of persons engaged (EMP) to measure labor input. The number of abnormal values for compensation of employees (COMP), which is larger than value-added (VA), was 483, whereas that for labor compensation (LAB) was 1,827. Therefore, we used COMP rather than LAB to measure labor compensation, and if COMP was greater than VA, we set them equal. There were too many negative and zero values for capital compensation (CAP); hence, we used the difference between one and CAP, which is in line with the assumption of constant returns to scale.

The WIOT allowed us to calculate the total output of the world resulting from the unit final demand of sector j in country s (i.e., the Leontief inverse). By matching WIOT and SEA, we further obtained the total labor and capital inputs of GVC resulting from the unit final demand of sector j in country s, from which we calculated the Leontief-based GVC TFP.

6.2 World GVC TFP

Figure 6-1 shows the evolution of the world's aggregate TFP based on the Domar approach and that based on the Leontief approach. The Domar aggregation based on sectoral TFP was equal to the aggregate Domar-based GVC TFP. And also, the aggregate Domar-based GVC TFP is, 28.03% on average during 2000-2014, larger than the aggregate Leontief-based GVC TFP, which agrees with the theoretical predictions and simulations.

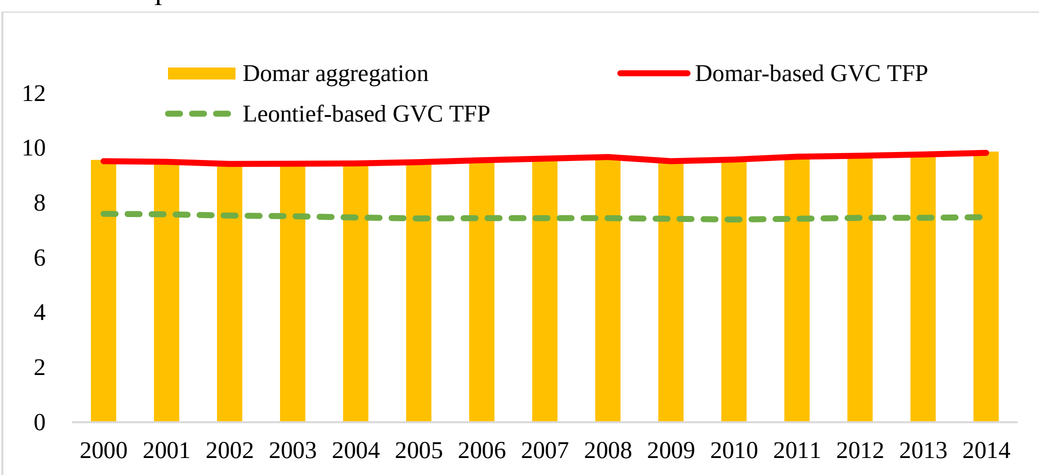


Figure 6-1: GVC TFP (World): Domar *v.s.* Leontief

To investigate the missing productivity in conventional approaches, including the aggregate TFP based on share (output or value added) weight. And also, we display the results based on APF. Figure 6-2 shows that all the aggregate TFPs based on conventional approaches and APF were smaller than the aggregate Domar-based GVC TFP, which aligns with our theoretical analysis and the simulation results. Therefore, the conventional approaches and APF miss significant productivity measurements: roughly half of the aggregate Domar-based GVC TFP during 2000-2014.

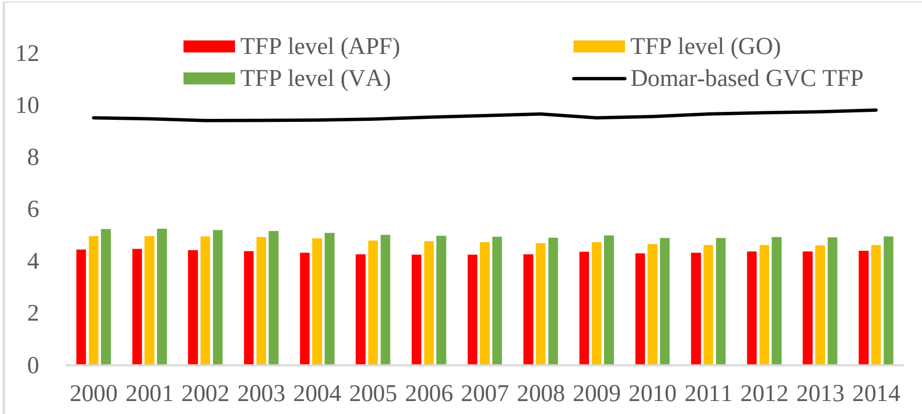


Figure 6-2: GVC TFP v.s. conventional approaches and APF (World)

6.3 Country-Level GVC TFP

Compared with the world's TFP, national TFP might be of more interests to scholars and decision makers. Therefore, we further calculate the aggregate TFP at the country level (or region level). As we mentioned above, Domar aggregation based on sectoral TFP at the country level is problematic because it neglects the endogeneity of imported intermediate inputs. The national TFP based on conventional approaches fails to consider the endogeneity of both domestic and imported intermediate inputs, which might be more problematic than Domar aggregation. However, the Domar-based GVC TFP at the country level is free of the endogeneity issue.

Table 6-3 lists the GVC TFP growth rates of selected countries. For example, those of China and Russia are higher than other developed countries, such as the US, Germany, and Japan (prior to the 2008 financial crisis). However, after 2008, the advantages of China and Russia disappeared, which is likely the result of a strong anti-globalization push made by developed countries. Table 6-4 lists the foreign contributions to the GVC TFP levels (%) of selected countries (2000–2014). Foreign contributions to China's GVC TFP experienced an inverted U shape over the sample period, with 2007 as the turning point. Foreign contributions to India's GVC TFP also stopped growing after 2007. However, foreign contributions to developed countries kept growing. All of these findings provide evidence of the negative impact of anti-globalization on developing countries.

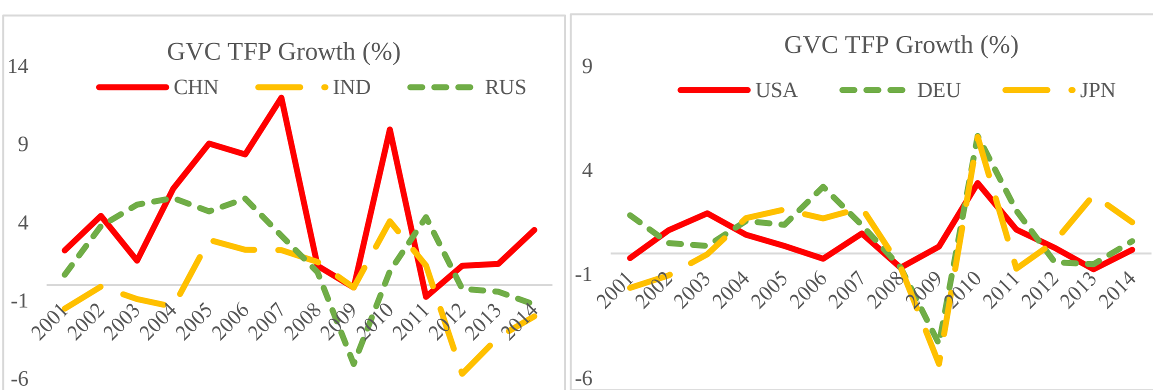


Figure 6-3: GVC TFP Growth (%) of selected countries (2000-2014)

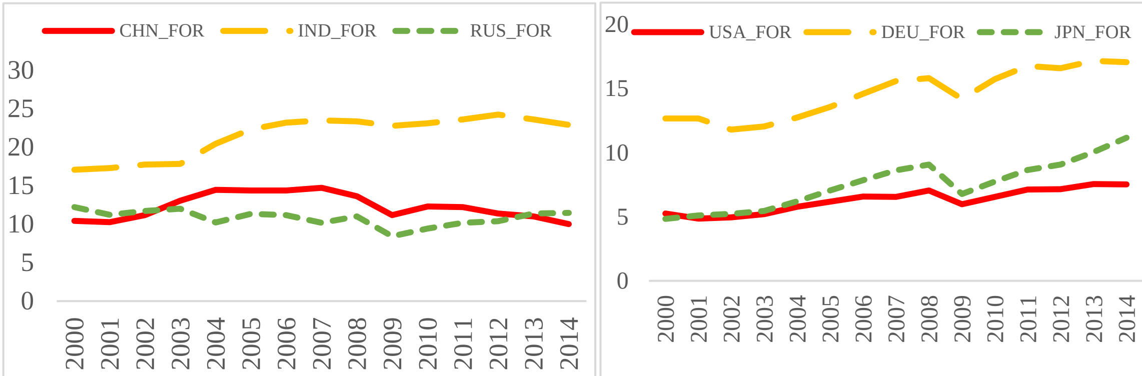


Figure 6-4: Foreign contribution to GVC TFP level (%) of selected countries (2000-2014)

Based on the Domar-based GVC TFP, we can further identify the country origins of the GVC TFP of a specific country. From this, we can answer the question, "Which country contributes more to the international competitiveness of a specific country?"

Table 6-1 shows each country's contribution to the GVC TFP level of the US. From this, it can be seen that Canada, Japan, and Germany have long been the three most important contributors to the US' GVC TFP. The contributions of China, Russia, India, and South Korea grew dramatically, and China became the most important foreign contributor to the US' GVC TFP in 2014.

Table 6-2 lists each country's contribution to the GVC TFP level of Japan. Compared with the US, foreign contributions grew dramatically, especially those from China. The US, China, and Australia have long been the top three contributors to the GVC TFP of Japan. The contributions from China, Russia, and India increased many times during 2000–2014, and China became the most important foreign contributor to their GVC TFP in 2014.

Table 6-1: Each country's contribution to the GVC TFP level of the US (%)

2000		2014	
USA	93.3386	USA	90.0230
ROW	1.5898	ROW	2.4440
CAN	1.2060	**CHN**	**1.3584**
JPN	0.6979	CAN	1.3472
DEU	0.4303	DEU	0.6281
GBR	0.3953	JPN	0.5889
FRA	0.2984	GBR	0.4435
MEX	0.2724	MEX	0.4283
CHN	**0.1961**	**KOR**	**0.3535**
ITA	0.1809	FRA	0.3020
KOR	**0.1681**	ITA	0.2126
TWN	0.1497	NLD	0.1886
NLD	0.1307	BRA	0.1855
BRA	0.0992	TWN	0.1690
AUS	0.0788	**RUS**	**0.1642**
ESP	0.0774	BEL	0.1358
BEL	0.0758	CHE	0.1168
SWE	0.0742	ESP	0.1032
CHE	0.0726	**IND**	**0.0933**
RUS	**0.0714**	AUS	0.0830
IRL	0.0662	IRL	0.0816
NOR	0.0532	SWE	0.0726
FIN	0.0409	AUT	0.0559
IND	**0.0339**	NOR	0.0557
AUT	0.0321	FIN	0.0529
TUR	0.0316	TUR	0.0508
IDN	0.0299	IDN	0.0496
DNK	0.0275	POL	0.0406
POL	0.0170	DNK	0.0369
CZE	0.0111	CZE	0.0287
HUN	0.0091	HUN	0.0185
PRT	0.0086	PRT	0.0181
LUX	0.0068	ROU	0.0151
GRC	0.0060	LUX	0.0117
ROU	0.0053	SVK	0.0078
CYP	0.0049	GRC	0.0077
HRV	0.0030	LTU	0.0057
SVN	0.0023	BGR	0.0054
SVK	0.0021	SVN	0.0050
MLT	0.0015	HRV	0.0035
LVA	0.0011	EST	0.0034
LTU	0.0008	LVA	0.0020
EST	0.0007	CYP	0.0012
BGR	0.0007	MLT	0.0010

Table 6-2: Each country's contribution to the GVC TFP level of Japan (%)

2000		2014	
JPN	**93.7262**	**JPN**	**85.3558**
ROW	**2.1645**	**ROW**	**6.2179**
USA	1.2316	**CHN**	**2.0334**
CHN	**0.3748**	USA	1.2519
AUS	0.3060	AUS	0.9158
KOR	0.2561	KOR	0.6267
DEU	0.2538	**RUS**	**0.4745**
CAN	0.2437	DEU	0.4356
GBR	0.2347	TWN	0.3297
TWN	0.1722	GBR	0.2907
IDN	0.1469	IDN	0.2821
FRA	0.1357	CAN	0.2787
RUS	**0.0852**	FRA	0.2095
ITA	0.0835	BRA	0.1511
NLD	0.0760	ITA	0.1356
CHE	0.0663	NLD	0.1117
BRA	0.0623	CHE	0.1040
BEL	0.0478	ESP	0.0901
SWE	0.0472	BEL	0.0777
NOR	0.0448	**IND**	**0.0725**
DNK	0.0352	SWE	0.0687
ESP	0.0347	NOR	0.0683
FIN	0.0248	DNK	0.0583
IRL	0.0242	FIN	0.0560
IND	**0.0228**	AUT	0.0446
MEX	0.0223	IRL	0.0383
AUT	0.0182	MEX	0.0326
TUR	0.0125	TUR	0.0325
POL	0.0087	POL	0.0310
LUX	0.0071	LUX	0.0225
GRC	0.0068	CZE	0.0190
CZE	0.0050	ROU	0.0135
HUN	0.0043	GRC	0.0131
PRT	0.0042	PRT	0.0118
ROU	0.0026	HUN	0.0115
HRV	0.0022	SVK	0.0058
SVN	0.0010	EST	0.0048
SVK	0.0009	BGR	0.0046
EST	0.0007	HRV	0.0043
CYP	0.0007	SVN	0.0040
LVA	0.0006	LTU	0.0040
LTU	0.0006	LVA	0.0030
MLT	0.0004	CYP	0.0019
BGR	0.0004	MLT	0.0014

Table 6-3: Each country's contribution to the GVC TFP level of China (%)

2000		2014	
CHN	87.8275	CHN	89.8796
ROW	3.9580	ROW	4.4115
JPN	2.0045	JPN	0.7454
USA	1.1826	USA	0.7272
TWN	0.8744	KOR	0.7180
KOR	0.8266	DEU	0.5461
DEU	0.5591	AUS	0.4880
FRA	0.3940	TWN	0.4093
AUS	0.3363	RUS	0.2591
GBR	0.3258	FRA	0.2206
RUS	0.2242	BRA	0.1969
CAN	0.1762	GBR	0.1877
ITA	0.1637	CAN	0.1414
IDN	0.1595	NLD	0.1271
NLD	0.1257	ITA	0.1086
SWE	0.0997	IDN	0.1061
BRA	0.0833	BEL	0.0745
BEL	0.0805	CHE	0.0717
CHE	0.0780	ESP	0.0639
ESP	0.0679	IND	0.0595
FIN	0.0641	SWE	0.0567
IND	0.0419	AUT	0.0430
AUT	0.0410	NOR	0.0424
NOR	0.0407	DNK	0.0419
ROU	0.0369	FIN	0.0410
IRL	0.0368	TUR	0.0362
DNK	0.0346	POL	0.0317
MEX	0.0326	IRL	0.0295
TUR	0.0280	CZE	0.0234
POL	0.0213	MEX	0.0218
LUX	0.0157	LUX	0.0147
CZE	0.0135	HUN	0.0134
HUN	0.0122	ROU	0.0116
GRC	0.0082	PRT	0.0112
PRT	0.0075	GRC	0.0087
HRV	0.0062	BGR	0.0068
SVN	0.0027	SVK	0.0067
SVK	0.0022	SVN	0.0040
BGR	0.0013	LTU	0.0034
EST	0.0011	HRV	0.0033
LTU	0.0011	EST	0.0025
LVA	0.0010	LVA	0.0021
MLT	0.0009	CYP	0.0012
CYP	0.0009	MLT	0.0009

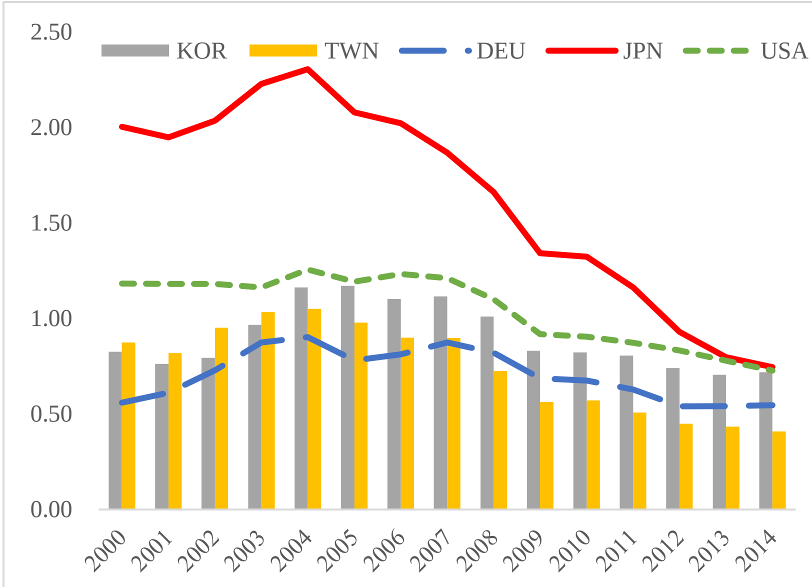


Figure 6-5: Contributions to the GVC TFP level of China (%)

6.4 Aggregate TFP at the Country-Sector Level

To further illuminate the gap of GVC and sectoral TFPs among countries, we calculated the TFP at the country-sector level. GVC TFP is more closely associated with relative-price international competitiveness, compared with sectoral TFP (Gu & Yan, 2017; Timmer & Ye, 2020). Figure 6-6 shows the results for the computer, electronic, and optical product sector. China maintained an international competitive advantage over India and Russia in terms of GVC TFP, despite bearing no advantages in sectoral TFP. This means that the final goods produced in China benefited the world more than those produced in India or Russia. This provides answers to the question, "Which country in which the final goods are produced is better at promoting global productivity?"

The GVC TFP of China kept growing during the given period, whereas that of the US decreased. Although the sectoral TFP of computers in the US was far greater than that in China, the US was eventually surpassed in GVC TFP. Thus, GVC integration provides a new metric by which developing countries can "catch up" with developed economies in terms of international competition. Furthermore, Japan, and Germany achieved increasing advantages over the US in terms of the GVC TFP of computers, indicating the decreasing competitiveness of final products made in the US.

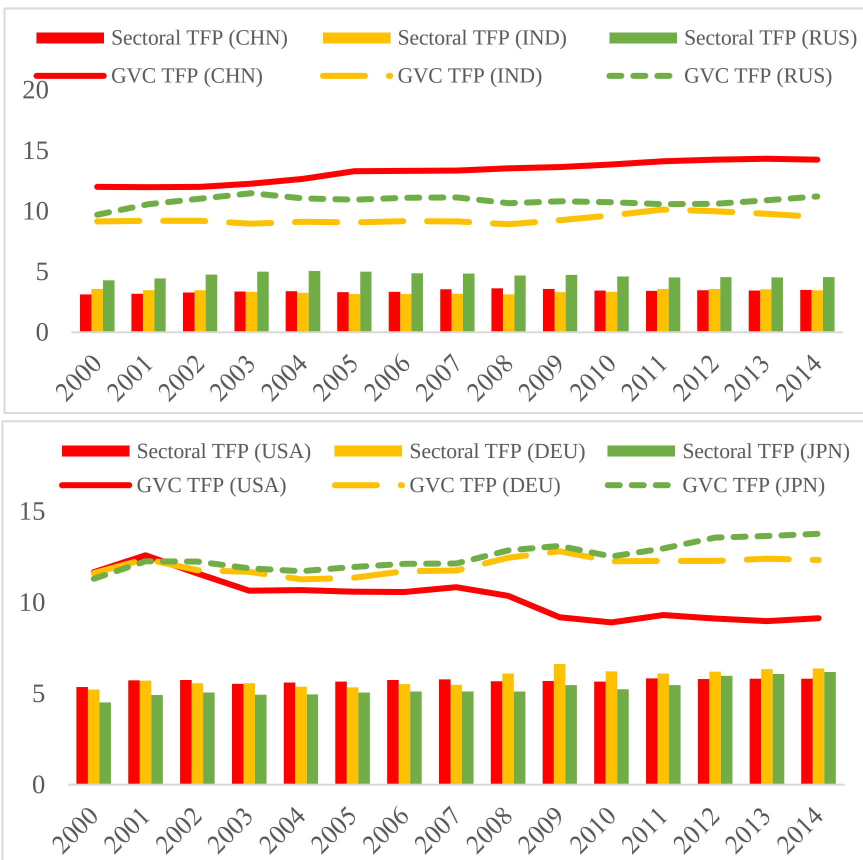


Figure 6-6: GVC TFP levels of computer (electronic and optical products) in selected countries (2000–2014)

Next, we further identify the country origins of the GVC TFP of specific country-sectors. Table 6-3 lists each country's contribution to the GVC TFP level of the computer sector in the US.

It can be seen that Japan, Canada, and South Korea have long been the three most important contributors. Contributions from China, Russia, Brazil, and India grew dramatically, and China became the most important foreign contributor to the GVC TFP in 2014.

Table 6-4 lists each country's contribution to the GVC TFP level of the computer sector in Japan. Compared with the US, foreign contributions to Japan grew dramatically, especially from China. The US, China, and South Korea have long been the most important contributors, but the contributions from China, Russia, and India increased greatly during 2000–2014. Notably, China became the most important foreign contributor to the GVC TFP of Japan in 2014. Interestingly, contributions from the US, the UK, and Mexico decreased dramatically.

Table 6-3: Each country's contribution to the GVC TFP level of the computer sector in the US (%)

2000		2014	
USA	87.6803	USA	86.8753
ROW	3.0751	ROW	3.3016
JPN	2.2397	**CHN**	**3.1847**
CAN	1.1531	JPN	1.0702
KOR	0.8797	CAN	0.8641
TWN	0.6843	KOR	0.7570
DEU	0.6263	DEU	0.6419
MEX	0.5882	MEX	0.5801
CHN	**0.5294**	TWN	0.3806
GBR	0.5153	GBR	0.3591
FRA	0.4085	FRA	0.2734
ITA	0.2219	ITA	0.1825
CHE	0.1500	CHE	0.1790
NLD	0.1266	NLD	0.1587
IRL	0.1260	**RUS**	**0.1364**
SWE	0.1194	**BRA**	**0.1231**
AUS	0.1069	AUS	0.0969
BRA	**0.0969**	BEL	0.0932
ESP	0.0875	ESP	0.0807
BEL	0.0865	SWE	0.0719
RUS	**0.0823**	**IND**	**0.0695**
IDN	0.0673	IRL	0.0657
AUT	0.0441	AUT	0.0591
NOR	0.0432	IDN	0.0546
FIN	0.0429	NOR	0.0466
DNK	0.0376	TUR	0.0430
IND	**0.0349**	DNK	0.0421
TUR	0.0294	FIN	0.0384
POL	0.0213	POL	0.0383
PRT	0.0151	CZE	0.0272
HUN	0.0145	HUN	0.0205
CZE	0.0125	PRT	0.0153
LUX	0.0108	ROU	0.0147
MLT	0.0106	LUX	0.0137
ROU	0.0072	SVK	0.0078
GRC	0.0069	GRC	0.0076
CYP	0.0039	BGR	0.0059
HRV	0.0037	SVN	0.0054
SVN	0.0032	EST	0.0034
SVK	0.0027	LTU	0.0033
LVA	0.0013	HRV	0.0032
LTU	0.0011	LVA	0.0017
BGR	0.0009	MLT	0.0016
EST	0.0008	CYP	0.0011

Table 6-4: Each country's contribution to the GVC TFP level of the computer sector in Japan (%)

2000		2014	
JPN	**89.5445**	**JPN**	**81.7051**
ROW	**3.1093**	**ROW**	**6.4635**
USA	**2.5025**	**CHN**	**4.8955**
KOR	0.8476	KOR	1.2501
CHN	**0.7559**	**TWN**	**1.0817**
TWN	**0.7134**	**USA**	**1.0387**
DEU	0.4247	DEU	0.5317
GBR	**0.3189**	AUS	0.4806
FRA	0.2332	**RUS**	**0.2990**
AUS	0.2214	**GBR**	**0.2751**
CAN	0.1674	CHE	0.2504
IDN	0.1512	IDN	0.2313
CHE	0.1302	FRA	0.2173
ITA	0.1116	CAN	0.1577
RUS	**0.0833**	ITA	0.1316
SWE	0.0796	NLD	0.1201
NLD	0.0784	**BRA**	**0.1119**
IRL	0.0740	ESP	0.0860
BRA	**0.0640**	BEL	0.0730
BEL	0.0629	**IND**	**0.0647**
MEX	**0.0620**	SWE	0.0623
ESP	0.0484	NOR	0.0512
NOR	0.0310	AUT	0.0506
DNK	0.0294	IRL	0.0445
AUT	0.0269	DNK	0.0412
FIN	0.0252	POL	0.0405
IND	**0.0239**	FIN	0.0390
TUR	0.0162	**MEX**	**0.0382**
POL	0.0117	TUR	0.0339
LUX	0.0093	CZE	0.0260
HUN	0.0082	LUX	0.0219
CZE	0.0072	PRT	0.0142
PRT	0.0060	HUN	0.0139
GRC	0.0056	ROU	0.0123
ROU	0.0039	GRC	0.0102
HRV	0.0029	SVK	0.0069
SVN	0.0016	BGR	0.0068
SVK	0.0015	SVN	0.0046
MLT	0.0014	HRV	0.0040
LTU	0.0008	LTU	0.0037
CYP	0.0007	EST	0.0032
LVA	0.0007	LVA	0.0023
EST	0.0007	MLT	0.0022
BGR	0.0006	CYP	0.0015

6.5 Decomposition of aggregate productivity growth

Section 4.3 provides the decomposition framework of aggregate GVC TFP growth based on Jorgenson's framework. It can be decomposed into aggregate Domar-based GVC TFP and three reallocation effects. The three reallocation effects include Intra-sector, Intra-GVC, Inter-GVC reallocation effect (Inter GVC). Table 6-5a and Table 6-5b show the results of decompositions.

The APG_{PPFGVC} after the financial crisis in 2008 was negative in many years, which is in line with the economic downturn following the financial crisis. The intra-sector (within a GVC) reallocation effect was overall negative, indicating that the reallocation among different channels of intermediate delivery is difficult. In comparison, there were more positive values in intra-GVC and inter-GVC reallocation effects. The information transmission is common within a GVC, which promotes the intra-GVC reallocation. Due to asset specificity, job-hopping and capital reallocation across GVCs (perhaps within the same sector) are highly possible. Despite the significance of the reallocation within and across GVCs, they have long been neglected by studies based on sectoral TFP.

By further decomposing the intra-GVC and inter-GVC reallocation effects into labor reallocation and capital reallocation, we found that there are more positive values for capital than labor, which indicates that capital flow was smoother than labor mobility.

Tabel 6-5a: Decomposition of aggregate productivity growth (%)

	APG_{PPFGVC}					APG_{PPFGVC}	
		$APG_{DomarGVC}$	Intra-GVC Intra-Sector	Intra-GVC Inter-Sector	Inter-GVC	RE_Y	APG_{APFGVC}
2001	0.3261	0.0392	-0.8571	1.4171	-0.2732	0.2880	0.0380
2002	0.5680	0.7490	-0.8380	0.7301	-0.0731	0.2024	0.3656
2003	0.0176	0.7834	-0.7945	0.2823	-0.2536	-0.5520	0.5696
2004	1.5069	1.4531	-0.6770	-0.2066	0.9374	0.2739	1.2330
2005	1.7241	2.0794	-1.6005	0.3392	0.9061	0.3136	1.4105
2006	2.2247	1.6622	-1.0602	0.6154	1.0073	0.3029	1.9218
2007	3.2511	2.2310	-1.2481	1.6567	0.6115	0.5047	2.7464
2008	3.0659	-0.3412	-1.3196	1.2119	3.5148	3.0570	0.0088
2009	-2.0841	-2.2515	-2.5500	2.8753	-0.1578	1.1670	-3.2510
2010	4.3846	4.6128	-2.3436	1.0666	1.0488	0.5201	3.8646
2011	0.7334	0.8433	-0.9276	0.1432	0.6744	0.2697	0.4637
2012	-1.5740	-0.3139	-0.7397	0.4678	-0.9882	0.2971	-1.8711
2013	-1.5781	0.1138	-2.2842	1.1946	-0.6024	-0.5318	-1.0463
2014	-0.2392	0.7812	-1.0651	0.2619	-0.2173	0.2264	-0.4657

Tabel 6-5b: Labor and capital reallocation effect (%)

Year	Intra-GVC			Inter-GVC		
		L	K		L	K
2001	1.4171	0.7382	0.6790	-0.2732	-0.4724	0.1992
2002	0.7301	0.2714	0.4588	-0.0731	-0.0660	-0.0071
2003	0.2823	-0.1024	0.3847	-0.2536	0.1414	-0.3950
2004	-0.2066	-0.2995	0.0929	0.9374	0.6852	0.2522
2005	0.3392	0.0754	0.2637	0.9061	0.5205	0.3856
2006	0.6154	0.3836	0.2318	1.0073	0.5744	0.4329
2007	1.6567	1.2631	0.3937	0.6115	-0.0100	0.6216
2008	1.2119	0.6235	0.5884	3.5148	0.7413	2.7735
2009	2.8753	1.3878	1.4875	-0.1578	-0.8382	0.6804
2010	1.0666	0.6030	0.4636	1.0488	0.4747	0.5740
2011	0.1432	0.0557	0.0875	0.6744	0.2650	0.4094
2012	0.4678	0.1502	0.3176	-0.9882	-1.4573	0.4691
2013	1.1946	0.5149	0.6798	-0.6024	-1.0031	0.4007
2014	0.2619	0.0116	0.2503	-0.2173	-0.7410	0.5236

7 Concluding Remarks

The rise of GVCs has rendered the "made in" sentiment an archaic symbol of a bygone era because most products, especially manufactured goods, are now perceived as "made in the world." The traditional Jorgenson accounting method based on country- and sector-level TFP is limited in explaining widespread international production fragmentation. This paper provides a systematic framework for measuring GVC TFP by explicitly considering intermediate inputs as an endogenous variable. Based on theoretical derivations, scenario analyses, simulations, and our recursive approach, we provide the following major findings:

(1) We clarified the gap between the Domar- and Leontief-based GVC TFP, which serves as an excellent bridge for understanding the relationship between sectoral and GVC TFP. Domar aggregation based on sectoral TFP has been a widely used method of calculating aggregate TFP in the literature. However, few scholars have noticed that the Domar approach can also be used to analyze GVC TFP, and the differences between Domar-based GVC TFP and Leontief-based GVC TFP remain unclear. We found that the intermediate products of a sector delivered to other sectors or the same sector through different channels are assumed to be heterogeneous in the Domar-based GVC TFP, whereas they are homogenous in Leontief-based GVC TFP. Based on the WIOD, the aggregate Domar-based GVC TFP was 28.03% larger on average than the aggregate Leontief-based GVC TFP during 2000–2014. This is a breakthrough revelation.

(2) We integrated GVC TFP into Jorgenson's framework. Jorgenson provides a classical framework for decomposing aggregate sectoral TFP growth. Based on this, we decomposed aggregate GVC TFP growth into aggregate Domar-based GVC TFP and three reallocation effects: intra-sector, intra-GVC, inter-GVC reallocation effects. The intra- and inter-GVC reallocation effects were further decomposed into labor and capital types. The intra- and inter-GVC reallocation effect has long been neglected by sector-level analysis studies. Hence, both GVC and sectoral TFP are now unified in Jorgenson's framework, and the empirical findings show the huge potential for global labor and capital reallocations to promote global TFP growth.

(3) We pointed out the knife-edge feature of the Domar aggregation based on sectoral TFP. That is, the Domar aggregation is valid only in closed economies. Pertaining to the national TFP of an open economy, it fails to properly capture foreign value chains. However, measuring national TFP in an open economy is a common approach in academic and policy studies. Furthermore, we located the missing productivity of conventional share-weighted sectoral TFP approaches and APF approach, which fail to resolve endogeneity issues. Importantly, the missing productivity accounts for about 50% of the aggregate Domar-based GVC TFP during 2000–2014.

(4) Apart from our theoretical findings, we also made some interesting empirical findings. First, anti-globalization does more harm for developing countries than developed countries in terms of GVC TFP growth and the foreign contributions to GVC TFP. The contribution from China dramatically improved the GVC TFPs of the US and Japan. Second, in terms of the computer, electronic, and optical product sector, rather than India or Russia, China (where the final goods were produced) was better at promoting global productivity. Although the sectoral TFP of computers in the US was far higher than that in China, China has surpassed the US in GVC TFP.

Thus, GVC integration provides a new metric that developing countries can leverage to catch up with developed economies in terms of international competitiveness.

In summary, this paper answered, “Which country in which final goods are produced is better for promoting global productivity” and “Which country contributes more to the international competitiveness of a specific country?” The answers that we provided are expected to be important to the further development of global and national economies. Moreover, we can also delineate the specific country-sector origins of cutthroat technologies, which will provide even more practical economic and policymaking guidance. However, we will leave this effort to future research.

References

1. Abramovitz M. Resource and output trends in the United States since 1870. American Economic Review, Papers and Proceedings, Vol. XLVI, May 1956, pp. 5-23, reprinted as National Bureau of Economic Research, Occasional Paper 52 (New York, 1956).

2. Ahmad N, Ribarsky J. Trade in value added, jobs and investment. paper presented at the 33rd general conference of International Association of Research in Income and Wealth, Rotterdam, Netherlands, 2014.

3. Altomonte C, Ottaviano G I P. The role of international production sharing in EU productivity and competitiveness[J]. European Investment Bank Papers, 2011, 16(1): 62-89.

4. Aulin-Ahmavaara P. Effective rates of sectoral productivity change[J]. Economic Systems Research, 1999, 11(4): 349-363.

5. Baily M N, Hulten C, Campbell D, et al. Productivity dynamics in manufacturing plants[J]. Brookings papers on economic activity (Microeconomics), 1992, 1992: 187-267.

6. Carter A P. Changes in the structure of the American economy, 1947 to 1958 and 1962[J]. The Review of Economics and Statistics, 1967, 49: 209-224.

7. Cas A, Rymes T K. On concepts and measures of multifactor productivity in Canada, 1961-1980 [J]. Cambridge University Press, Cambridge, UK, 1991.

8. Cobb C W, Douglas P H. A theory of production[J]. The American economic review, 1928, 18(1): 139-165.

9. Diewert W E. Exact and superlative index numbers[J]. Journal of econometrics, 1976, 4(2): 115-145.

10. Diewert W E, Nakamura A O. Essays in index number theory. Volume I, Chapter 2 Contributions to Economic Analysis Series, No.217, North-Holland, Amsterdam, 1993.

11. Domar E D. On the measurement of technological change[J]. The Economic Journal, 1961, 71(284): 709-729.

12. Durand R. Canadian input–output-based multi-factor productivity accounts[J]. Economic Systems Research, 1996, 8(4): 367-390.

13. Esfahani M, Fernald J G, Hobijn B. World productivity: 1996-2014[J]. Federal Reserve Bank of San Francisco Working Paper Series 2020-17, March 2020.

14. Foster L, Haltiwanger J, Syverson C. Reallocation, firm turnover, and efficiency: Selection on productivity or profitability?[J]. American Economic Review, 2008, 98(1): 394-425.

15. Gollop F M. Accounting for intermediate input: The link between sectoral and aggregate measures of productivity growth. In The meaning and interpretation of productivity, Albert Rees and John Kendrick, eds., 318-33. Washington: National Academy of Sciences, 1979.

16. Griliches Z. Specification bias in estimates of production functions. Journal of Farm Economics, Vol. XXXIX, February 1957, pp. 8-20.

17. Griliches Z, Regev H. Firm productivity in Israeli industry 1979-1988[J]. Journal of Econometrics, 1995, 65:175-203.

18. Grover A G, Maloney W F. Proximity without productivity: Agglomeration effects with plant-level output and price data. Policy Research Working Paper Series 9977, The World Bank, 2022.

19. Gu W, Yan B. Productivity growth and international competitiveness[J]. Review of Income and Wealth, 2017, 63: S113-S133.

20. Gullickson W, Harper M J. Possible measurement bias in aggregate productivity growth[J]. Monthly Labor Review, 1999, 122 (February): 47-67.

21. Haltiwanger J. Firm dynamics and productivity: TFPQ, TFPR, and demand-side factors[J]. Economía, 2016, 17(1): 3-26.

22. Hulten C R. Growth accounting with intermediate inputs[J]. The Review of Economic Studies, 1978, 45(3): 511-518.

23. Hulten C R. Divisia index. In: Palgrave Macmillan (eds) The New Palgrave Dictionary of Economics. Palgrave Macmillan, London, 1987.

24. Hsieh C T, Klenow P J. Misallocation and manufacturing TFP in China and India[J]. The Quarterly Journal of Economics, 2009, 124(4): 1403-1448.

25. Jorgenson D W. Capital theory and investment behaviour[J]. American Economic Review, 1963, 53(2): 247–59.

26. Jorgenson, Dale W. Productivity, Volume 1: Postwar US economic growth. MIT Press Books, The MIT Press, edition 1, volume 1, number 0262100495, December, 1995a.

27. Jorgenson, Dale W. Productivity, Volume 2: International comparisons of economic growth," MIT Press Books, The MIT Press, edition 1, volume 2, December, 1995b.

28. Jorgenson, Dale W. Energy prices and productivity growth. Scandinavian Journal of Economics, 1981, 83(2): 165-179.

29. Jorgenson, Dale W. The role of energy in productivity growth. American Economic Review, 1984, 74(2): 26-30.

30. Jorgenson D W, Gollop F, Fraumeni B. Productivity and US economic growth, Cambridge, Mass: Harvard University Press, 1987.

31. Jorgenson D W, Griliches Z. The explanation of productivity change[J]. The Review of Economic Studies, 1967, 34(3): 249-283.

32. Kendrick J W. Productivity trends in the United States. National Bureau of Economic Research. Princeton: Princeton University Press, 1961.

33. Kendrick J W. Postwar productivity trends in the United States: 1948–1969. New York: National Bureau of Economic Research, 1973.

34. Keynes J M. The General Theory of Employment, Interest and Money. London: Macmillan, 1936.

35. Leontief W. Structural change. In Studies in the structure of the American economy, ed. Wassily Leontief. New York: Oxford University Press, 1953.

36. Melvin J R. Intermediate goods and technical change. Economica, 1969, 36:401-407.

37. Nishimizu M. Total factor productivity analysis: A disaggregated study of the postwar Japanese economy with explicit consideration of intermediate inputs, and comparison with the United States. Unpublished PhD dissertation, Johns Hopkins University, 1974.

38. Oliner S D, Sichel D E, Stiroh K J. Explaining a productive decade[J]. Brookings Papers on Economic Activity, 2007, 2007(1): 81-137.

39. Olley S, Pakes A. The dynamics of productivity in the telecommunications equipment industry[J]. Econometrica, 1996, 64(6):1263-1297.

40. OECD. The future of productivity. Organisation for Economic Cooperation and Development (OECD) Publishing, Paris, 2015.

41. OECD. Measuring productivity - OECD manual: Measurement of aggregate and industry-level productivity growth. Organisation for Economic Cooperation and Development (OECD) Publishing, Paris, 2001.

42. Ren R, Sun L L. Total factor productivity growth in China industries: 1981–2000. presented at the 5th International Input–Output Conference, Beijing, China, June 27–July 1, 2005.

43. Rymes T K. On the concepts of capital and technical change, Cambridge University Press, Cambridge, UK, 1971.

44. Rymes T K. The measurement of capital and total factor productivity in the context of the Cambridge theory of capital. Review of Income and Wealth, 18, 79–108, 1972.

45. Saia A, Andrews D, Albrizio S. Public Policy and Spillovers from the Global Productivity Frontier: Industry Level Evidence[J]. Economics Department Working Paper No. 1238, OECD, 2015.

46. Schreyer P. OECD productivity manual: A guide to the measurement of industry-level and aggregate productivity growth[M]. Organisation for Economic Co-operation and Development (OECD), 2001.

47. Solow R M. Technical change and the aggregate production function[J]. The review of Economics and Statistics, 1957: 312-320.

48. Statistics Canada. Aggregate productivity measures 1994, Catalogue No.15-204E, Statistics Canada, Ottawa, 1994.

49. Syverson C. Market structure and productivity: A concrete example[J]. Journal of political Economy, 2004, 112(6): 1181-1222.

50. Timmer M P, Erumban A, Los B, et al. New measures of European competitiveness: a global value chain perspective[J]. World Input-Output Database, Working Paper, 2012, 9: 2012a.

51. Timmer M, Ye X. Productivity and substitution patterns in global value chains[M]//The Oxford Handbook of Productivity Analysis. Oxford University Press, 2018: 699-724.

52. Timmer M P, Ye X. Accounting for growth and productivity in global value chains[M]. In B. Fraumeni (Ed.), Measuring Economic Growth and Productivity: Foundations, KLEMS Production Models, and Extensions. Academic Press, 2020: 413-426.

53. Tinbergen J. Zur Theorie def Langfristigen Wirtschaftsentwicklung. WeltWirtschaftliches Archive, 1942, 55(1):511-549. Aslo, “On the theory of trend movements”, in: Selected Papers of Jan Tinbergen, North-Holland, 1959.

54. Triplett J E. Economic statistics, the new economy, and the productivity slowdown[J]. Business Economics, 1999, 34: 13-17.

55. Van Ark B, O'Mahoney M, Timmer M P. The productivity gap between Europe and the United States: trends and causes[J]. Journal of Economic Perspectives, 2008, 22(1): 25-44.

56. Watanabe T. A note on measuring sector input productivity. Review of Income and Wealth, 17 (Dec. 1971), 335-340.

57. Wolff E N. Productivity measurement within an input-output framework[J]. Regional Science and Urban Economics, 1994, 24(1): 75-92.

58. Williamson, O. E. (1979). Transaction-cost economics: the governance of contractual relations. The journal of Law and Economics, 22(2), 233-261.

59. Wu H X. Losing Steam?—An industry origin analysis of China's productivity slowdown[M]//Measuring Economic Growth and Productivity. Academic Press, 2020: 137-167.

Appendix I. TFP measures

I.1 Snake v.s. Spider

The difference between sectoral TFP and GVC TFP could be seen in Figure I-1 and Table I-1. Figure I-1 depicts a simplified GVC, where y refers to the final product, and x_{ij} (l_{ij}, k_{ij}) refers to the output (e.g., labor or capital services) of sector i resulting from the final demand of sector j. The GVC TFP starts from the final product of Sector 1[1], which gives rise to changes in outputs and inputs in various sectors via their input–output relations. However, sectoral TFP is concerned only with the inputs and outputs of the sector itself.

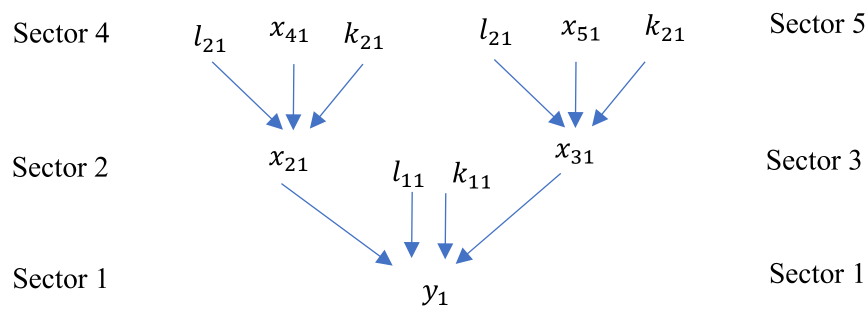


Figure I-1: Simplified GVC

Table I-1 presents a simplified input–output table. Although the calculation of sectoral TFP requires only information from one column (dotted-line box), GVC TFP requires information from the entire matrix (area in blue).

Table I-1: Simplified input-output table

Output / Input		Intermediate use				Final use	Gross output
	Sector	1	2	⋯	n	Y	X
Intermediate input	1	X_{11}	X_{12}	⋯	X_{1n}	Y_1	X_1
	2	X_{21}	X_{22}	⋯	X_{2n}	Y_2	X_2
	⋮	⋮	⋮		⋮	⋮	⋮
	n	X_{n1}	X_{n2}	⋯	X_{nn}	Y_n	X_n
Primary input	L	L_1	L_2	⋯	L_n		
	K	K_1	K_2	⋯	K_n		
Value Added	V	V_1	V_2	⋯	V_n		
Gross Input	X	X_1	X_2	⋯	X_n		

I.2 Sectoral TFP

(1) Output-based sectoral TFP

Assume the output production function of sector i to be Cobb-Douglas:

[1] This means that if a country only produces intermediate goods, there will be no GVC TFP for the country.

$$x_i = F_i(A_i; l_i; k_i; m_i) = A_i {l_i}^{\alpha_i} {k_i}^{\beta_i} {m_i}^{\gamma_i} \tag{I-1}$$

The sectoral TFP growth based on gross output production function could be given as:

$$\dot{A_i} = \dot{x_i} - \frac{P_i^M m_i}{P_i x_i}\dot{m_i} - \frac{W_i l_i}{P_i x_i}\dot{l_i} - \frac{R_i k_i}{P_i x_i}\dot{k_i} \tag{I-2}$$

If the (direct) sectoral origins of the intermediates are considered, the growth rate of sectoral TFP would be given as:

$$\dot{A_i} = \dot{x_i} - \sum_{j=1}^{n} \frac{P_j x_{ij}}{P_i x_i}\dot{x_{ij}} - \frac{W_i l_i}{P_i x_i}\dot{l_i} - \frac{R_i k_i}{P_i x_i}\dot{k_i} \tag{I-3}$$

where x_{ij} denotes the output of sector j used as intermediate by sector i.

(2) Value added-based sectoral TFP

Assume the value added production function of sector i to be Cobb-Douglas:

$$v_i = F_i(A_i; l_i, k_i) = A_i {l_i}^{\alpha_i} {k_i}^{\beta_i} \tag{I-4}$$

The sectoral TFP growth based on value added production function could be given as:

$$\dot{A_{vi}} = \dot{v_i} - \frac{W_i l_i}{P_i^V v_i}\dot{l_i} - \frac{R_i k_i}{P_i^V v_i}\dot{k_i} \tag{I-5}$$

It is worth noting that the value added production function itself is problematic: it is not in line with the producer's behavior and thus a lack of microeconomic foundations; it fails to include the contribution of intermediate inputs and thus overestimates TFP (Griliches, 1957; Domar, 1961); it relies on the strict assumption on the separability between intermediates and primary inputs (Gollop, 1979).

(3) Sectoral TFP: output v.s. value added

$$P_i \Delta x_i = P_i^V \Delta v_i + \sum_{j=1}^{n} P_j \Delta x_{ij} \tag{I-6}$$

$$\Rightarrow \frac{P_i^V v_i}{P_i x_i}\dot{v}_i = \dot{x}_i - \sum_{j=1}^{n} \frac{P_j x_{ij}}{P_i x_i}\dot{x}_{ij} \tag{I-7}$$

Substituting (I-7) into (I-3) delivers:

$$\dot{A_i} = \frac{P_i^V v_i}{P_i x_i}\dot{v}_i - \frac{W_i l_i}{P_i x_i}\dot{l_i} - \frac{R_i k_i}{P_i x_i}\dot{k_i} = \frac{P_i^V v_i}{P_i x_i}(\dot{v}_i - \frac{W_i l_i}{P_i^V v_i}\dot{l_i} - \frac{R_i k_i}{P_i^V v_i}\dot{k_i}) = \frac{P_i^V v_i}{P_i x_i}\dot{A_{vi}} \tag{I-8}$$

Therefore, the ratio of sectoral TFP based on output production function to that based on value added production function equals the ratio of value added to output (Bruno, 1978; Gollop, 1979; OECD, 2001).

I.3 Aggregate TFP

(1) Aggregate sectoral TFP: output v.s. value added

Domar weight is proposed by Domar (1961), and then developed by Hulten (1978). Hulten (1978) claims that Domar weight captures both the contribution of TFP to final demand and that to intermediate inputs, which further deliver to the sectors using these intermediate inputs. It has been widely used to measure aggregate TFP in the literature (Jorgenson et al., 1987; Gullickson and Harper 1999; Triplett and Bosworth, 2004).

Based on Domar (1961) and Hulten (1978), we could express Domar weight as follows

$$D_i = \frac{P_i x_i}{\sum_{=1}^{n} P_i^V v_i} \tag{I-9}$$

With some transformations, we have

$$\sum_{i=1}^{n} \frac{P_i^V v_i}{\sum_{=1}^{n} P_i^V v_i} \dot{v}_i = \sum_{i=1}^{n} \frac{P_i x_i}{\sum_{=1}^{n} P_i^V v_i} \frac{P_i^V v_i}{P_i x_i} \dot{v}_i = \sum_{i=1}^{n} D_i \frac{P_i^V v_i}{P_i x_i} \dot{v}_i \tag{I-10}$$

$$\sum_{i=1}^{n} \frac{R_i k_i}{\sum_{i=1}^{n} P_i^V v_i} \dot{k}_i = \sum_{i=1}^{n} \frac{P_i x_i}{\sum_{i=1}^{n} P_i^V v_i} \frac{R_i k_i}{P_i x_i} \dot{k}_i = \sum_{i=1}^{n} D_i \frac{R_i k_i}{P_i x_i} \dot{k}_i \tag{I-11}$$

$$\sum_{i=1}^{n} \frac{W_i l_i}{\sum_{i=1}^{n} P_i^V v_i} \dot{l}_i = \sum_{i=1}^{n} \frac{P_i x_i}{\sum_{i=1}^{n} P_i^V v_i} \frac{W_i l_i}{P_i x_i} \dot{l}_i = \sum_{i=1}^{n} D_i \frac{W_i l_i}{P_i x_i} \dot{l}_i \tag{I-12}$$

Substitute (5-12)~ (5-14) into Equation (5-10), we could rewrite aggregate sectoral TFP growth as:

$$\dot{A}^{PPF} = \sum_{i=1}^{n} D_i \frac{P_i^V v_i}{P_i x_i} \dot{v}_i - \sum_{i=1}^{n} D_i \frac{R_i k_i}{P_i x_i} \dot{k}_i - \sum_{i=1}^{n} D_i \frac{W_i l_i}{P_i x_i} \dot{l}_i = \sum_{i=1}^{n} D_i \dot{A}_\iota = \sum_{i=1}^{n} \frac{P_i^V v_i}{\sum_{i=1}^{n} P_i^V v_i} \dot{A_{v\iota}} \tag{I-13}$$

In addition, since the output share is smaller Domar share, the APG weighted by output share (Watanabe, 1971) is smaller than that weighted by Domar share.

$$\sum_{i=1}^{n} \frac{P_i x_i}{\sum_{i=1}^{n} P_i x_i} \frac{\dot{A}_\iota}{A_i} < \frac{\dot{A}^D}{A^D} \tag{I-14}$$

(2) Aggregate Production Possibility Frontier

Considering the stringent assumptions by the APF approach, Jorgenson *et al*. (1987) propose an aggregate production possibility frontier (APPF) approach incorporating Domar weight. The approach uses output-based sectoral TFP, and thus does not require the existence of value added function; it aggregates the sectoral TFP with a Domar weight, and thus does not require the production function to be identical; capitals (or labors) are aggregated using Törnqvist index, and thus the heterogeneity of primary factors are considered.

$$APG_{PPFSector} = \sum_{j=1}^{n} \frac{V_j}{\sum_{j=1}^{n} V_j} \dot{v}_j - \frac{L}{V} \dot{l} - \frac{L}{V} \dot{k} \tag{I-15}$$

$$\dot{x}_j = s_j^M \dot{m}_j + s_j^V \dot{v}_j = s_j^M \dot{m}_j + s_j^L \dot{l}_j + s_j^K \dot{k}_j + v_j^T \Rightarrow \dot{v}_j = \frac{s_j^L}{s_j^V} \dot{l}_j + \frac{s_j^K}{s_j^V} \dot{k}_j + \frac{1}{s_j^V} v_j^T$$

$$\sum_{j=1}^{n} \omega_j^V \dot{v}_j = \sum_{j=1}^{n} \frac{\omega_j^V}{s_j^V} v_j^T + \sum_{j=1}^{n} \omega_j^V \frac{s_j^L}{s_j^V} \dot{l}_j + \sum_{j=1}^{n} \omega_j^V \frac{s_j^K}{s_j^V} \dot{k}_j$$

$$\Rightarrow APG_{PPFSector} = \sum_{j=1}^{n} \frac{\omega_j^V}{s_j^V} v_j^T + \left(\sum_{j=1}^{n} \omega_j^V \frac{s_j^L}{s_j^V} \dot{l}_j - \frac{wl}{p^V v} \dot{l}\right) + \left(\sum_{j=1}^{n} \omega_j^V \frac{s_j^K}{s_j^V} \dot{k}_j - \frac{rk}{p^V v} \dot{k}\right) \quad \text{(I-16)}$$

Where,

$\frac{\omega_j^V}{s_j^V} = \left. \frac{P_j^V v_j}{\sum_{j=1}^{n} P_j^V v_j} \middle/ \frac{p_j^V v_j}{p_j x_j} \right. = \frac{P_j x_j}{\sum_{j=1}^{n} P_j^V v_j}$ is the Domar weight.

s_j^V is the nominal share of value-added in gross output of a sector. $s_j^V = \frac{P_j^V v_j}{P_j x_j}$. Similarly, $s_j^M = \frac{P_j^m m_j}{P_j x_j}$, $s_j^L = \frac{W_j l_j}{P_j x_j}$, $s_j^K = \frac{R_j k_j}{P_j x_j}$

ω_j^V is the nominal share of value-added of a sector in total value-added of the economy.

$$\omega_j^V = \frac{P_j^V v_j}{\sum_{j=1}^{n} P_j^V v_j}$$

$$\dot{l}_j = \sum_h \frac{W_h l_{hj}}{\sum_h W_h l_{hj}} \dot{l}_{ij},\ \dot{k}_j = \sum_g \frac{R_g k_{gj}}{\sum_g R_g k_{gj}} \dot{k}_{gj},\ \dot{m}_j = \sum_i \frac{P_i^M m_{ij}}{\sum_{i=1}^{n} P_i^M m_{ij}} \dot{m}_{ij}$$

All the weights or shares are two-period averages, which has been omitted the for notational convenience[1].

Appendix II. Leontief-Based GVC TFP: Bottom-Up Derivations

I.1 Basic Settings

Following the canonical work of Solow (1957), we estimate TFP based on the Cobb–Douglas production function.

$$x = F(A, l, k, m) = AF(\, l, k, m) = A l^{\alpha} k^{\beta} m^{\gamma} \quad \text{(II-1)}$$

For simplicity, our analysis is based on the following assumptions:

(1) Hicks-neutral technology progress: $F(A, l, k, m) = AF(\, l, k, m)$;

(2) Perfect competition, hence factor elasticity, equals factor share: $\alpha = \frac{wl}{px}, \beta = \frac{rk}{px}, \gamma = \frac{p_m m}{px}$;

(3) Constant returns to scale, hence output elasticities, sums to a unit: $\alpha + \beta + \gamma = 1$;

(4) Input prices used by all downstream industries are identical;

(5) Products for intermediate and final uses are separable, and their prices are the same.

[1] Törnqvist Index is a "superlative" index to approximate the Divisia index in empirical analysis. See Diewert & Nakamura (1993) for detailed introduction of the history of index numbers.

Assumptions (1) and (3) indicate that the production function is homothetically separable, and Assumptions (1) and (2) are the necessary conditions for producer equilibrium. Hence, we require two steps to calculate the aggregate GVC TFP. The first integrates different production stages into a complete value chain, and the second aggregates different value chains into a whole economy.

I.2 Integration: from a production stage to a whole value chain

Scenario 1: one stage

$$y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1}$$

Then logarithm of GVC TFP could be expressed as:

$$lnA_1^{GVC} = lnA_1 \tag{II-2}$$

$$\text{or, } lnA_1^{GVC} = lny_1 - \alpha_1 lnl_1 - \beta_1 lnk_1 \tag{II-3}$$

Scenario 2: two stages, one final sector + one intermediate sector (with input-output relation)

$$y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{21}^{\gamma_1}$$

$$x_{21} = A_2 l_2^{\alpha_2} k_2^{\beta_2}$$

$$\Rightarrow y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1}\left[A_2 l_2^{\alpha_2} k_2^{\beta_2}\right]^{\gamma_1} = A_1 A_2^{\gamma_1}\left[l_1^{\alpha_1} l_2^{\alpha_2\gamma_1}\right]\left[k_1^{\beta_1} k_2^{\beta_2\gamma_1}\right]$$

Then logarithm of GVC TFP could be expressed as:

$$lnA_1^{GVC} = lnA_1 + \gamma_1 lnA_2 \tag{II-4}$$

$$\text{or, } lnA_1^{GVC} = lny_1 - (\alpha_1 lnl_1 + \gamma_1\alpha_2 lnl_2\,) - (\beta_1 lnk_1 + \gamma_1\beta_2 lnk_2) \tag{II-5}$$

where $\gamma_1 = \frac{P_2 x_{21}}{P_1 y_1}$

Scenario 3: three stages:

$$y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{21}^{\gamma_1}$$

$$x_{21} = A_2 (l_{21})^{\alpha_2} (k_{21})^{\beta_2} (x_{31})^{\gamma_2}$$

$$\Rightarrow y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} \cdot \left[A_2 (l_{21})^{\alpha_2} (k_{21})^{\beta_2} (x_{31})^{\gamma_2}\right]^{\gamma_1}$$
$$= \left(A_1 A_2^{\gamma_1}\right) \cdot \left[l_1^{\alpha_1} (l_{21})^{\alpha_2\gamma_1}\right]\left[k_1^{\beta_1} (k_{21})^{\beta_2\gamma_1}\right](x_{31})^{\gamma_1\gamma_2}$$

$$x_{31} = A_3 (l_{31})^{\alpha_3} (k_{31})^{\beta_3}$$

$$\Rightarrow y_1 = \left(A_1 A_2^{\gamma_1}\right) \cdot \left[k_1^{\alpha_1} (l_{21})^{\alpha_2\gamma_1}\right]\left[k_1^{\beta_1} (k_{21})^{\beta_2\gamma_1}\right] \cdot \left[A_3 (l_{31})^{\alpha_3} (k_{31})^{\beta_3}\right]^{\gamma_1\gamma_2}$$

$$= \left(A_1 A_2^{\gamma_1} A_3^{\gamma_1\gamma_2}\right) \cdot \left[l_1^{\alpha_1}(l_{21})^{\alpha_2\gamma_1}(l_{31})^{\alpha_3\gamma_1\gamma_2}\right] \cdot \left[k_1^{\beta_1}(k_{21})^{\beta_2\gamma_1}(k_{31})^{\beta_3\gamma_1\gamma_2}\right] \quad \text{(II-6)}$$

Then logarithm of GVC TFP could be expressed as:

$$lnA_1^{GVC} = lnA_1 + \gamma_1 lnA_2 + \gamma_1\gamma_2 lnA_3 \quad \text{(II-7)}$$

or, $lnA_1^{GVC} = lny_1 - (\alpha_1 lnl_1 + \gamma_1\alpha_2 lnl_{21} + \gamma_1\gamma_2\alpha_3 lnl_{31}) - (\beta_1 lnk_1 + \gamma_1\beta_2 lnk_{21} + \gamma_1\gamma_2\beta_3 lnk_{31})$ (II-8)

where $\gamma_1 = \frac{P_2 x_{21}}{P_1 y_1}$, $\gamma_1\gamma_2 = \frac{P_2 y_{21}}{P_1 y_1} \cdot \frac{P_3 x_{31}}{P_2 y_{21}} = \frac{P_3 x_{31}}{P_1 y_1}$

Scenario 4: n stages

Then, we could obtain the equation in n stages.

$$y_1 = \prod_{i=1}^{n} A_i^{\prod_{j=1}^{i}\gamma_{j-1}} \cdot \prod_{i=1}^{n}(l_{i1})^{\alpha_i \prod_{j=1}^{i}\gamma_{j-1}} \cdot \prod_{i=1}^{n}(k_{i1})^{\beta_i \prod_{j=1}^{i}\gamma_{j-1}} \quad \text{(II-9)}$$

Taking logarithm on both sides delivers:

$$lny_1 = \sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} \cdot lnA_i + \sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} \cdot \alpha_i lnl_{i1} + \sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} \cdot \beta_i lnk_{i1} \quad \text{(II-10)}$$

where $\gamma_0 = 1$; $\prod_{j=1}^{i}\gamma_{j-1} = \gamma_1\gamma_2 \ldots \gamma_{i-1} = \frac{p_i x_{i1}}{p_1 y_1}$; $\prod_{j=1}^{i}\gamma_{j-1} \cdot \alpha_i = \frac{p_i x_{i1}}{p_1 y_1} \cdot \frac{w_i l_{i1}}{p_i x_{i1}} = \frac{w_i l_{i1}}{p_1 y_1}$

Therefore, we yield the equation of logarithm of GVC TFP with sector 1 as the final good:

$$lnA_1^{GVC} = \sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} \cdot lnA_i = \sum_{i=1}^{n}\frac{p_i x_{i1}}{p_1 y_1} \cdot lnA_i$$

$$= lnA_1 + \gamma_1 lnA_2 + \gamma_1\gamma_2 lnA_3 + \gamma_1\gamma_2 \ldots \gamma_{n-1} lnA_n > lnA_1 \quad \text{(II-11)}$$

To note that $\sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} = 1 + \gamma_1 + \gamma_1\gamma_2 + \cdots + \prod_{j=1}^{n-1}\gamma_j > 1$, which is weighted sum, as mentioned in Domar (1961), rather than weighted mean, where the weights sum to unity.

$$\text{or, } lnA_1^{GVC} = lny_1 - \sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} \cdot \alpha_i lnl_{i1} - \sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1} \cdot \beta_i lnk_{i1}$$

$$= lny_1 - \sum_{i=1}^{n}\frac{w_i l_{i1}}{p_1 y_1} lnl_{i1} - \sum_{i=1}^{n}\frac{r_i k_{i1}}{p_1 y_1} lnk_{i1} \quad \text{(II-12)}$$

Jorgenson et al. (1987) take the ratio of weighted sum to simple addition of factors as the quality of factors. If the quality of factors is included in technology change, then we could obtain a measurement of GVC TFP based on **simple addition**.

$$lnA_1^{GVC} = lny_1 - \bar{\alpha} \cdot ln\sum_{i=1}^{n} l_{i1} - \bar{\beta} \cdot ln\sum_{i=1}^{n} k_{i1}$$

$$= lny_1 - \frac{\sum_{i=1}^{n} w_i l_{i1}}{p_1 y_1} \cdot ln\sum_{i=1}^{n} l_{i1} - \frac{\sum_{i=1}^{n} r_i k_{i1}}{p_1 y_1} \cdot ln\sum_{i=1}^{n} k_{i1} \quad \text{(II-13)}$$

where $\bar{\alpha} = \frac{\sum_{i=1}^{n} w_i l_{i1}}{p_1 y_1} = \frac{w_1 l_1 + w_2 l_{21} + \cdots + w_n l_{n1}}{p_1 y_1}$; $\bar{\beta} = \frac{\sum_{i=1}^{n} r_i k_{i1}}{p_1 y_1} = \frac{r_1 k_1 + r_2 k_{21} + \cdots + r_n k_{n1}}{p_1 y_1}$

$ln \sum_{i=1}^{n} l_i (ln \sum_{i=1}^{n} k_i$) is the logarithm of total labor (capital) input along the global value chain resulting from the final output of sector j. $\bar{\alpha}$ ($\bar{\beta}$) refers to the output elasticity of labor (capital) of the global value chain, which equals the capital's (labor's) output share according to the perfect competition assumption. Simple addition, without using any weight, of the primary inputs along the global value chain makes sense because they contribute to an integrated process.

I.3 Aggregation: from value chain to whole economy

Scenario 1: one stage, with two final goods (no input-output relation)

$$y_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1}; y_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} \quad \text{(II-14)}$$

$$\Rightarrow lnA^{LeontiefGVC} = \frac{P_1 y_1}{P_1 y_1 + P_2 y_2} lnA_1^{LeontiefGVC} + \frac{P_2 y_2}{P_1 y_1 + P_2 y_2} lnA_2^{LeontiefGVC} \quad \text{(II-15)}$$

Scenario 2: n stages, with n sectors (complex input-output relations)

Then, we could obtain aggregate GVC TFP:

$$lnA^{LeontiefGVC} = \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} lnA_j^{LeontiefGVC}$$

$$= \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \cdot lny_j - \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \cdot \sum_{i=1}^{n} \frac{W_i l_{ij}}{P_j y_j} lnl_{ij} - \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \cdot \sum_{i=1}^{n} \frac{R_i k_{ij}}{P_j y_j} lnk_{ij}$$

$$= \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \cdot lny_j - \sum_{j=1}^{n} \frac{\sum_{i=1}^{n} W_i l_{ij} \cdot lnl_{ij}}{\sum_{j=1}^{n} P_j y_j} - \sum_{j=1}^{n} \frac{\sum_{i=1}^{n} R_i k_{ij} \cdot lnk_{ij}}{\sum_{j=1}^{n} P_j y_j}$$

$$= \sum_{j=1}^{n} \frac{P_j y_j}{\sum_{j=1}^{n} P_j y_j} \cdot lny_j - \frac{\sum_{j=1}^{n} \sum_{i=1}^{n} W_i l_{ij} \cdot lnl_{ij}}{\sum_{j=1}^{n} P_j y_j} - \frac{\sum_{j=1}^{n} \sum_{i=1}^{n} R_i k_{ij} \cdot lnk_{ij}}{\sum_{j=1}^{n} P_j y_j} \quad \text{(II-16)}$$

where l_{ij} refers to the embodied sector i's labor services for producing final goods j.

Then we consider an **open economy**, where $lnA_{js}^{LeontiefGVC}$ is the GVC TFP of sector j in country s. Then the world TFP would be

$$lnA^{LeontiefGVC} = \sum_{j=1}^{n} \sum_{s=1}^{m} \frac{P_{js} y_{js}}{\sum_{j=1}^{n} \sum_{s=1}^{m} P_{js} y_{js}} lnA_{js}^{LeontiefGVC} \quad \text{(II-17)}$$

Where $\sum_{j=1}^{n} \sum_{s=1}^{m} P_{js} y_{js}$ is the world GDP (value of final output). $P_{js} y_{js}$ refers to the value of final output of sector j in country s. Though all sectors are considered to have only one type of direct intermediate input in the above models, the situation involving various types of intermediate inputs could also be extended in similar ways.

I.4 Weights for integration and aggregation

The weight for integration is $\prod_{j=1}^{i}\gamma_{j-1}=\frac{P_{i-1}^{M}M_{i-1}}{P_1Y_1}=\frac{P_iy_{i1}}{P_1y_1}$, and $\sum_{i=1}^{n}\prod_{j=1}^{i}\gamma_{j-1}>1$, while the weight for aggregation is $\frac{P_jy_j}{\sum_{j=1}^{n}P_jy_j}$, and $\sum_{j=1}^{n}\frac{P_jy_j}{\sum_{j=1}^{n}P_jy_j}=1$. At first sight, the two weights are different. The rationality behind, however, are the same. For simplicity, we assume there are only two sectors:

Suppose that the output of sector 2 is used as the only intermediate inputs by sector 1.

$$y_1=A_1l_1^{\alpha_1}k_1^{\beta_1}x_{21}^{\gamma_1}\,;\ x_{21}=A_2l_2^{\alpha_2}k_2^{\beta_2} \tag{II-18}$$

Then $lnA_1^{LeontiefGVC}=\frac{P_1y_1}{P_1y_1}lnA_1+\frac{P_2x_{21}}{P_1y_1}lnA_2$. This is a kind of integration because both sectors are only part of the production process.

Further, we suppose that both sectors are final sectors, without input-output relation with each other.

$$y_1=A_1l_1^{\alpha_1}k_1^{\beta_1};\ y_2=A_2l_2^{\alpha_2}k_2^{\beta_2} \tag{II-19}$$

Then $lnA^{LeontiefGVC}=\frac{P_1y_1}{P_1y_1+P_2y_2}lnA_1^{LeontiefGVC}+\frac{P_2y_2}{P_1y_1+P_2y_2}lnA_2^{LeontiefGVC}$, this is a kind of aggregation because both sectors involve integrated production process.

The two weights are different. Whereas the sum of integration weights is greater than 1 ($\frac{P_1y_1}{P_1y_1}+\frac{P_2y_{21}}{P_1y_1}$>1), that of aggregation weights equals 1 ($\frac{P_1y_1}{P_1y_1+P_2y_2}+\frac{P_2y_2}{P_1y_1+P_2y_2}=1$).

However, the two weights share a similar logic. While integration weight could be explained as the ratio between output value final to the production stage and output value final to the whole production process (one value chain), aggregation weight could be explained as the ratio between output value final to the whole production process and output value final to the whole economy (including all value chains).

Appendix III. GVC TFP: Domar *v.s.* Leontief

III.1 Two sectors

$$x_1 = F_1(A_1, l_1, k_1, x_{11}, x_{21})\ ;\ x_2 = F_2(A_2, l_2, k_2, x_{12}) \tag{III-1}$$

1. $\boldsymbol{AP_{LeontiefGVC}}$

$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\boldsymbol{z} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$, where $\boldsymbol{x}$ refers to the vector of output, $\boldsymbol{y}$ is the vector of final demand, $\boldsymbol{z}$ is the matrix of intermediate input, $\sum_j x_{ij} + y_i = x_i$

$y_1 = x_1 - x_{11} - x_{12} = x_1 - a_{11}x_1 - a_{12}x_2;$

$y_2 = x_2 - x_{21} - x_{22} = x_2 - a_{21}x_1 - a_{22}x_2$

We assume that the prices of output and intermediate input are equal to one. Then we have

$\Rightarrow \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where $a_{ij} = \frac{x_{ij}}{x_j}$

$\Rightarrow \boldsymbol{I} - \boldsymbol{A} = \begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix}$

Then the Leontief inverse matrix can be expressed as

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = (\boldsymbol{I} - \boldsymbol{A})^{-\mathbf{1}} = \frac{(\boldsymbol{I}-\boldsymbol{A})^*}{|\boldsymbol{I}-\boldsymbol{A}|} = \frac{1}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}} \begin{bmatrix} 1 - a_{22} & a_{12} \\ a_{21} & 1 - a_{11} \end{bmatrix} \tag{III-2}$$

$$\boldsymbol{C'} = \frac{1}{(1-a_{11})(1-a_{22}) - a_{12}a_{21}} \begin{bmatrix} 1 - a_{22} & a_{21} \\ a_{12} & 1 - a_{11} \end{bmatrix} \tag{III-3}$$

$\Rightarrow \boldsymbol{C\widehat{Y}} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_1 & \\ & y_2 \end{bmatrix} = \begin{bmatrix} c_{11}y_1 & c_{12}y_2 \\ c_{21}y_1 & c_{22}y_2 \end{bmatrix}$

$$\boldsymbol{C_l\widehat{Y}} = \begin{bmatrix} \frac{l_1}{x_1}c_{11} & \frac{l_1}{x_1}c_{12} \\ \frac{l_2}{x_2}c_{21} & \frac{l_2}{x_2}c_{22} \end{bmatrix} \begin{bmatrix} y_1 & \\ & y_2 \end{bmatrix} = \begin{bmatrix} \frac{l_1}{x_1}c_{11}y_1 & \frac{l_1}{x_1}c_{12}y_2 \\ \frac{l_2}{x_2}c_{21}y_1 & \frac{l_2}{x_2}c_{22}y_2 \end{bmatrix} \Rightarrow \boldsymbol{C_L\widehat{Y}} = \begin{bmatrix} \frac{w_1l_1}{x_1}c_{11}y_1 & \frac{w_1l_1}{x_1}c_{12}y_2 \\ \frac{w_2l_2}{x_2}c_{21}y_1 & \frac{w_2l_2}{x_2}c_{22}y_2 \end{bmatrix} \tag{III-4}$$

$$\boldsymbol{C_k\widehat{Y}} = \begin{bmatrix} \frac{k_1}{x_1}c_{11} & \frac{k_1}{x_1}c_{12} \\ \frac{k_2}{x_2}c_{21} & \frac{k_2}{x_2}c_{22} \end{bmatrix} \begin{bmatrix} y_1 & \\ & y_2 \end{bmatrix} = \begin{bmatrix} \frac{k_1}{x_1}c_{11}y_1 & \frac{k_1}{x_1}c_{12}y_2 \\ \frac{k_2}{x_2}c_{21}y_1 & \frac{k_2}{x_2}c_{22}y_2 \end{bmatrix} \Rightarrow \boldsymbol{C_K\widehat{Y}} = \begin{bmatrix} \frac{r_1k_1}{x_1}c_{11}y_1 & \frac{r_1k_1}{x_1}c_{12}y_2 \\ \frac{r_2k_2}{x_2}c_{21}y_1 & \frac{r_2k_2}{x_2}c_{22}y_2 \end{bmatrix} \tag{III-5}$$

$$lnA_j^{LeontiefGVC} = lny_j - \sum_{i=1}^{2} \frac{w_il_i}{x_i} c_{ij}y_j * lnl_{ij} - \sum_{i=1}^{2} \frac{r_ik_1}{x_i} c_{ij}y_j * lnk_{ij} \ (j=1,2) \tag{III-6}$$

$$\Rightarrow AP_{LeontiefGVC} = \sum_{j=1}^{2} \frac{y_j}{\sum_{j=1}^{2} y_j} lnA_j^{LeontiefGVC} \tag{III-7}$$

2. $\boldsymbol{AP_{Domar}}$

(1) $\boldsymbol{AP_{DomarSector}}$

$$lnA_1 = lnx_1 - \frac{W_1 l_1}{x_1} * lnl_1 - \frac{R_1 k_1}{x_1} * lnk_1 - \frac{x_{11}}{x_1} * lnx_{11} - \frac{x_{21}}{x_1} * lnx_{21} \quad \text{(III-8)}$$

$$lnA_2 = lnx_2 - \frac{W_2 l_2}{x_2} * lnl_2 - \frac{R_2 k_2}{x_2} * lnk_2 - \frac{x_{12}}{x_2} * lnx_{12} - \frac{x_{22}}{x_2} * lnx_{22} \quad \text{(III-9)}$$

Therefore, we have the Domar aggregation based on sectoral TFP:

$$\boldsymbol{AP_{DomarSector}} = \frac{x_1}{y_1+y_2} lnA_1 + \frac{x_2}{y_1+y_2} lnA_2 = \frac{1}{y_1+y_2}(x_1, x_2)\begin{bmatrix} lnA_1 \\ lnA_2 \end{bmatrix} \quad \text{(III-10)}$$

Where

$$\begin{bmatrix} lnA_1 \\ lnA_2 \end{bmatrix} = \begin{bmatrix} lnx_1 \\ lnx_2 \end{bmatrix} - \begin{bmatrix} \frac{W_1 l_1}{x_1} & \\ & \frac{W_2 l_2}{x_2} \end{bmatrix}\begin{bmatrix} lnl_1 \\ lnl_2 \end{bmatrix} - \begin{bmatrix} \frac{R_1 k_1}{x_1} & \\ & \frac{R_2 k_2}{x_2} \end{bmatrix}\begin{bmatrix} lnk_1 \\ lnk_2 \end{bmatrix} - \begin{bmatrix} \frac{x_{11}}{x_1} & \\ & \frac{x_{21}}{x_1} \end{bmatrix}\begin{bmatrix} lnx_{11} \\ lnx_{21} \end{bmatrix} - \begin{bmatrix} \frac{x_{12}}{x_2} & \\ & \frac{x_{22}}{x_2} \end{bmatrix}\begin{bmatrix} lnx_{12} \\ lnx_{22} \end{bmatrix}$$

(2) $\boldsymbol{AP_{DomarGVC}}$

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}}\ ;\ x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} x_{12}^{\gamma_{12}} \quad \text{(III-11)}$$

$\delta_{ij} = \frac{x_{ij}}{x_i}, \gamma_{ij} = a_{ij} = \frac{x_{ij}}{x_j}$

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}} = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} (\delta_{21} A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} x_{12}^{\gamma_{12}})^{\gamma_{21}} \quad \text{(III-12)}$$

$$\Rightarrow \boldsymbol{lnx_1} = \boldsymbol{lnA_1} + \alpha_1 lnl_1 + \beta_1 lnk_1 + \gamma_{11} ln\delta_{11} + \boldsymbol{\gamma_{11} lnx_1} + \gamma_{21} ln\delta_{21} + \boldsymbol{\gamma_{21} lnA_2} + \gamma_{21}\alpha_2 lnl_2 + \gamma_{21}\beta_2 lnk_2 + \gamma_{21}\gamma_{12} ln\delta_{12} + \boldsymbol{\gamma_{21}\gamma_{12} lnx_1} + \gamma_{21}\gamma_{22} ln\delta_{22} + \boldsymbol{\gamma_{21}\gamma_{22} lnx_2} \quad \text{(III-13)}$$

$$\Rightarrow lnA_1^{DomarGVC} = lnA_1 + \gamma_{11} lnA_1^{DomarGVC} + \gamma_{21} lnA_2 + \gamma_{21}\gamma_{22} lnA_2^{DomarGVC} + \gamma_{21}\gamma_{12} lnA_1^{DomarGVC} \quad \text{(III-14)}$$

$$\Rightarrow (1 - \gamma_{11} - \gamma_{21}\gamma_{12}) lnA_1^{DomarGVC} - \gamma_{21}\gamma_{22} lnA_2^{DomarGVC} = lnA_1 + \gamma_{21} lnA_2 \quad \text{(III-15)}$$

$$x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} x_{12}^{\gamma_{12}} = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{22}^{\gamma_{22}} (\delta_{12} A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}})^{\gamma_{12}} \quad \text{(III-16)}$$

$$\Rightarrow \boldsymbol{lnx_2} = \boldsymbol{lnA_2} + \alpha_2 lnl_2 + \beta_2 lnk_2 + \gamma_{22} ln\delta_{22} + \boldsymbol{\gamma_{22} lnx_2} + \gamma_{12} ln\delta_{21} + \boldsymbol{\gamma_{12} lnA_1} + \gamma_{12}\alpha_1 lnl_1 + \gamma_{12}\beta_1 lnk_1 + \gamma_{11}\gamma_{12} ln\delta_{11} + \gamma_{11}\gamma_{12}\boldsymbol{lnx_1} + \gamma_{12}\gamma_{21} ln\delta_{21} + \boldsymbol{\gamma_{12}\gamma_{21} lnx_2} \quad \text{(III-17)}$$

$$\Rightarrow lnA_2^{DomarGVC} = lnA_2 + \gamma_{22} lnA_2^{DomarGVC} + \gamma_{12} lnA_1 + \gamma_{11}\gamma_{12} lnA_1^{DomarGVC} + \gamma_{12}\gamma_{21} lnA_2^{DomarGVC} \tag{III-18}$$

$$\Rightarrow -\gamma_{12}\gamma_{11} lnA_1^{DomarGVC} + (1-\gamma_{22}-\gamma_{12}\gamma_{21})\, lnA_2^{DomarGVC} = \gamma_{12} lnA_1 + lnA_2 \tag{III-19}$$

$$\begin{bmatrix} (1-\gamma_{11}-\gamma_{21}\gamma_{12}) & -\gamma_{21}\gamma_{22} \\ -\gamma_{12}\gamma_{11} & (1-\gamma_{22}-\gamma_{12}\gamma_{21}) \end{bmatrix} \begin{bmatrix} lnA_1^{DomarGVC} \\ lnA_2^{DomarGVC} \end{bmatrix} = \begin{bmatrix} 1 & \gamma_{21} \\ \gamma_{12} & 1 \end{bmatrix} \begin{bmatrix} lnA_1 \\ lnA_2 \end{bmatrix} \tag{III-20}$$

$$\boldsymbol{ED} = \boldsymbol{TS} \tag{III-21}$$

$$\begin{aligned} |\boldsymbol{E}| &= (1-\gamma_{11}-\gamma_{21}\gamma_{12})(1-\gamma_{22}-\gamma_{12}\gamma_{21}) - \gamma_{21}\gamma_{22}\gamma_{12}\gamma_{11} \\ &= (1-\gamma_{12}\gamma_{21})[(1-\gamma_{11})(1-\gamma_{22}) - \gamma_{12}\gamma_{21}] \end{aligned}$$

$$\boldsymbol{E^{-1}} = \frac{\boldsymbol{E}^*}{|\boldsymbol{E}|} = \frac{1}{|\boldsymbol{E}|} \begin{bmatrix} 1-\gamma_{22}-\gamma_{12}\gamma_{21} & \gamma_{21}\gamma_{22} \\ \gamma_{12}\gamma_{11} & 1-\gamma_{11}-\gamma_{21}\gamma_{12} \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{E^{-1}T} &= \frac{1}{|\boldsymbol{E}|} \begin{bmatrix} 1-\gamma_{22}-\gamma_{12}\gamma_{21} & \gamma_{21}\gamma_{22} \\ \gamma_{12}\gamma_{11} & 1-\gamma_{11}-\gamma_{21}\gamma_{12} \end{bmatrix} \begin{bmatrix} 1 & \gamma_{21} \\ \gamma_{12} & 1 \end{bmatrix} = \\ &\frac{1}{|\boldsymbol{E}|} \begin{bmatrix} 1-\gamma_{22}-\gamma_{12}\gamma_{21}+\gamma_{12}\gamma_{21}\gamma_{22} & \gamma_{21}-\gamma_{21}\gamma_{12}\gamma_{21} \\ \gamma_{12}-\gamma_{21}\gamma_{12}\gamma_{12} & \gamma_{21}\gamma_{12}\gamma_{11}+1-\gamma_{11}-\gamma_{21}\gamma_{12} \end{bmatrix} = \\ &\frac{1}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \begin{bmatrix} 1-a_{22} & a_{21} \\ a_{12} & 1-a_{11} \end{bmatrix} = \boldsymbol{C'} \end{aligned} \tag{III-22}$$

$$\Rightarrow \boldsymbol{D} = \boldsymbol{C'S}$$

$$lnA_1^{DomarGVC} = \frac{(1-a_{22})lnA_1 + a_{21}lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \tag{III-23}$$

$$lnA_2^{DomarGVC} = \frac{a_{12}lnA_1 + (1-a_{11})lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \tag{III-24}$$

$$\Rightarrow AP_{DomarGVC} = \frac{y_1}{y_1+y_2} lnA_1^{DomarGVC} + \frac{y_2}{y_1+y_2}\, lnA_2^{DomarGVC} = \frac{1}{y_1+y_2}(y_1, y_2) \begin{bmatrix} lnA_1^{DomarGVC} \\ lnA_2^{DomarGVC} \end{bmatrix} \tag{III-25}$$

$$\mathbf{CY} = \mathbf{X} \Rightarrow \boldsymbol{C} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow (y_1, y_2)\boldsymbol{C'} = (x_1, x_2) \tag{III-26}$$

$$(y_1, y_2) \frac{(1-a_{12}a_{21})}{[(1-a_{11})(1-a_{22})-a_{12}a_{21}](1-a_{12}a_{21})} \begin{bmatrix} (1-a_{22}) & a_{21} \\ a_{12} & (1-a_{11}) \end{bmatrix} \begin{bmatrix} lnA_1 \\ lnA_2 \end{bmatrix} = (x_1, x_2) \begin{bmatrix} lnA_1 \\ lnA_2 \end{bmatrix} \tag{III-27}$$

$$\Rightarrow lnA_1^{DomarGVC} = \frac{(1-a_{22})lnA_1 + a_{21}lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \tag{III-28}$$

$$lnA_2^{DomarGVC} = \frac{a_{12}lnA_1+(1-a_{11})lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \qquad \text{(III-29)}$$

where,

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = (\boldsymbol{I}-\boldsymbol{A})^{-\mathbf{1}} = \frac{(\boldsymbol{I}-\boldsymbol{A})^*}{|\boldsymbol{I}-\boldsymbol{A}|} = \frac{1}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \begin{bmatrix} 1-a_{22} & a_{12} \\ a_{21} & 1-a_{11} \end{bmatrix}$$

Proof 1: $\boldsymbol{AP_{DomarSector} = AP_{DomarGVC}}$

$$\boldsymbol{AP_{DomarGVC}} = \frac{y_1}{y_1+y_2} lnA_1^{DomarGVC} + \frac{y_2}{y_1+y_2} lnA_2^{DomarGVC}$$

$$= \frac{x_1-a_{12}x_2-a_{11}x_1}{y_1+y_2} \frac{(1-a_{22})lnA_1+a_{21}lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}}$$

$$+ \frac{x_2-a_{21}x_1-a_{22}x_2}{y_1+y_2} \frac{a_{12}lnA_1+(1-a_{11})lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}}$$

$$= \frac{(x_1-a_{12}x_2-a_{11}x_1)(1-a_{22})+(x_2-a_{21}x_1-a_{22}x_2)a_{12}}{(y_1+y_2)[(1-a_{11})(1-a_{22})-a_{12}a_{21}]} lnA_1$$

$$+ \frac{(x_1-a_{12}x_2-a_{11}x_1)a_{21}+(x_2-a_{21}x_1-a_{22}x_2)(1-a_{11})}{(y_1+y_2)[(1-a_{11})(1-a_{22})-a_{12}a_{21}]} lnA_2$$

$$= \frac{x_1[(1-a_{11})(1-a_{22})-a_{12}a_{21}]}{(y_1+y_2)[(1-a_{11})(1-a_{22})-a_{12}a_{21}]} lnA_1$$

$$+ \frac{x_2[(1-a_{11})(1-a_{22})-a_{12}a_{21}]}{(y_1+y_2)[(1-a_{11})(1-a_{22})-a_{12}a_{21}]} lnA_2 = \frac{x_1}{y_1+y_2} lnA_1 + \frac{x_2}{y_1+y_2} lnA_2$$

$$\boldsymbol{= AP_{DomarSector}}$$

Proof 2 : $\boldsymbol{AP_{DomarGVC} > AP\_LeontiefGVC}$

$$lnA_1^{LeontiefGVC} = lny_1 - [c_{11} * (\frac{W_1l_1}{x_1} lnl_{11} + \frac{R_1k_1}{x_1} lnk_{11}) + c_{21}(\frac{W_2l_2}{x_2} lnl_{21} + \frac{R_2k_2}{x_2} * lnk_{21})]$$

$$= lny_1 - \frac{(1-a_{22})\left(\frac{W_1l_1}{x_1}*lnl_{11}+\frac{R_1k_1}{x_1}*lnk_{11}\right)+a_{21}\left(\frac{W_2l_2}{x_2}*lnl_{21}+\frac{R_2k_2}{x_2}*lnk_{21}\right)}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} = lny_1 -$$

$$\frac{(1-a_{22})(lnx_1-a_{11}lnx_{11}-a_{21}lnx_{21}-lnA_1)+a_{21}(lnx_2-a_{12}lnx_{12}-a_{22}lnx_{22}-lnA_2)}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} = lny_1 -$$

$$\frac{(1-a_{22})(lnx_1-a_{11}lnx_{11}-a_{21}lnx_{21})+a_{21}(lnx_2-a_{12}lnx_{12}-a_{22}lnx_{22})}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} + \frac{(1-a_{22})lnA_1+a_{21}lnA_2}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} \qquad \text{(III-30)}$$

$$lnA_1^{DomarGVC} - lnA_1^{LeontiefGVC} = \frac{(1-a_{22})(lnx_1-a_{11}lnx_{11}-a_{21}lnx_{21})+a_{21}(lnx_2-a_{12}lnx_{12}-a_{22}lnx_{22})}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} - lny_1 =$$

$$\frac{(1-a_{22})(lnx_1-a_{11}lnx_{11})-a_{12}a_{21}lnx_{12}+a_{21}[lnx_2-a_{22}lnx_{22}-(1-a_{22})lnx_{21}]}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} - ln(x_1-x_{11}-x_{12}) =$$

$$\frac{-a_{11}(1-a_{22})ln\delta_{11}-a_{12}a_{21}ln\delta_{12}+a_{21}[1-a_{22}ln\delta_{22}-(1-a_{22})ln\delta_{21}]}{(1-a_{11})(1-a_{22})-a_{12}a_{21}} - ln(1-\delta_{11}-\delta_{12}) > 0 \qquad \text{(III-31)}$$

$$\Rightarrow lnA_1^{DomarGVC} > lnA_1^{LeontiefGVC}$$

$$\Rightarrow \boldsymbol{AP_{DomarGVC} > AP_{Leontief}}$$

III.2 Three sectors

$$x_1 = F_1(A_1, l_1, k_1, x_{11}, x_{21}, x_{31}); x_2 = F_2\big(A_2, l_2, k_2, x_{12}, x_{22,}x_{32}\big); x_3 = F_3\big(A_3, l_3, k_3, x_{13}, x_{23,}x_{33}\big) \quad \text{(III-32)}$$

For convenience's sake, we use a Cobb-Douglas function.

$$x_1 = A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}} x_{31}^{\gamma_{31}}; x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{12}^{\gamma_{12}} x_{22}^{\gamma_{22}} x_{32}^{\gamma_{32}}; x_3 = A_3 l_3^{\alpha_3} k_3^{\beta_3} x_{13}^{\gamma_{13}} x_{23}^{\gamma_{23}} x_{33}^{\gamma_{33}} \quad \text{(III-33)}$$

1. $\boldsymbol{AP_{Leontief}}$

$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, $\boldsymbol{z} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$, where $\boldsymbol{x}$ refers to the vector of output, $\boldsymbol{y}$ is the vector of final demand, $\boldsymbol{z}$ is the matrix of intermediate input, $\sum_j x_{ij} + y_i = x_i$

$$y_1 = x_1 - x_{11} - x_{12} - x_{13} = x_1 - a_{11}x_1 - a_{12}x_2 - a_{13}x_3,$$

$$y_2 = x_2 - x_{21} - x_{22} - x_{23} = x_2 - a_{21}x_1 - a_{22}x_2 - a_{23}x_3$$

$$y_3 = x_3 - x_{31} - x_{32} - x_{33} = x_3 - a_{31}x_1 - a_{32}x_2 - a_{33}x_3$$

We assume that the prices of output and intermediate input are equal to one. Then we have

$\Rightarrow \boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, where $a_{ij} = \frac{x_{ij}}{x_j}$

$$\Rightarrow \boldsymbol{I} - \boldsymbol{A} = \begin{bmatrix} 1 - a_{11} & -a_{12} & -a_{13} \\ -a_{21} & 1 - a_{22} & -a_{23} \\ -a_{31} & -a_{32} & 1 - a_{33} \end{bmatrix}$$

The Leontief inverse matrix can be expressed as

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \frac{(\boldsymbol{I}-\boldsymbol{A})^*}{|\boldsymbol{I}-\boldsymbol{A}|} =$$

$$\frac{1}{|\boldsymbol{I}-\boldsymbol{A}|} \begin{bmatrix} (1-a_{22})(1-a_{33}) - a_{23}a_{32} & a_{12}(1-a_{33}) + a_{13}a_{32} & a_{12}a_{23} + a_{13}(1-a_{22}) \\ a_{21}(1-a_{33}) + a_{23}a_{31} & (1-a_{11})(1-a_{33}) - a_{13}a_{31} & a_{23}(1-a_{11}) + a_{13}a_{21} \\ a_{21}a_{32} + (1-a_{22})a_{31} & a_{32}(1-a_{11}) + a_{12}a_{31} & (1-a_{11})(1-a_{22}) - a_{12}a_{21} \end{bmatrix}$$

(III-34)

$$\Rightarrow \boldsymbol{C\hat{Y}} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} y_1 & & \\ & y_2 & \\ & & y_3 \end{bmatrix} = \begin{bmatrix} c_{11}y_1 & c_{12}y_2 & c_{13}y_3 \\ c_{21}y_1 & c_{22}y_2 & c_{23}y_3 \\ c_{31}y_1 & c_{32}y_2 & c_{33}y_3 \end{bmatrix}$$

$$\boldsymbol{C_L\hat{Y}} = \begin{bmatrix} \frac{W_1l_1}{x_1}c_{11}y_1 & \frac{W_1l_1}{x_1}c_{12}y_2 & \frac{W_1l_1}{x_1}c_{13}y_3 \\ \frac{W_2l_2}{x_2}c_{21}y_1 & \frac{W_2l_2}{x_2}c_{22}y_2 & \frac{W_2l_2}{x_2}c_{23}y_3 \\ \frac{W_3l_3}{x_3}c_{31}y_1 & \frac{W_3l_3}{x_3}c_{32}y_2 & \frac{W_3l_3}{x_3}c_{33}y_3 \end{bmatrix}, \boldsymbol{C_K\hat{Y}} = \begin{bmatrix} \frac{R_1k_1}{x_1}c_{11}y_1 & \frac{R_1k_1}{x_1}c_{12}y_2 & \frac{R_1k_1}{x_1}c_{13}y_3 \\ \frac{R_2k_2}{x_2}c_{21}y_1 & \frac{R_2k_2}{x_2}c_{22}y_2 & \frac{R_2k_2}{x_2}c_{23}y_3 \\ \frac{R_3k_3}{x_3}c_{31}y_1 & \frac{R_3k_3}{x_3}c_{32}y_2 & \frac{R_3k_3}{x_3}c_{33}y_3 \end{bmatrix} \quad \text{(III-35)}$$

$$lnA_j^{LeontiefGVC} = lny_j - \sum_{i=1}^{3}\frac{W_il_i}{x_i}c_{ij}y_j * lnl_i - \sum_{i=1}^{3}\frac{R_ik_1}{x_i}c_{ij}y_j * lnk_i \ (j=1,2,3) \quad \text{(III-36)}$$

$$\Rightarrow AP_{LeontiefGVC} = \sum_{j=1}^{3}\frac{y_j}{\sum_{i=1}^{3}y_j}lnA_j^{LeontiefGVC} \quad \text{(III-37)}$$

2. $\boldsymbol{AP_{Domar}}$

(1) $\boldsymbol{AP_{DomarSector}}$

$$lnA_1 = lnx_1 - \frac{W_1l_1}{x_1} * lnl_1 - \frac{R_1k_1}{x_1} * lnk_1 - \sum_{i=1}^{3}\frac{x_{i1}}{x_1} * lnx_{i1} \quad \text{(III-38)}$$

$$lnA_2 = lnx_2 - \frac{W_2l_2}{x_2} * lnl_2 - \frac{R_2k_2}{x_2} * lnk_2 - \sum_{i=1}^{3}\frac{x_{i2}}{x_2} * lnx_{i2} \quad \text{(III-39)}$$

$$lnA_3 = lnx_3 - \frac{W_3l_3}{x_3} * lnl_3 - \frac{R_3k_3}{x_3} * lnk_3 - \sum_{i=1}^{3}\frac{x_{i3}}{x_3} * lnx_{i3} \quad \text{(III-40)}$$

$$\Rightarrow \boldsymbol{AP_{DomarSector}} = \sum_{i=1}^{3}\frac{x_i}{\sum_{i=1}^{3}y_i}lnA_i$$

(2) $\boldsymbol{AP_{DomarGVC}}$

$$x_1 = A_1l_1^{\alpha_1}k_1^{\beta_1}x_{11}^{\gamma_{11}}x_{21}^{\gamma_{21}}x_{31}^{\gamma_{31}};\ x_2 = A_2l_2^{\alpha_2}k_2^{\beta_2}x_{12}^{\gamma_{12}}x_{22}^{\gamma_{22}}x_{32}^{\gamma_{32}};\ x_3 = A_3l_3^{\alpha_3}k_3^{\beta_3}x_{13}^{\gamma_{13}}x_{23}^{\gamma_{23}}x_{33}^{\gamma_{33}} \quad \text{(III-41)}$$

$$\delta_{ij} = \frac{x_{ij}}{x_i}, \gamma_{ij} = a_{ij} = \frac{x_{ij}}{x_j}$$

$$x_1 = A_1l_1^{\alpha_1}k_1^{\beta_1}x_{11}^{\gamma_{11}}x_{21}^{\gamma_{21}}x_{31}^{\gamma_{31}} =$$
$$A_1l_1^{\alpha_1}k_1^{\beta_1}(\delta_{11}x_1)^{\gamma_{11}}(\delta_{21}A_2l_2^{\alpha_2}k_2^{\beta_2}x_{12}^{\gamma_{12}}x_{22}^{\gamma_{22}}x_{32}^{\gamma_{32}})^{\gamma_{21}}(\delta_{31}A_3l_3^{\alpha_3}k_3^{\beta_3}x_{13}^{\gamma_{13}}x_{23}^{\gamma_{23}}x_{33}^{\gamma_{33}})^{\gamma_{31}} \quad \text{(III-42)}$$

$$\Rightarrow \boldsymbol{lnx_1 = lnA_1} + \alpha_1lnl_1 + \beta_1lnk_1 + \gamma_{11}ln\delta_{11} + \boldsymbol{\gamma_{11}lnx_1}$$

$$+\gamma_{21}ln\delta_{21} + \boldsymbol{\gamma_{21}lnA_2} + \gamma_{21}\alpha_2lnl_2 + \gamma_{21}\beta_2lnk_2 + \gamma_{21}\gamma_{12}ln\delta_{12} + \boldsymbol{\gamma_{21}\gamma_{12}lnx_1} + \gamma_{21}\gamma_{22}ln\delta_{22} + \boldsymbol{\gamma_{21}\gamma_{22}lnx_2} + \gamma_{21}\gamma_{32}ln\delta_{32} + \boldsymbol{\gamma_{21}\gamma_{32}lnx_3}$$

$+\gamma_{31}ln\delta_{31} + \boldsymbol{\gamma_{31}lnA_3} + \gamma_{31}\alpha_3 lnl_3 + \gamma_{31}\beta_3 lnk_3 + \gamma_{31}\gamma_{13}ln\delta_{13} + \boldsymbol{\gamma_{31}\gamma_{13}lnx_1} +$
$\gamma_{31}\gamma_{23}ln\delta_{23} + \boldsymbol{\gamma_{31}\gamma_{23}lnx_2} + \gamma_{31}\gamma_{33}ln\delta_{33} + \boldsymbol{\gamma_{31}\gamma_{33}lnx_3}$ (III-43)

$\Rightarrow lnA_1^{DomarGVC} = lnA_1 + \gamma_{11}lnA_1^{DomarGVC} + \gamma_{21}lnA_2 + \gamma_{21}\gamma_{12}lnA_1^{DomarGVC} +$
$\gamma_{21}\gamma_{22}lnA_2^{DomarGVC} + \gamma_{21}\gamma_{32}lnA_3^{DomarGVC} + \gamma_{31}lnA_3 + \gamma_{31}\gamma_{13}lnA_1^{DomarGVC} +$
$\gamma_{31}\gamma_{23}lnA_2^{DomarGVC} + \gamma_{31}\gamma_{33}lnA_3^{DomarGVC}$ (III-44)

$\Rightarrow (1-\gamma_{11}-\gamma_{21}\gamma_{12}-\gamma_{31}\gamma_{13})lnA_1^{DomarGVC} - (\gamma_{21}\gamma_{22}+\gamma_{31}\gamma_{23})lnA_2^{DomarGVC} -$
$(\gamma_{21}\gamma_{32}+\gamma_{31}\gamma_{33})lnA_3^{DomarGVC} = lnA_1 + \gamma_{21}lnA_2 + \gamma_{31}lnA_3$ (III-45)

$x_2 = A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{12}^{\gamma_{12}} x_{22}^{\gamma_{22}} x_{32}^{\gamma_{32}} =$
$A_2 l_2^{\alpha_2} k_2^{\beta_2} (\delta_{12} A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}} x_{31}^{\gamma_{31}})^{\gamma_{12}} (\delta_{22} x_{22})^{\gamma_{22}} (\delta_{32} A_3 l_3^{\alpha_3} k_3^{\beta_3} x_{13}^{\gamma_{13}} x_{23}^{\gamma_{23}} x_{33}^{\gamma_{33}})^{\gamma_{32}}$ (III-44)

$\Rightarrow \boldsymbol{lnx_2} = \boldsymbol{lnA_2} + \alpha_2 lnl_2 + \beta_2 lnk_2 + \gamma_{22}ln\delta_{22} + \boldsymbol{\gamma_{22}lnx_2}$

$+\gamma_{12}ln\delta_{12} + \boldsymbol{\gamma_{12}lnA_1} + \gamma_{12}\alpha_1 lnl_1 + \gamma_{12}\beta_1 lnk_1 + \gamma_{12}\gamma_{11}ln\delta_{11} + \gamma_{12}\gamma_{11}\boldsymbol{lnx_1} + \gamma_{12}\gamma_{21}ln\delta_{21}$
$+ \gamma_{12}\gamma_{21}\boldsymbol{lnx_2} + \gamma_{12}\gamma_{31}ln\delta_{31} + \gamma_{12}\boldsymbol{\gamma_{31}lnx_3}$

$+\gamma_{32}ln\delta_{32} + \gamma_{32}\boldsymbol{lnA_3} + \gamma_{32}\alpha_3 lnl_3 + \gamma_{32}\beta_3 lnk_3 + \gamma_{32}\gamma_{13}ln\delta_{13} + \gamma_{32}\boldsymbol{\gamma_{13}lnx_1} +$
$\gamma_{32}\gamma_{23}ln\delta_{23} + \gamma_{32}\boldsymbol{\gamma_{23}lnx_2} + \gamma_{32}\gamma_{33}ln\delta_{33} + \gamma_{32}\boldsymbol{\gamma_{33}lnx_3}$ (III-46)

$\Rightarrow lnA_2^{DomarGVC} = lnA_2 + \gamma_{22}lnA_2^{DomarGVC} + \gamma_{12}lnA_1 + \gamma_{12}\gamma_{11}lnA_1^{DomarGVC} +$
$\gamma_{12}\gamma_{21}lnA_2^{DomarGVC} + \gamma_{12}\gamma_{31}lnA_3^{DomarGVC} + \gamma_{32}lnA_3 + \gamma_{32}\gamma_{13}lnA_1^{DomarGVC} +$
$\gamma_{32}\gamma_{23}lnA_2^{DomarGVC} + \gamma_{32}\gamma_{33}lnA_3^{DomarGVC}$ (III-47)

$\Rightarrow (1-\gamma_{22}-\gamma_{12}\gamma_{21}-\gamma_{32}\gamma_{23})lnA_2^{DomarGVC} - (\gamma_{12}\gamma_{11}+\gamma_{32}\gamma_{13})lnA_1^{DomarGVC} -$
$(\gamma_{12}\gamma_{31}+\gamma_{32}\gamma_{33})lnA_3^{DomarGVC} = lnA_2 + \gamma_{12}lnA_1 + \gamma_{32}lnA_3$ (III-48)

$x_3 = A_3 l_3^{\alpha_3} k_3^{\beta_3} x_{13}^{\gamma_{13}} x_{23}^{\gamma_{23}} x_{33}^{\gamma_{33}} =$
$A_3 l_3^{\alpha_3} k_3^{\beta_3} (\delta_{13} A_1 l_1^{\alpha_1} k_1^{\beta_1} x_{11}^{\gamma_{11}} x_{21}^{\gamma_{21}} x_{31}^{\gamma_{31}})^{\gamma_{13}} (\delta_{23} A_2 l_2^{\alpha_2} k_2^{\beta_2} x_{12}^{\gamma_{12}} x_{22}^{\gamma_{22}} x_{32}^{\gamma_{32}})^{\gamma_{23}} (\delta_{33} x_3)^{\gamma_{33}}$ (III-49)

$\Rightarrow \boldsymbol{lnx_3} = \boldsymbol{lnA_3} + \alpha_3 lnl_3 + \beta_3 lnk_3 + \gamma_{33}ln\delta_{33} + \boldsymbol{\gamma_{33}lnx_3}$

$+\gamma_{13}ln\delta_{13} + \boldsymbol{\gamma_{13}lnA_1} + \gamma_{13}\alpha_1 lnl_1 + \gamma_{13}\beta_1 lnk_1 + \gamma_{13}\gamma_{11}ln\delta_{11} + \boldsymbol{\gamma_{13}\gamma_{11}lnx_1} + \gamma_{13}\gamma_{21}ln\delta_{21}$
$+ \boldsymbol{\gamma_{13}\gamma_{21}lnx_2} + \gamma_{13}\gamma_{31}ln\delta_{31} + \boldsymbol{\gamma_{13}\gamma_{31}lnx_3}$

$+\gamma_{23}ln\delta_{23} + \boldsymbol{\gamma_{23}lnA_2} + \gamma_{23}\alpha_2 lnl_2 + \gamma_{23}\beta_2 lnk_2 + \gamma_{23}\gamma_{12}ln\delta_{12} + \boldsymbol{\gamma_{23}\gamma_{12}lnx_1} +$
$\gamma_{23}\gamma_{22}ln\delta_{22} + \boldsymbol{\gamma_{23}\gamma_{22}lnx_2} + \gamma_{23}\gamma_{32}ln\delta_{32} + \boldsymbol{\gamma_{23}\gamma_{32}lnx_3}$ (III-50)

$$\Rightarrow lnA_3^{DomarGVC} = lnA_3 + \gamma_{33} lnA_3^{DomarGVC} + \gamma_{13} lnA_1 + \gamma_{13}\gamma_{11} lnA_1^{DomarGVC} + \gamma_{13}\gamma_{21} lnA_2^{DomarGVC} + \gamma_{13}\gamma_{31} lnA_3^{DomarGVC} + \gamma_{23} lnA_2 + \gamma_{23}\gamma_{12} lnA_1^{DomarGVC} + \gamma_{23}\gamma_{22} lnA_2^{DomarGVC} + \gamma_{23}\gamma_{32} lnA_3^{DomarGVC} \quad \text{(III-51)}$$

$$\Rightarrow (1-\gamma_{33}-\gamma_{13}\gamma_{31}-\gamma_{23}\gamma_{32}) lnA_3^{DomarGVC} - (\gamma_{13}\gamma_{11}+\gamma_{23}\gamma_{12}) lnA_1^{DomarGVC} - (\gamma_{13}\gamma_{21}+\gamma_{23}\gamma_{22}) lnA_2^{DomarGVC} = lnA_3 + \gamma_{13} lnA_1 + \gamma_{23} lnA_2 \quad \text{(III-52)}$$

$$\begin{bmatrix} (1-\gamma_{11}-\gamma_{21}\gamma_{12}-\gamma_{31}\gamma_{13}) & -(\gamma_{21}\gamma_{22}+\gamma_{31}\gamma_{23}) & -(\gamma_{21}\gamma_{32}+\gamma_{31}\gamma_{33}) \\ -(\gamma_{12}\gamma_{11}+\gamma_{32}\gamma_{13}) & (1-\gamma_{22}-\gamma_{12}\gamma_{21}-\gamma_{32}\gamma_{23}) & -(\gamma_{12}\gamma_{31}+\gamma_{32}\gamma_{33}) \\ -(\gamma_{13}\gamma_{11}+\gamma_{23}\gamma_{12}) & -(\gamma_{13}\gamma_{21}+\gamma_{23}\gamma_{22}) & (1-\gamma_{33}-\gamma_{13}\gamma_{31}-\gamma_{23}\gamma_{32}) \end{bmatrix} \begin{bmatrix} lnA_1^{DomarGVC} \\ lnA_2^{DomarGVC} \\ lnA_3^{DomarGVC} \end{bmatrix}$$
$$= \begin{bmatrix} lnA_1 + \gamma_{21} lnA_2 + \gamma_{31} lnA_3 \\ \gamma_{12} lnA_1 + lnA_2 + \gamma_{32} lnA_3 \\ \gamma_{13} lnA_1 + \gamma_{23} lnA_2 + lnA_3 \end{bmatrix} = \begin{bmatrix} 1 & \gamma_{21} & \gamma_{31} \\ \gamma_{12} & 1 & \gamma_{32} \\ \gamma_{13} & \gamma_{23} & 1 \end{bmatrix} \begin{bmatrix} lnA_1 \\ lnA_2 \\ lnA_3 \end{bmatrix}$$

$$\boldsymbol{ED} = \boldsymbol{TS} \Rightarrow \boldsymbol{D} = \boldsymbol{E}^{-1}\boldsymbol{TS} = \boldsymbol{C}'\boldsymbol{S} \quad \text{(III-53)}$$

where $\boldsymbol{C}' =$

$$\frac{\mathbf{1}}{|\boldsymbol{I}-\boldsymbol{A}|}\begin{bmatrix} (1-a_{22})(1-a_{33})-a_{23}a_{32} & a_{21}(1-a_{33})+a_{23}a_{31} & a_{21}a_{32}+(1-a_{22})a_{31} \\ a_{12}(1-a_{33})+a_{13}a_{32} & (1-a_{11})(1-a_{33})-a_{13}a_{31} & a_{32}(1-a_{11})+a_{12}a_{31} \\ a_{12}a_{23}+a_{13}(1-a_{22}) & a_{23}(1-a_{11})+a_{13}a_{21} & (1-a_{11})(1-a_{22})-a_{12}a_{21} \end{bmatrix}$$

$$|\boldsymbol{I}-\boldsymbol{A}| = (1-a_{11})(1-a_{22})(1-a_{33}) - (1-a_{11})a_{23}a_{32} - (1-a_{22})a_{13}a_{31} - (1-a_{33})a_{12}a_{21} - a_{12}a_{23}a_{31} - a_{13}a_{21}a_{32}$$

The Domar-based GVC TFP could also be calculated with the help of Leontief inverse:

$$\mathbf{CY} = \mathbf{X} \Rightarrow \boldsymbol{C}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow (y_1, y_2, y_3)\boldsymbol{C}'\begin{bmatrix} lnA_1 \\ lnA_2 \\ lnA_3 \end{bmatrix} = (x_1, x_2, x_3)\begin{bmatrix} lnA_1 \\ lnA_2 \\ lnA_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} lnA_1^{DomarGVC} \\ lnA_2^{DomarGVC} \\ lnA_3^{DomarGVC} \end{bmatrix} = \boldsymbol{C}'\begin{bmatrix} lnA_1 \\ lnA_2 \\ lnA_3 \end{bmatrix} \quad \text{(III-54)}$$

$$\Rightarrow AP_{DomarGVC} = \sum_{i=1}^{3} \frac{y_i}{\sum_{i=1}^{3} y_i} lnA_i^{DomarGVC}$$

It is not difficult to prove that $AP_{DomarSector} = AP_{DomarGVC} > AP_{LeontiefGVC}$