Chapter 2 Equilibrium locations of upstream and downstream firms

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Chapter 2 Equilibrium Locations of Upstream and Downstream Firms

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Abstract

This paper explores the interaction between upstream firms and downstream firms in a two-region general equilibrium model. In many countries, lower tariff rates are set for intermediate manufactured goods and higher tariff rates are set for final manufactured goods. The derived results imply that such settings of tariff rates tend to preserve a symmetric spread of upstream and downstream firms, and continuing tariff reduction may cause core-periphery structures. In the case in which the circular causality between upstream and downstream firms is focused as agglomeration forces, the present model is fully solved. Thus, we find that (1) the present model displays, at most, three interior steady states, (2) when the asymmetric steady-states exist, they are unstable and (3) location displays hysteresis when the transport costs of intermediate manufactured goods are sufficiently high.

Keywords: Spatial economics, upstream and downstream firms

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1. Introduction

The difference in the tariff rates of products may show that each country intends to promote local production. Kumagai and Kuroiwa (2009) examined the average effective applied tariff rate in intra-regional trade in East Asia in 2006. The effective applied rate is the tariff rate which is actually applied to imported goods from East Asian countries, weighted by the import value from each country. In Vietnam, Lao PDR, Thailand and Brunei, the tariff rate on consumption goods is twice the tariff rate on parts and components. In Korea, China, Malaysia, Indonesia, Taiwan, the Philippines and Myanmar, the tariff rate on consumption goods is four times as high as the tariff rate on parts and components. This implies that most countries prefer to maintain higher tariff rates on consumption goods. Furthermore, these tendencies were clear in the automobile industry as well. To clarify the effects of such difference in tariff rates not only on local production but also on the spread of economic activity, a model is needed which includes the transport costs of intermediate goods and the transport costs of final consumption goods.

Only a small number of papers focus on the agglomeration and dispersion of upstream and downstream firms because of the complexity of introducing two types of firms with increasing returns to scale technologies. The pioneering study of Venables (1996) illustrated figuers to show the interaction of upstream and downstream firms in a two-region general equilibrium model. Using a two-region general equilibrium model with upstream and downstream firms and focusing on the agglomeration, Amiti (2005) showed the tension between factors in Heckscher-Ohlin model and new economic geography model. To avoid complexity, Krugman and Venables (1995) developed a two-region general equilibrium model with one mass of manufactured products used for intermediary and household consumption, examined the emergence of the core-periphery structure, and showed the convergence of real wages between countries with a decrease in the transport cost of the manufacturing sector. To develop a more analytical model, Ottaviano and Robert-Nicoud (2006) developed a fully solvable general equilibrium model with one mass of manufactured products. The advance made by Robert-Nicoud (2004) and Ottaviano and Robert-Nicoud (2006), respectively, was to show that the same key features are shared by the most prevalent New Economic Geography models, such as Krugman (1991) and Krugman and Venables (1995) and the models in which firms use unskilled workers as variable costs and the rest of the inputs as fixed costs. Although we assume that the intermediate goods are incurred as fixed costs of downstream firms, Ottaviano and Robert-Nicoud (2006) write that to use intermediate goods in fixed costs is ea (not so) special case.f To realize the same merits as Ottaviano and Robert-Nicoud (2006) in their analysis on the instability of two asymmetric steady-states, this chapter extends Ottaviano and Robert-Nicoud (2006) to introduce upstream firms and downstream firms in a model. Such an extension allows us to obtain solvable results even when upstream firms produce intermediary goods and downstream firms produce consumption goods for households in a two-region general equilibrium model.

The spatial configuration of economic activities is decided by the balance of the agglomeration force and dispersion force. The source of agglomeration force is forward and backward linkages, which bring the circular causality. In our model, there are two circular causalities: the interaction between upstream firms and downstream firms and the interaction between downstream firms and households. However, it is difficult to focus on two circular causalities at once; we set a fixed number of skilled workers and allow these workers to earn in the agricultural sector, although such case emerges when the same wage rates in the manufacturing sector and the agricultural sector. As a result, the mechanism of our model changes depending on the supply of skilled workers. When the supply of skilled workers is scarce, all skilled workers are in the manufacturing sector and the wage rates increase. In this extreme case, this scarcity works as a dispersion force. Only the circular causality between downstream firms and households causes industrial agglomeration. In contrast, when the supply of skilled workers is abundant, there are some skilled workers who are not working in the manufacturing sector. Thus, the regional income in a region does not change. As a result, only the circular causality between upstream firms and downstream firms works. In the followings, only the latter is fully solved.

The remaining portion of this chapter is organized as follows. Section 2 explains the model, which is solved in Section 3. Section 4 examines the Pareto dominant allocation. Section 5 presents the conclusion.

2. Assumptions

The economic space is composed of two regions, called north and south, which are symmetric in terms of tastes, technology and endowments. The economy has two sectors: agriculture (herein referred to as A) and manufacturing (herein referred to as M). The economy is endowed with two factors of production: entrepreneurs (H) and workers (L).

Workers are not inter-regionally mobile, and regions are endowed with equal supplies of the immobile factor, L, thus $L = L^* = L^w/2$. Some workers (L) are entrepreneurs (H), who can be regarded as human capital, and regions are also endowed with equal supplies of entrepreneurs, $\overline{H}$, thus $\overline{H} = \overline{H}^* = \overline{H}^w/2$. Entrepreneurs are free to choose to utilize their human capital or not.

Preferences are identical across all workers and are described by the upper tier consisting of a Cobb-Douglas 'nest' of consumption of the agricultural goods and a composite of all manufactured consumption goods,

$$U = C_\mu C_M^\mu C_A^{1-\mu},\ C_M \equiv \left(\int_{i=0}^{m+m^*} c_i^{1-1/\sigma_c} di \right)^{1/(1-1/\sigma_c)}\ ;\ 0 < \mu < 1 < \sigma_c \qquad (1)$$

where $C_\mu \equiv \mu^\mu (1-\mu)^{\mu-1}$ is constant, and C_M and C_A are, respectively, consumption of the CES composite of manufactured consumption goods varieties and consumption of A. Furthermore, m and m^* are the mass (number) of north and south varieties in manufactured consumption goods, μ is the expenditure share in industrial varieties, and σ_c is the constant elasticity of substitution between the varieties. The corresponding indirect utility functions for typical northern entrepreneurs and workers are ω_H and ω, where

$$\omega_H \equiv \frac{w_H}{P_c^\mu},\ \omega \equiv \frac{w}{P_c^\mu},\ P_c \equiv (\Delta_c)^{-1/(\sigma_c-1)}, \Delta_c = \int_{i=0}^{m^w} p_{ci}^{1-\sigma} di \qquad (2)$$

w_H is the northern wage for entrepreneurs and w_H is the northern wage for workers. Expressions for the corresponding southern values are isomorphic. Below, isomorphic expressions are omitted.

The agricultural good is homogeneous and produced using workers only. The agricultural sector uses a constant-returns technology under conditions of perfect competition. The specific cost function is w. Both regions produce an agricultural good. Trade in the homogeneous A good is costless.

The manufacturing firms are under the condition of monopolistic competition and use the increasing returns to scale technology. As for the technology of downstream firms, fixed and marginal input requirements are satisfied by different factors: the marginal cost is incurred in terms of labor only; the fixed cost is incurred in terms of the differentiated varieties of the manufactured intermediate goods. In symbols, the cost function of the typical northern downstream firm l is given by

$$C_c(c_l) = f_c P_{int} + a_c c_l w, P_{int} \equiv \Delta_{int}^{-1/(\sigma_{int}-1)}; \Delta_{int} \equiv \int_{k=0}^{n^w} p_{intk}^{1-\sigma_{int}} dk \tag{3}$$

where a_c is the marginal labor requirements in downstream firms, and P_{int} is the upstream firm's price index. Trade in industrial goods is subject to iceberg trade costs; a firm wishing to sell one unit of its good in the other region must ship $\tau_c \geq 1$ units since $\tau_c - 1$ units melt in transit.

As for the technology of upstream firms, the marginal cost is also incurred in terms of labor only, whereas the fixed cost is incurred in terms of one entrepreneur. The cost function of the typical northern upstream firm j is given by

$$C_{int}(q_j) = f_{int} w_H + w a_{int} q_j \tag{4}$$

where q_j is the demand for variety j on intermediate goods. Also, a_{int} is the marginal labor requirements in upstream firms. Trade in industrial goods is subject to iceberg trade costs; a firm wishing to sell one unit of its good in the other region must ship $\tau_{int} \geq 1$ units since $\tau_{int} - 1$ units melt in transit.

Because entrepreneurs and workers are free to choose between the agricultural and manufacturing sectors, the wage rates must be the same in the two sectors within each region, provided that the two sectors exist and entrepreneurs work in the two sectors in the given region. Entrepreneurs' decisions to work as a worker or an entrepreneur are based on the nominal wage difference. The wage gap between them is expressed as $\ln(w_H/w)$. Long-run equilibrium wages for entrepreneursf capital satisfy

$$\begin{aligned} &\ln(w_H/w) = 0, \quad n \in (0, \bar{H}/f_{int}), \\ &\ln(w_H/w) > 0, \quad n = \bar{H}/f_{int}, \\ &\ln(w_H/w) < 0, \quad n = 0. \end{aligned} \tag{5}$$

The flow of upstream firms is expressed as follows:

$$\dot{n} = n\left(\bar{H}/f_{int} - n\right) \ln\left(\frac{w_H}{w}\right), \quad \dot{n^*} = n^*\left(\bar{H}^*/f_{int} - n^*\right) \ln\left(\frac{w_H^*}{w^*}\right). \tag{6}$$

Downstream firms decide to enter when current profits are positive and exit when they are negative. The flow of downstream firms is regulated as follows:

$$\dot{m} = m \ln\left(\frac{\pi_c}{P_{int} f_c}\right), \quad \dot{m}^* = m^* \ln\left(\frac{\pi_c^*}{P_{int}^* f_c}\right) \tag{7}$$

where π_c and π_c^* are operating profits and $P_{int} f_c$ and $P_{int}^* f_c$ are fixed costs.

3. Instantaneous Equilibrium

The equilibrium taking as a given the spread of manufacturing firms is analyzed.

The agricultural good is costlessly traded between regions and is chosen as the numéraire, and thus the price of the agricultural good is one. The equalization of the price of the agricultural good causes the wage for workers to also be one in both regions.

From utility maximization, northern consumption of a variety j is $c_j = p_{cj}^{-\sigma_c}(\mu E_c/\Delta_c n^w)$, where $E_c = w_H H + w(L - H)$ is northern expenditure for manufactured consumption goods. From monopolistic competition, mill prices for all upstream firms are as follow:

$$p_c = \frac{a_c}{1 - (1/\sigma_c)}, \; p_c^* = \tau_c \frac{a_c}{1 - (1/\sigma_c)} \tag{8}$$

where p_c and p_c^* are the consumer prices for a typical north-made variety in the northern market and the southern market, respectively. Likewise, northern demand of a variety l of manufactured intermediate goods is $q_l = p_{intl}(E_{int}/\Delta_{int} n^w)$, where $E_{int} = m P_{int} f_c$ is northern expenditure for manufactured intermediate goods. Mill prices for all downstream firms are as follow:

$$p_{int} = \frac{a_{int}}{1 - (1/\sigma_{int})}, \; p_{int}^* = \tau_{int} \frac{a_{int}}{1 - (1/\sigma_{int})} \tag{9}$$

where p_{int} and p_{int}^* are the intermediate manufactured goods prices for a typical north-made variety in the northern market and the southern market, respectively.

Using the demand functions and mill prices, the profit function for downstream firms (Π_c and Π_c^*) are as follow:

$$\Pi_c = \frac{\mu}{\sigma_c}\left(\frac{E_c}{\Delta_c} + \phi_c \frac{E_c^*}{\Delta_c^*}\right) - (\Delta_{int})^{-\frac{1}{\sigma_{int}-1}} f_c, \tag{10}$$

$$\Pi_c^* = \frac{\mu}{\sigma_c}\left(\phi_c\frac{E_c}{\Delta_c} + \frac{E_c^*}{\Delta_c^*}\right) - (\Delta_{int}^*)^{-\frac{1}{\sigma_{int}-1}} f_c; \tag{11}$$

$$\phi_c \equiv \tau_c^{1-\sigma_c},\ \phi_{int} \equiv \tau_{int}^{1-\sigma_{int}} \tag{12}$$

$$E_c \equiv w(L-H) + w_H H,\ E_c^* \equiv w^*(L^*-H^*) + w_H^* H^*, \tag{13}$$

$$\Delta_c \equiv m\left(\frac{a_c}{1-(1/\sigma_c)}\right)^{1-\sigma_c} + \phi_c m^* \left(\frac{a_c}{1-(1/\sigma_c)}\right)^{1-\sigma_c}, \tag{14}$$

$$\Delta_c^* \equiv \phi_c m\left(\frac{a_c}{1-(1/\sigma_c)}\right)^{1-\sigma_c} + m^* \left(\frac{a_c}{1-(1/\sigma_c)}\right)^{1-\sigma_c}, \tag{15}$$

$$\Delta_{int} \equiv n\left(\frac{a_{int}}{1-(1/\sigma_{int})}\right)^{1-\sigma_{int}} + \phi_{int} n^* \left(\frac{a_{int}}{1-(1/\sigma_{int})}\right)^{1-\sigma_{int}}, \tag{16}$$

$$\Delta_{int}^* \equiv \phi_{int} n\left(\frac{a_{int}}{1-(1/\sigma_{int})}\right)^{1-\sigma_{int}} + n^* \left(\frac{a_{int}}{1-(1/\sigma_{int})}\right)^{1-\sigma_{int}}. \tag{17}$$

The value of ϕ_c and ϕ_{int}, respectively, ranges from $\phi_c = 0$ and $\phi_{int} = 0$, with prohibitive trade costs to $\phi_c = 1$ and $\phi_{int} = 1$, with zero trade costs.

Using the demand functions, mill prices, and the zero profit condition of downstream firms, the profit functions for upstream firms (Π_{int} and Π_{int}^*) are expressed as follow:

$$\Pi_{int} = \frac{\mu}{\sigma_{int}\sigma_c}\left(\frac{m}{\Delta_{int}}\left(\frac{E_c}{\Delta_c} + \frac{\phi_c E_c^*}{\Delta_c^*}\right) + \frac{m^*\phi_{int}}{\Delta_{int}^*}\left(\frac{\phi_c E_c}{\Delta_c} + \frac{E_c^*}{\Delta_c^*}\right)\right) - w_H f_{int}, \tag{18}$$

$$\Pi_{int}^* = \frac{\mu}{\sigma_{int}\sigma_c}\left(\frac{m\phi_{int}}{\Delta_{int}}\left(\frac{E_c}{\Delta_c} + \frac{\phi_c E_c^*}{\Delta_c^*}\right) + \frac{m^*}{\Delta_{int}^*}\left(\frac{\phi_c E_c}{\Delta_c} + \frac{E_c^*}{\Delta_c^*}\right)\right) - w_H^* f_{int}. \tag{19}$$

Therefore, the zero profit condition of upstream firms implies that the nominal wage rates for entrepreneurs are:

$$w_H = \frac{(\varpi_B\varpi_C - \varpi_A\varpi_D)(L-H)H^* + \varpi_A(L-H)b + \varpi_B(L^*-H^*)b}{(\varpi_A\varpi_D - \varpi_B\varpi_C)HH^* - b(\varpi_A H + \varpi_D H^*) + b^2}, \tag{20}$$

$$w_H^* = \frac{(\varpi_B\varpi_C - \varpi_A\varpi_D)(L^*-H^*)H + \varpi_C(L-H)b + \varpi_D(L^*-H^*)b}{(\varpi_A\varpi_D - \varpi_B\varpi_C)HH^* - b(\varpi_A H + \varpi_D H^*) + b^2} \tag{21}$$

where

$$b \equiv f_{int}\sigma_c\sigma_{int}/\mu,$$

$$\varpi_A \equiv \frac{m}{\Delta_{int}\Delta_c} + \frac{m^*\phi_{int}\phi_c}{\Delta_{int}^*\Delta_c}, \;\; \varpi_B \equiv \frac{m\phi_c}{\Delta_{int}\Delta_c^*} + \frac{m^*\phi_{int}}{\Delta_{int}^*\Delta_c^*},$$
$$\varpi_C \equiv \frac{m\phi_{int}}{\Delta_{int}\Delta_c} + \frac{m^*\phi_c}{\Delta_{int}^*\Delta_c}, \;\; \varpi_D \equiv \frac{m\phi_{int}\phi_c}{\Delta_{int}\Delta_c^*} + \frac{m^*}{\Delta_{int}^*\Delta_c^*}.$$

We make the usual choice of units for lightening the following analysis. We normalize a_c to $1-(1/\sigma_c)$, a_{int} to $1-(1/\sigma_{int})$, f_c to one, and f_{int} to one.

4. Equilibrium

4.1. Agglomeration and Dispersion

The equilibrium resulting from the free choice between the agricultural sector and manufacturing firms is analyzed in ten cases. As Baldwin (2001) used, local stability is evaluated by linearizing (6) and (7) around an equilibrium point $\bar{m}, \bar{m^*}, \bar{n}, \bar{n^*}$. The elements of the Jacobian matrix can be expressed as in Appendix 1. From Hatman-Grobman theorem (see Hartman 1964: 244, Theorem 7.1), which shows that linearization preserves the qualitative properties of the nonlinear system in the neighborhood of the equilibrium point when no eigenvalue of the Jacobian matrix has real parts equal to zero, equilibrium is stable when the Jacobian matrix has eigenvalues all with negative real parts.

Agglomeration and abundant supply of entrepreneurs

The manufacturing sector is supposed to be concentrated in the north, with $n^* = m^* = 0$. Since entrepreneurs are abundant, some entrepreneurs are in the agricultural sector in the north. All entrepreneurs in the south are also in the agricultural sector. Thus, the wage rate of entrepreneurs is one in both regions. Setting $\bar{m}^* = \bar{n}^* = H^* = 0$ and using (10) and (20), the zero profit condition of downstream firms and the equality of wage rates between workers and entrepreneurs implies the following number of upstream firms and downstream firms in the core:

$$m = \frac{\mu}{\sigma_c}(2L)^{\frac{\sigma_{int}}{\sigma_{int}-1}} b^{-\frac{1}{\sigma_{int}-1}} \equiv m_1,\ n = H = \frac{2L}{b}, \tag{22}$$

and the operating profits and fixed costs for upstream firms and the wage rate for entrepreneurs are as follow.

$$\pi_c = \frac{\mu}{\sigma_c}\frac{2L}{m},\ P_{int} = n^{-\frac{1}{\sigma_{int}-1}},\ w_H = \frac{2L/n - 1}{b-1}. \tag{23}$$

Because the entrepreneurs are abundant, we have $2L/b < \bar{H}$.

Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.1, we find that the case when all manufacturing firms are in the north under

$2L/b < \bar{H}$ is a spatial equilibrium under the following condition:

$$\frac{1}{2}\left(\phi_c + 1/\phi_c\right) < \phi_{int}^{-\frac{1}{\sigma_{int}-1}}. \tag{24}$$

The condition (24) shows the case when downstream firms do not emerge in the periphery. This implies that agglomeration is sustainable whenever transport costs of intermediate goods are high enough and transport costs of final goods are low enough. As explained in Fujita et al. (1999: 249), the right-hand side of the inequality shows the forward linkages between upstream firms and downstream firms. That is, downstream firms need to bear a disadvantage to operate in the region where there are no upstream firms. On the other hand, the left-hand side of the inequality shows the backward linkages between downstream firms and households. The first term in the parenthesis shows that downstream firms have a disadvantage when transporting to the other region, and the second term in the parenthesis shows that transport costs moderate price competition in the periphery.

Insert Figure 1 around here

To understand the relation with the results of the standard core-periphery model, Figure 1 is helpful. The condition (24) is depicted as a curve in Figure 1. The core-periphery structure is sustainable in the domain under the curve. The diagonal line shows the pair of transport costs when both transport costs take the same value. Supposing that both transport costs fall along the diagonal line from the origin of Figure 1, the core-periphery pattern becomes stable after transport costs decrease adequately, as discussed in Fujita et al. (1999).

Agglomeration and scarce supply of entrepreneurs

The manufacturing sector is supposed to be concentrated in the north, with $n^* = m^* = 0$. Since entrepreneurs are scarce, the number of upstream firms is decided by the number of entrepreneurs in the north. All entrepreneurs in the south are in the agricultural sector. Setting $\bar{m}^* = \bar{n}^* = H^* = 0$ and using (10) and (20), the zero profit condition of downstream firms and the scarcity of entrepreneurs implies the following number of upstream firms and downstream firms in the core:

$$m = \frac{\mu}{\sigma_c}\frac{b}{b-1}(2L - \bar{H})\bar{H}^{\frac{1}{\sigma_{int}-1}} \equiv m_2,\ n = \bar{H}, \tag{25}$$

and the operating profits and fixed costs for upstream firms, and wage rate for entrepreneurs, are as follows

$$\pi_c = \frac{\mu}{\sigma_c}\frac{(2L - \bar{H})b}{(b-1)m},\ P_{int} = n^{-\frac{1}{\sigma_{int}-1}},\ w_H = \frac{2L - \bar{H}}{(b-1)\bar{H}}. \tag{26}$$

Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.2., we find that the case when all manufacturing firms are in the north under $2L/b \geq \bar{H}$ is a spatial equilibrium under the following condition:

$$\phi_{int} < \frac{(b-1)\bar{H}}{2L - \bar{H}}, \tag{27}$$

and

$$\frac{\phi_c[(b+1)L - b\bar{H}] + (b-1)L/\phi_c}{b(2L - \bar{H})} < \phi_{int}{}^{-\frac{1}{\sigma_{int}-1}}. \tag{28}$$

The first condition (27) shows the case when upstream firms do not emerge in the periphery. The right-hand side of (27) is the nominal wage rate of entrepreneurs in the south (the periphery) divided by that of entrepreneurs in the north (the core). That is, upstream firms emerge in the periphery when transport costs are low enough that the decrease in profit caused by the additional transport cost can be covered by the increase in profit due to the relatively lower nominal wage rates of entrepreneurs in the periphery. The second condition (28) is similar to (24). The minor difference in them stems from the asymmetric market size in (28). In Figure 2, (27) is depicted as a horizontal line, whereas (28) is depicted as a curve. As in Figure 1, the diagonal line shows the pairs of transport costs when both transport costs take the same value. Supposing that both transport costs fall along the diagonal line from the origin of Figure 1, the core-periphery pattern becomes stable after transport costs decrease adequately because downstream firms lose the incentive to emerge in the periphery. Further decrease in transport costs causes the emergence of upstream firms in the periphery.

Insert Figure 2 around here

Dispersion and abundant supply of entrepreneurs

The manufacturing sector is supposed to be spread symmetrically, with $n = n^*$ and $m = m^*$. Since entrepreneurs are abundant, some entrepreneurs are in the agricultural sector in both regions. Thus, the wage rate of entrepreneurs is one in both regions. The zero profit condition for upstream firms implies that $w_H = (L - H)/(b-1)H$. Since w_H is one when entrepreneurs are abundant, the number of upstream firms in a region is $n = H = L/b$, which implies $L/b < \bar{H}$, whereas the number of downstream firms in a region is

$$m = \frac{\mu}{\sigma_c} \left(\frac{1 + \phi_{int}}{b} \right)^{1/(\sigma_{int}-1)} L^{\sigma_{int}/(\sigma_{int}-1)} \equiv m_3. \tag{29}$$

Examining the instance when all eigenvalues are negative in the above setting in Appendix B.3, we find that the case when upstream firms and downstream firms spread symmetrically under $L/b < \bar{H}$ is spatial equilibrium under the following condition:

$$(1 - \phi_c)^2 - \frac{4\phi_c}{(\sigma_{int} - 1)} > 0. \tag{30}$$

The first and the second term on the left-hand side of (30), respectively, show dispersion force and agglomeration force. When the first term is larger than the second, a symmetric pattern of firms exists. The first term is the effects from an upstream firm on other upstream firms and those from a downstream firm on other downstream firms, whereas the second term is the effects from an upstream firm on downstream firms and those from a downstream firm on upstream firms. Surprisingly, transport costs for intermediate goods are not related to the stability of the symmetric pattern. This is because the effects of transport costs on both forces are the same and so are canceled out from (30). From (30), the dispersion force decreases with the fall of the transport costs, whereas agglomeration forces increase with the rise of transport costs. Thus, the symmetric pattern is stable when $\phi_c \in (0, \phi_c^s)$, where

ϕ_c^s is defined from (30) as

$$\phi_c^s = \frac{\sqrt{\sigma_{int}} - 1}{\sqrt{\sigma_{int}} + 1}.$$

As intermediate goods become more substitutable, the agglomeration forces become weaker. As $\partial\phi_c^s/\sigma_{int}$ is always positive, an increase in substitutable intermediate goods causes a larger ϕ_c^s.

Dispersion and scarce supply of entrepreneurs

The manufacturing sector is supposed to be spread symmetrically, with $n = n^*$ and $m = m^*$. Since entrepreneurs are scarce, the number of upstream firms is decided by the number of entrepreneurs in both regions. The zero profit condition for upstream firms implies that $w_H = (L - \bar{H})/(b-1)\bar{H}$ and $w_H^* = (L - \bar{H}^*)/(b-1)\bar{H}^*$. Since entrepreneurs are scarce, the number of upstream firms in a region is $n = n^* = \bar{H}$. Since we consider the case in which $w_H \geq 1$, we have $L/b \geq \bar{H}$. The zero profit condition for downstream firms implies the following number of downstream firms in a region:

$$m = \frac{\mu}{\sigma_c}(L - \bar{H})\bar{H}^{\frac{1}{\sigma_{int}-1}}(1 + \phi_{int})^{\frac{1}{\sigma_{int}-1}} \equiv m_4. \tag{31}$$

Examining the instance when all eigenvalues are negative in the above setting in Appendix B.4, we find that the case when upstream firms and downstream firms spread symmetrically under $L/b \geq \bar{H}$ is always a spatial equilibrium.

As an agglomeration force, circular causality exists between entrepreneurs and final manufactured goods. That is, the increase in downstream firms raises wage rates for entrepreneurs; the rise in wage rates for entrepreneurs provides a larger demand for final manufactured goods. However, these agglomeration forces are surpassed by dispersion forces which emerge from the harsh price competition among downstream firms. The other agglomeration forces engendered by the interaction between downstream firms and upstream firms do not exert an effect due to the lack of additional entrepreneurs.

Two agglomerations by one type of firm and abundant supply of entrepreneurs

Upstream firms are supposed to exist only in a region with $m^* = 0$, whereas downstream firms are supposed to exist only in the other region with $n = 0$. Since entrepreneurs are abundant, some entrepreneurs are in the agricultural sector in both regions. Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.5, we find that the case in which all upstream firms are in one region and all downstream firms are in the other region under $2L/b < \bar{H}$ is not a spatial equilibrium. This is because, due to the positive transport costs for intermediate goods, downstream firms emerge in the region where all downstream firms locate to save on transport costs for intermediate goods, and furthermore, upstream firms emerge in the region where all upstream firms locate.

Two agglomerations by one type of firms and scarce supply of entrepreneurs

Upstream firms are supposed to exist only in a region with $m^* = 0$, whereas downstream firms are supposed to exist only in the other region with $n = 0$. Since entrepreneurs are scarce, the number of upstream firms is decided by the number of entrepreneurs in a region. Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.6, we find that the case in which all upstream firms are in one region and all downstream firms are in the other region under $2L/b \geq \bar{H}$ is not a spatial equilibrium.

Agglomeration of upstream firms, dispersion of downstream firms, and abundant supply of entrepreneurs

Upstream firms are supposed to be located in a region, and downstream firms are supposed to be located symmetrically. This means $n > 0$, $n^* = 0$ and $m = m^*$. Since entrepreneurs are abundant, some entrepreneurs are in the agricultural sector in both regions. Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.7, we find that the case in which upstream firms locate only in a region and downstream firms spread evenly under $2L/b < \bar{H}$ is not a spatial equilibrium. In this configuration of firms, downstream firms have an incentive to emerge in the region where downstream firms do not locate because the wage rates for entrepreneurs are always higher than that for workers.

Agglomeration of upstream firms, dispersion of downstream firms, and scarce supply of entrepreneurs

Downstream firms are supposed to exist only in a region with $n^* = 0$, whereas upstream firms are supposed to spread evenly into the two regions with $m = m^*$. Since entrepreneurs are scarce, the number of upstream firms is decided by the number of entrepreneurs in a region. Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.8, we find that the case in which upstream firms disperse symmetrically and all downstream firms are in a region under $2L/b \geq \bar{H}$ is not a spatial equilibrium.

Dispersion of upstream firms, agglomeration of downstream firms, and abundant supply

of entrepreneurs

Upstream firms are supposed to be located symmetrically, and downstream firms are supposed to be located only in a region. That means $m > 0$, $m^* = 0$ and $n = n^*$. Since entrepreneurs are abundant, some entrepreneurs are in the agricultural sector in both regions. Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.9, we find that the case in which upstream firms locate symmetrically and downstream firms locate only in a region with $w^H = 1$ is not a spatial equilibrium. This is because downstream firms always have an incentive to emerge in the region where there are no downstream firms.

Dispersion of upstream firms, agglomeration of downstream firms, and scarce supply of entrepreneurs

Upstream firms are supposed to exist only in a region with $m^* = 0$, whereas downstream firms are supposed to spread evenly into the two regions with $n = n^*$. Since entrepreneurs are scarce, the number of upstream firms is decided by the number of entrepreneurs in both regions. Examining the instance when all eigenvalues are negative in the above setting as in Appendix B.10, we find that the case in which all upstream firms are in one region and downstream firms disperse symmetrically with $w^H \geq 1$ is not a spatial equilibrium.

4.2. Partial Agglomeration

Here, we start to examine the interior solutions, which refer to the solutions without agglomeration by any firms and which are associated with uneven distribution of firms. To this aim, we rewrite the entire system in terms of ratios, in keeping with the method used in Robert-Nicoud (2004).

We define $\tilde{\Delta}_c$ and $\tilde{\Delta}_{int}$ as

$$\tilde{\Delta}_c \equiv \frac{\Delta_c}{\Delta_c^*} = \frac{m + \phi_c m^*}{\phi_c m + m^*} \tag{32}$$

$$\tilde{\Delta}_{int} \equiv \frac{\Delta_{int}}{\Delta_{int}^*} = \frac{n + \phi_{int} n^*}{\phi_{int} n + n^*} \tag{33}$$

Solving (32) and (33), we have

$$\frac{m}{m^*} = \frac{\phi_c - \tilde{\Delta}_c}{\tilde{\Delta}_c \phi_c - 1} \tag{34}$$

$$\frac{n}{n^*} = \frac{\phi_{int} - \tilde{\Delta}_{int}}{\tilde{\Delta}_{int} \phi_{int} - 1} \tag{35}$$

We can easily verify that functions (34) and (35) are bijectional, using (32) and (33), and are, respectively, continuous between $\phi_c < \tilde{\Delta}_c < 1/\phi_c$ and between $\phi_{int} < \tilde{\Delta}_{int} < 1/\phi_{int}$.

To clarify the interaction between upstream firms and downstream firms, we focus on the case in which all entrepreneurs receive the same wage rates as workers in agricultural sector. That is, the regional income becomes the same in the two regions. In this case, mobile expenditure by downstream firms is defined as the ratio of the mobile regional expenditure of downstream firms in Region 1 to the mobile regional expenditure of downstream firms in Region 2, η_{int} as follows:

$$\eta_{int} \equiv \frac{m}{m^*} \frac{1 + \phi_c \tilde{\Delta}_c}{\phi_c + \tilde{\Delta}_c} \tag{36}$$

Substituting (34) into (36), we have

$$\eta_{int} = \frac{\phi_c - \tilde{\Delta}_c}{\tilde{\Delta}_c \phi_c - 1} \frac{1 + \phi_c \tilde{\Delta}_c}{\phi_c + \tilde{\Delta}_c} \tag{37}$$

From the zero profit condition of upstream firms, we have as follows:

$$\ln\left(\frac{\eta_{int}/\tilde{\Delta}_{int} + \phi_{int}}{\eta_{int}\phi_{int}/\tilde{\Delta}_{int} + 1}\right) = 0;. \tag{38}$$

Thus, we have $\eta_{int} = \tilde{\Delta}_{int}$. Similarly, from the zero profit condition of downstream firms, we have as follows:

$$\ln\left(\frac{1 + \phi_c \tilde{\Delta}_c}{\phi_c + \tilde{\Delta}_c}\right) = -\frac{1}{\sigma_{int} - 1} \ln\left(\tilde{\Delta}_{int}\right) \tag{39}$$

Substituting the right-hand side of (39) into (37) and using $\eta_{int} = \tilde{\Delta}_{int}$, we have

$$\ln(\tilde{\Delta}_{int}) = \frac{\sigma_{int} - 1}{\sigma_{int}} \ln\left(\frac{\phi_c - \tilde{\Delta}_c}{\tilde{\Delta}_c \phi_c - 1}\right) \tag{40}$$

Since $\partial\tilde{\Delta}_{int}/\partial\tilde{\Delta}_c > 0$ in (40), we find that there exists no partial agglomeration when $0 < m/m^* < 0.5$ and $0.5 < n/n^* < 1$ and when $0.5 < m/m^* < 1$ and $0 < n/n^* < 0.5$.

Using $\eta_{int} = \tilde{\Delta}_{int}$ and substituting (37) into (39), we have

$$-\sigma_{int} \ln\left(\frac{1 + \phi_c \tilde{\Delta}_c}{\phi_c + \tilde{\Delta}_c}\right) - \ln\left(\frac{\phi_c - \tilde{\Delta}_c}{\phi_c \tilde{\Delta}_c - 1}\right) = 0 \tag{41}$$

The function always becomes zero when $\tilde{\Delta}_c = 1$. Thus, there exists an equilibrium in the symmetric case. The first derivative of (41) with $\tilde{\Delta}_c$ has two zeros when the symmetric pattern is stable and one zero when transport costs of final goods are at the threshold value such that the symmetric pattern is stable or not, $\phi_c = \phi_c^s$. The first derivative of (41) with $\tilde{\Delta}_c$ has no zero when the symmetric pattern is unstable. Thus, there exist two unstable asymmetric equilibriums at most, which are regarded as two partial agglomerations. On the core-periphery structure, setting $\tilde{\Delta}_c = \phi_c$ and $Del\tilde{t}a_{int} = \phi_{int}$ in (39), we find the condition in which core-periphery patterns become a spatial equilibrium when the left-hand side of the derived equation becomes smaller than the right-hand side of the derived equation. Thus, substituting $\tilde{\Delta}_c = \phi_c$, $\tilde{\Delta_{int}} = \phi_{int}$ and then $\phi_c = \phi_c^s$ into the inequality, which is related with (39), we have a condition in which the core-periphery structure is sustainable when $\phi_c = \phi_c^s$ is an equilibrium as follows:

$$0 < \phi_{int} < \left(\frac{\sigma_{int} - 1}{\sigma_{int} + 1} \right)^{\sigma_{int} - 1} \equiv \phi_{int}^h. \tag{42}$$

This implies that location displays hysteresis when the transport costs of intermediate goods are high enough.

To summarize the result of the present section:

Proposition 1 (Equilibrium). When $0 < \bar{H}/L < 1/b$, symmetric equilibrium exists and is stable when $\phi_c \in (0,1)$ and $\phi_{int} \in (0,1)$. Two core-periphery equilibriums are stable when the following conditions are satisfied:

$$\phi_{int} < \min \left\{ \frac{(b-1)\bar{H}}{2L - \bar{H}}, \left\{ \frac{\phi_c[(b+1)L - b\bar{H}] + (b-1)L/\phi_c}{b(2L - \bar{H})} \right\}^{-(\sigma_{int}-1)} \right\} \tag{43}$$

When $1/b < \bar{H}/L < 2/b$ or $1/b < \bar{H}/L < 1$ under $b < 2$, symmetric equilibrium exists and is stable when $\phi_c \in (0, \phi_c^s)$. Two core-periphery equilibriums exist and are stable when (43) is satisfied. When $2/b < \bar{H}/L < 1$, the model displays at most three interior equilibriums. Symmetric equilibrium exists and is stable when $\phi_c \in (0, \phi_c^s)$ and $\phi_{int} \in$

$(0,1)$. *When two partial agglomerations exist, symmetric equilibrium exists and these two partial agglomerations are unstable. Location displays hysteresis when* $0 < \phi_{int} < \phi_{int}^{h}$. *Two core-periphery equilibriums are stable when the following conditions are satisfied:*

$$\phi_{int} < \left[\frac{\phi_c + 1/\phi_c}{2}\right]^{-(\sigma_{int}-1)}. \tag{44}$$

5.Pareto Dominant Allocation

In the previous section, we found that spatial equilibriums exist in the core-periphery pattern and symmetric pattern depending on the size of $\bar{H}/L$. Thus, we can focus only on these patterns as spatial equilibriums when examining the Pareto dominant allocation. We evaluate the indirect utility level of workers and entrepreneurs in the two regions.

When $0 < \bar{H}/L < 1/b$, the indirect utility level of entrepreneurs in the core under the core-periphery structure (V_{CH}^{A1}) in terms of the indirect utility level of entrepreneurs under the symmetric pattern (V_H^{S1}) is expressed as follows:

$$\frac{V_{CH}^{A1}}{V_H^{S1}} = \left[\frac{2L-\bar{H}}{L-\bar{H}}\right]^{1+\frac{\mu}{\sigma_c-1}} \left[\frac{1}{(1+\phi_c)(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}}\right]^{\frac{\mu}{\sigma_c-1}}. \tag{45}$$

The indirect utility level of entrepreneurs in the periphery under the core-periphery structure (V_H^{S1}) in terms of indirect utility level of entrepreneurs under the symmetric pattern (V_H^{S1}) is expressed as follows:

$$\frac{V_P^{A1}}{V_H^{S1}} = \frac{(b-1)\bar{H}}{L-\bar{H}} \left[\frac{\phi_c}{1+\phi_c}\frac{2L-\bar{H}}{L-\bar{H}}\frac{1}{(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}}\right]^{\frac{\mu}{\sigma_c-1}}. \tag{46}$$

The indirect utility level of workers in the core under the core-periphery structure (V_C^{A1}) in terms of indirect utility level of entrepreneurs under the symmetric pattern (V_H^{S1}) is expressed as follows:

$$\frac{V_C^{A1}}{V^{S1}} = \left[\frac{1}{1+\phi_c} \frac{2L-\bar{H}}{L-\bar{H}} \frac{1}{(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}} \right]^{\frac{\mu}{\sigma_c-1}} \tag{47}$$

The indirect utility level of workers in the periphery under the core-periphery structure (V_P^{A1}) in terms of indirect utility level of entrepreneurs under the symmetric pattern (V_H^{S1}) is expressed as follows:

$$\frac{V_P^{A1}}{V^{S1}} = \left[\frac{\phi_c}{1+\phi_c} \frac{2L-\bar{H}}{L-\bar{H}} \frac{1}{(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}} \right]^{\frac{\mu}{\sigma_c-1}} \tag{48}$$

We now find the following relations:

$$\frac{V_{CH}^{A1}}{V_H^{S1}} > \frac{V_C^{A1}}{V^{S1}} > \frac{V_P^{A1}}{V^{S1}} > \frac{V_P^{A1}}{V_H^{S1}} \tag{49}$$

Focusing on when $V_P^{A1}/V_H^{S1} < 1$ is satisfied to examine the Pareto dominant allocation, we find from (46) that agglomeration Pareto-dominates dispersion if trade costs of final goods are lower than the following level:

$$\phi_c = \frac{\left[\frac{L-\bar{H}}{(b-1)\bar{H}}\right]^{\frac{\sigma_c-1}{\mu}}}{\frac{2L-\bar{H}}{L-\bar{H}}(1+\phi_{int})^{-\frac{1}{(\sigma_{int}-1)}} - \left[\frac{L-\bar{H}}{(b-1)\bar{H}}\right]^{\frac{\sigma_c-1}{\mu}}} \equiv \phi_c^{Pareto1}. \tag{50}$$

When $1/b < \bar{H}/L < 2/b$ or $1/b < \bar{H}/L < 1$ under $b < 2$, the indirect utility level of entrepreneurs in the core under the core-periphery structure (V_{CH}^{A2}) in terms of indirect utility level under the symmetric pattern (V^{S2}) is expressed as follows:

$$\frac{V_{CH}^{A2}}{V^{S2}} = \left[\frac{2L-\bar{H}}{(b-1)\bar{H}}\right]^{1+\frac{\mu}{\sigma_c+1}} \left[\frac{b}{L\bar{H}}\right]^{\frac{\sigma_{int}\mu}{(\sigma_{int}-1)(\sigma_c-1)}} \left[\frac{1}{(1+\phi_c)(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}}\right]^{\frac{\mu}{\sigma_c-1}} \tag{51}$$

The indirect utility level in the core under the core-periphery structure (V_C^{A2}) in terms of indirect utility level under the symmetric pattern (V^{S2}) is expressed as follows:

$$\frac{V_c^{A2}}{V^{S2}} = \left[\frac{2L - \bar{H}}{(b-1)\bar{H}} \frac{1}{(1+\phi_c)(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}} \left(\frac{b}{\bar{H}L} \right)^{\frac{\sigma_{int}}{\sigma_{int}-1}} \right]^{\frac{\mu}{\sigma_c - 1}} \tag{52}$$

The indirect utility level in the periphery under the core-periphery structure (V_P^{A2}) in terms of indirect utility level under the symmetric pattern (V^{S2}) is expressed as follows:

$$\frac{V_P^{A2}}{V^{S2}} = \left[\frac{2L - \bar{H}}{(b-1)\bar{H}} \frac{\phi_c}{(1+\phi_c)(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}} \left(\frac{b}{\bar{H}L} \right)^{\frac{\sigma_{int}}{\sigma_{int}-1}} \right]^{\frac{\mu}{\sigma_c - 1}} \tag{53}$$

From the above equations, we find the following relation:

$$\frac{V_{CH}^{A2}}{V^{S2}} > \frac{V_c^{A2}}{V^{S2}} > \frac{V_P^{A2}}{V^{S2}} \tag{54}$$

To examine the Pareto dominant allocation from (53), we find that agglomeration Pareto-dominates dispersion if trade costs of final goods are lower than the following level:

$$\phi_c = \left[\frac{2L - \bar{H}}{(b-1)\bar{H}} \left(\frac{b}{\bar{H}L} \right)^{\frac{\sigma_{int}}{\sigma_{int}-1}} \frac{1}{(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}} - 1 \right]^{-1} \equiv \phi_c^{Pareto2}. \tag{55}$$

When $2/b < \bar{H}/L < 1$, the indirect utility level in the core under the core-periphery structure (V_C^{A3}) in terms of indirect utility level under the symmetric pattern (V^{S3}) is expressed as follows:

$$\frac{V_c^{A3}}{V^{S3}} = \left[\frac{2^{\frac{\sigma_{int}}{\sigma_{int}-1}}}{(1+\phi_c)(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}} \right]^{\frac{\mu}{\sigma_c - 1}}. \tag{56}$$

We easily find that $V_c^{A3}/V^{S3} > 1$. The indirect utility level in the periphery under the core-periphery structure (V_P^{A3}) in terms of indirect utility level under the symmetric pattern

(V^{S3}) is expressed as follows:

$$\frac{V_P^{A3}}{V^{S3}} = \left[\frac{\phi_c 2^{\frac{\sigma_{int}}{\sigma_{int}-1}}}{(1+\phi_c)(1+\phi_{int})^{\frac{1}{\sigma_{int}-1}}}\right]^{\frac{\mu}{\sigma_c-1}} \tag{57}$$

From the above equations, we find the following relation:

$$\frac{V_c^{A3}}{V^{S3}} > \frac{V_P^{A3}}{V^{S3}} \tag{58}$$

From (57), we find that agglomeration Pareto-dominates dispersion if trade costs of final goods are lower than the following level:

$$\phi_c = \left[\frac{2^{\frac{\sigma_{int}}{\sigma_{int}-1}}}{(1+\phi_{int})^{1/(\sigma_{int}-1)}}\right]^{-1} \equiv \phi_c^{Pareto3} \tag{59}$$

To summarize:

Proposition 2 (Pareto Dominant Allocation) Agglomeration Pareto-dominates dispersion if trade costs are low ($\phi_c > \phi_c^{Pareto1}$ when $0 < \bar{H}/L < 1/b$, $\phi_c > \phi_c^{Pareto2}$ when $1/b < \bar{H}/L < 2/b$ or $1/b < \bar{H}/L < 1$ under $b < 2$, and $\phi_c > \phi_c^{Pareto3}$ when $2/b < \bar{H}/L < 1$).

This result is the same as what has been established by Ottaviano and Robert-Nicoud (2006).

6.Conclusion

In many countries, lower tariff rates are set for intermediate manufactured goods, and higher tariff rates are set for final manufactured goods. The derived results imply that such setting of tariff rates tends to preserve the symmetric spread of upstream and downstream firms, and continued tariff reduction may cause core-periphery structures. When we focus on the interaction between upstream and downstream firms, the present model is fully solved. This means that (1) the present model displays, at most, three interior steady states, (2) when the asymmetric steady-states exist, they are unstable and (3) location displays hysteresis when the transport costs of intermediate goods are sufficiently high. Because this model is adequately simple to be used as an engine of larger models, new extensions will further develop the potential of New Economic Geography.

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Appendix A

The elements of the Jacobian matrix a_{ij} are as follow:

$$
\begin{aligned}
a_{11} &= \bar{m}\frac{\partial \ln(\frac{\pi_c}{P_{int}})}{\partial m} + \ln(\frac{\pi_c}{P_{int}}) \\
a_{12} &= \bar{m}\frac{\partial \ln(\frac{\pi_c}{P_{int}})}{\partial m^*} \\
a_{13} &= \bar{m}\frac{\partial \ln(\frac{\pi_c}{P_{int}})}{\partial n} \\
a_{14} &= \bar{m}\frac{\partial \ln(\frac{\pi_c}{P_{int}})}{\partial n^*} \\
a_{21} &= \bar{m^*}\frac{\partial \ln(\frac{\pi_c^*}{P_{int}^*})}{\partial m} \\
a_{21} &= \bar{m^*}\frac{\partial \ln(\frac{\pi_c^*}{P_{int}^*})}{\partial m^*} + \ln(\frac{\pi_c^*}{P_{int}^*}) \\
a_{21} &= \bar{m^*}\frac{\partial \ln(\frac{\pi_c^*}{P_{int}^*})}{\partial n} \\
a_{21} &= \bar{m^*}\frac{\partial \ln(\frac{\pi_c^*}{P_{int}^*})}{\partial n^*} \\
a_{31} &= \bar{n}(\bar{H} - \bar{n})\frac{\partial \ln(w_H/w)}{\partial m} \\
a_{32} &= \bar{n}(\bar{H} - \bar{n})\frac{\partial \ln(w_H/w)}{\partial m^*} \\
a_{33} &= \bar{n}(\bar{H} - \bar{n})\frac{\partial \ln(w_H/w)}{\partial n} + (\bar{H} - 2\bar{n})\ln(w_H/w) \\
a_{34} &= \bar{n}(\bar{H} - \bar{n})\frac{\partial \ln(w_H/w)}{\partial n^*} \\
a_{41} &= \bar{n^*}(\bar{H}^* - \bar{n}^*)\frac{\partial \ln(w_H^*/w^*)}{\partial m} \\
a_{42} &= \bar{n^*}(\bar{H}^* - \bar{n}^*)\frac{\partial \ln(w_H^*/w^*)}{\partial m^*} \\
a_{43} &= \bar{n^*}(\bar{H}^* - \bar{n}^*)\frac{\partial \ln(w_H^*/w^*)}{\partial n}
\end{aligned}
$$

$$a_{44} \quad = \quad \bar{n^*}(\bar{H}^* - \bar{n}^*)\frac{\partial \ln(w_H^*/w^*)}{\partial n^*} + (\bar{H}^* - 2\bar{n}^*)\ln(w_H^*/w^*)$$

Appendix B

B.1.

From $\pi_c/P_{int} = 1$ and $w_H/w = 1$, setting $\bar{m}^* = \bar{n}^* = H^* = 0$ and (22) and using Appendix A, we have the following Jacobian matrix:

$$J = \begin{pmatrix} \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ 0 & \ln(\pi_c^*/P_{int}^*) & 0 & 0 \\ \hat{n}\frac{\partial \ln(w_H/w)}{\partial m} & \hat{n}\frac{\partial \ln(w_H/w)}{\partial m^*} & \hat{n}\frac{\partial \ln(w_H/w)}{\partial n} & \hat{n}\frac{\partial \ln(w_H/w)}{\partial n^*} \\ 0 & 0 & 0 & \bar{H}\ln(w_H^*/w^*) \end{pmatrix} \tag{60}$$

where $\hat{n} \equiv \bar{n}(\bar{H} - \bar{n})$. Using (23) and the above Jacobian matrix, the characteristic equation of the system can be written as

$$\left(\lambda - \bar{H}\ln(\frac{w_H^*}{w^*})\right)\left(\lambda - \ln(\frac{\pi_c^*}{P_{int}^*})\right)\left(\lambda + 1\right)\left(\lambda + (\bar{H} - \bar{n})\frac{2L}{2L - \bar{n}}\right) = 0. \tag{61}$$

From the Hartman-Gobman theorem, the preservation of qualitative properties of the non-linear system in the neighbourhood of the equilibrium point is assured when the eigenvalue of the Jacobian matrix does not has zero or purely imaginary eigenvalues. Also, Routh-Hurwitz theorem shows that the necessary and sufficient condition for the stability of linear differential equations is when the real part of the eigenvalues of coefficient matrices is negative. Since w^* is one, the system become stable when $w_H^* < 1$ and $\pi_c^* - P_{int}^* < 0$. Using

(11) and $H = 2L/b$, the first requirement is met when $\phi_{int} < 1$, whereas, using (21), the second requirement is met when (24) is satisfied.

B.2.

From $\pi_c/P_{int} = 1$ and $n = \bar{H}$, setting $\bar{m}^* = \bar{n}^* = H^* = 0$ and (22), and using Appendix A, we have the following Jacobian matrix:

$$J = \begin{pmatrix} \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ 0 & \ln(\frac{\pi_c^*}{P_{int}^*}) & 0 & 0 \\ 0 & 0 & -\bar{H}\ln(w_H/w) & 0 \\ 0 & 0 & 0 & \bar{H}\ln(w_H^*/w^*) \end{pmatrix}. \quad (62)$$

Using (26) and the above Jacobian matrix, the characteristic equation of the system can be written as

$$\left(\lambda - \bar{H}\ln(\frac{w_H^*}{w^*})\right)\left(\lambda - \ln(\frac{\pi_c^*}{P_{int}^*})\right)\left(\lambda + 1\right)\left(\lambda + \bar{H}\ln\left(\frac{2L - \bar{H}}{(b-1)\bar{H}}\right)\right) = 0. \quad (63)$$

Thus, since w^* is one, the system is stable when $w_H^* < 1$ and $\pi_c^* - P_{int}^* < 0$. Using (11), the first requirement is met when (27) is satisfied, whereas using (21), the second requirement is met when (28) is satisfied.

B.3.

From $\pi_c/P_{int} = \pi_c^*/P_{int}^* = 1$ and $w_H/w = w_H^*/w^* = 1$, using Appendix A, the Jacobian matrix can be written as

$$\begin{pmatrix} \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m} & \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m^*} & \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n} & \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n^*} \\ \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial m} & \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial m^*} & \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial n} & \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial n^*} \\ \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial m} & \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial m^*} & \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial n} & \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial n^*} \end{pmatrix}. \quad (64)$$

The elements of the Jacobian matrix are derived from (10), (11), (20) and (21) as follows:

$$\frac{\partial \ln(\pi_c/P_{int})}{\partial m} = \frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m^*} = -\frac{(1+\phi_c^2)}{\bar{m}(1+\phi_c)^2}, \quad (65)$$

$$\frac{\partial \ln(\pi_c/P_{int})}{\partial m^*} = \frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m} = -\frac{2\phi_c}{\bar{m}(1+\phi_c)^2} \quad (66)$$

$$\frac{\partial \ln(\pi_c/P_{int})}{\partial n} = \frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n^*} = \frac{1}{\bar{n}(\sigma_{int}-1)(1+\phi_{int})}, \quad (67)$$

$$\frac{\partial \ln(\pi_c/P_{int})}{\partial n^*} = \frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n} = \frac{\phi_{int}}{\bar{n}(\sigma_{int}-1)(1+\phi_{int})}, \quad (68)$$

$$\frac{\partial \ln(w_H/w)}{\partial m} = \frac{\partial \ln(w_H^*/w^*)}{\partial m^*} = -\frac{\partial \ln(w_H/w)}{\partial m^*} = -\frac{\partial \ln(w_H^*/w^*)}{\partial m} = \frac{2L}{b-Z}\frac{\phi_c(1-\phi_{int})}{(1+\phi_c)^2(1+\phi_{int})\bar{n}^2\bar{m}}, \quad (69)$$

$$\frac{\partial \ln(w_H/w)}{\partial n} = \frac{\partial \ln(w_H^*/w^*)}{\partial n^*} = -\frac{b^2[b(1+\phi_c)(1+\phi_{int}^2)-(1-\phi_{int})(1-\phi_{int}\phi_c)]}{L(b-Z)(b-1)(1+\phi_c)(1+\phi_{int})^2}, \quad (70)$$

$$\frac{\partial \ln(w_H/w)}{\partial n^*} = \frac{\partial \ln(w_H^*/w^*)}{\partial n} = -\frac{b^2[2b\phi_{int}(1+\phi_c)-(1-\phi_{int})(\phi_c-\phi_{int})]}{L(b-Z)(b-1)(1+\phi_c)(1+\phi_{int})^2}, \quad (71)$$

where $Z \equiv (1-\phi_c)(1-\phi_{int})/[(1+\phi_c)(1+\phi_{int})]$. The characteristic equation of the system can be written as

$$(\lambda + 1/\bar{m})\left(\lambda + \frac{b^3(1+\phi_{int})^2 - b^2(1-\phi_{int})^2}{L(b-Z)(b-1)(1+\phi_{int})^2}\right)$$

$$\times \left\{ \lambda^2 + \left(\frac{(1-\phi_c)^2}{\bar{m}(1+\phi_c)^2} + \frac{b^2(1-\phi_{int})^2}{(b-Z)(1+\phi_{int})^2 L} \right) \lambda + \frac{b^2(1-\phi_{int})^2[(1-\phi_c)^2 - \frac{4\phi_c}{(\sigma_{int}-1)}]}{\bar{m}(b-Z)(1+\phi_c)^2(1+\phi_{int})^2 L} \right\} = 0. \quad (72)$$

The eigenvalues except in the last bracket are always negative. For the eigenvalue in the last bracket to be negative, the last term must be positive. Thus, this system is stable when (30) is met.

B.4.

From $\pi_c/P_{int} = \pi_c^*/P_{int}^* = 1$ and $w_H/w = w_H^*/w^* = 1$, using Appendix A, the Jacobian matrix can be written as

$$\begin{pmatrix} \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m} & \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m^*} & \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n} & \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n^*} \\ 0 & 0 & -\hat{H}\ln(w_H/w) & 0 \\ 0 & 0 & 0 & -\hat{H}^*\ln(w_H^*/w^*) \end{pmatrix}. \quad (73)$$

The characteristic equation of the system can be written as

$$\left(\lambda + \bar{H}\ln(\frac{w_H}{w})\right)\left(\lambda + \bar{H}^*\ln(\frac{w_H^*}{w^*})\right) \times \left\{ \lambda^2 - 2\frac{\partial \ln(\pi_c/P_{int})}{\partial m}\lambda + \left[\frac{\partial \ln(\pi_c/P_{int})}{\partial m} - \frac{\partial \ln(\pi_c/P_{int})}{\partial m^*}\right]\left[\frac{\partial \ln(\pi_c/P_{int})}{\partial m} + \frac{\partial \ln(\pi_c/P_{int})}{\partial m^*}\right] \right\} = 0. \quad (74)$$

The eigenvalues in the first and second bracket are negative when $w_H/w < 1$ and $w_H^*/w^* < 1$, which is satisfied when $L/b > \bar{H} = \bar{H}^*$. The last two eigenvalues becomes negative when the coefficient of the second term is positive and that of the last term is positive. Since the first bracket in the last term is $-(1-\phi_c)^2\bar{m}/\Delta_c^2$, we have the following relation

$$\frac{\partial(\pi_c/P_{int})}{\partial m} < \frac{\partial(\pi_c/P_{int})}{\partial m^*}. \tag{75}$$

Then, the above condition can be written as

$$2\frac{\partial \ln(\pi_c/P_{int})}{\partial m} < \frac{\partial \ln(\pi_c/P_{int})}{\partial m} + \frac{\partial \ln(\pi_c/P_{int})}{\partial m^*}. \tag{76}$$

Thus, the last two eigenvalues are negative when

$$\frac{\partial \ln(\pi_c/P_{int})}{\partial m} + \frac{\partial \ln(\pi_c/P_{int})}{\partial m^*} < 0 \Leftrightarrow 0 < b - (1-\phi_{int})/(1+\phi_{int}) \tag{77}$$

. Since $0 < \phi_{int} < 1$ and $b > 1$, the last two eigenvalues are always negative.

B.5.

The zero profit condition for upstream firms implies $w_H^* = (2L - H^*)/[(b-1)H^*]$. Since w_H is less than one when entrepreneurs are abundant, we have $2L/b < \bar{H}^*$.

Setting $n = 0$,$m^* = 0$, $w^*/w = 1$ and $\pi_c/P_{int} = 1$, the Jacobian matrix can be written as

$$J = \begin{pmatrix} \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ 0 & \ln(\pi_c^*/P_{int}^*) & 0 & 0 \\ 0 & 0 & \bar{H}\ln(w_H/w) & 0 \\ \hat{n}^*\frac{\partial \ln(w_H^*/w^*)}{\partial m} & \hat{n}^*\frac{\partial \ln(w_H^*/w^*)}{\partial m^*} & \hat{n}^*\frac{\partial \ln(w_H^*/w^*)}{\partial n} & \hat{n}^*\frac{\partial \ln(w_H^*/w^*)}{\partial n^*} \end{pmatrix}. \tag{78}$$

The characteristic equation of the system can be written as

$$\left(\lambda - \ln(\frac{\pi_c^*}{P_{int}^*})\right)\left(\lambda - \bar{H}\ln(w_H/w)\right) \times$$
$$\left\{\left[\lambda - \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m}\right]\left[\lambda - \bar{n}(\bar{H}-\bar{n})\frac{\partial (w^*/w)}{\partial n^*}\right] - \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n^*}\bar{n}(\bar{H}-\bar{n})\frac{\partial \ln(w^*/w)}{\partial m}\right\} = 0 \quad (79)$$

Using $\pi_c - P_{int} = 0$, we have

$$\pi_c^* - P_{int}^* = n^{*-\frac{1}{\sigma_{int}-1}} f\left\{\phi_{int}^{-\frac{1}{\sigma_{int}-1}}\left(\phi_c + \frac{1}{\phi_c}\right) - 1\right\}$$

Since $0 < \phi_{int} < 1$, $1/\phi_{int}$ becomes larger than one. From the inequality of the arithmetic and geometric means, $\phi_c + 1/\phi_c \geq 2$. Thus, we have $\pi_c^* - P_{int}^* > 0$. This means that the first eigenvalue is always positive.

Since the wage rates of entrepreneurs in the region where there are no upstream firms become $w^H = 1$, using $w = 1$, we have $w^H/w = 1$. Thus, the second eigenvalue is always positive.

B.6.

The zero profit condition for upstream firms implies $w_H^* = (2L - H^*)/[(b-1)H^*]$. Since w_H is more than one when entrepreneurs are scarce, using $n^* = \bar{H}^*$, we have $2L/b \geq \bar{H}^*$. Setting $m^* = 0$,$n = 0$ and $n^* = \bar{H}$, the characteristic equation of the system can be written as

$$\left(\lambda - \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m}\right)\left(\lambda - \bar{H}\ln(\pi_c^*/P_{int}^*)\right) \times$$

$$\left(\lambda - \bar{H}\ln(w_H/w)\right)\left(\lambda + \bar{H}\ln(w_H^*/w^*)\right) = 0 \quad (80)$$

Since the wage rates of entrepreneurs in the region where there are no upstream firms become $w^H = 1$, using $w = 1$, we have $w^H/w = 1$. Thus, the third eigenvalue is always positive.

B.7.

From the abundant supply of entrepreneurs, we have $w^H = w = 1$. Since the zero profit condition for upstream firms implies $w_H = (2L - H)/[(b-1)H]$, we have $2L/b < \bar{H}$. Setting $m^* = m$, $n > 0$ and $n^* = 0$, the Jacobian matrix can be written as

$$\begin{pmatrix} \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m} & \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m^*} & \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n} & \bar{m}^*\frac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n^*} \\ \hat{n}\frac{\partial \ln(w_H/w)}{\partial m} & \hat{n}\frac{\partial \ln(w_H/w)}{\partial m^*} & \hat{n}\frac{\partial \ln(w_H/w)}{\partial n} & \hat{n}\frac{\partial \ln(w_H/w)}{\partial n^*} \\ 0 & 0 & 0 & H\ln(w_H^*/w^*) \end{pmatrix}. \quad (81)$$

This system is stable when $w_H^*/w^* > 1$. Since $w_H = (2L - H)/[(b-1)H]$ and $w_H = W = 1$, we have $L/H = b/2$. Thus, we have:

$$w_H^* = \frac{1}{2}(\phi_{int} + 1/\phi_{int}).$$

From the inequality of the arithmetic and geometric means, $w_H^* > 1$ is implied. Thus, this system is not stable.

B.8.

The zero profit condition for upstream firms implies $w_H = (2L - H)/[(b-1)H]$. Since $w_H > 1$, we have $2L/b \geq \bar{H}$. Setting $m = m^*$, $n = \bar{H}$ and $n^* = 0$, the characteristic equation of the system can be written as

$$\begin{pmatrix} \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m} & \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial m^*} & \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n} & \bar{m}^*\dfrac{\partial \ln(\pi_c^*/P_{int}^*)}{\partial n^*} \\ 0 & 0 & -\bar{H}\ln(w_H/w) & 0 \\ 0 & 0 & 0 & \bar{H}\ln(w_H^*/w^*) \end{pmatrix}. \tag{82}$$

The third and forth eigenvalues are negative when $w_H^* < 1 < w^H$. Hence, we have

$$\frac{1}{2} < \frac{L}{bH} < \frac{b(1+\phi_c)\phi_{int} + (1-\phi_{int})\phi_c}{b(1+\phi_{int})(1+\phi_c) - (1-\phi_c)(1-\phi_{int})} \tag{83}$$

The inequality of the first and the third terms provide $b < 1$. The definition allows $b > 1$. Thus, the third or forth eigenvalue is positive.

B.9.

Setting $n^* = n \neq \bar{H}$, $m > 0$ and $m = 0$, the Jacobian matrix can be written as

$$J = \begin{pmatrix} \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial m^*} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n} & \bar{m}\dfrac{\partial \ln(\pi_c/P_{int})}{\partial n^*} \\ 0 & \ln(\pi_c^*/P_{int}^*) & 0 & 0 \\ \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial m} & \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial m^*} & \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial n} & \hat{n}\dfrac{\partial \ln(w_H/w)}{\partial n^*} \\ \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial m} & \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial m^*} & \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial n} & \hat{n}^*\dfrac{\partial \ln(w_H^*/w^*)}{\partial n^*} \end{pmatrix}. \tag{84}$$

The zero profit condition for downstream firms in the north implies that $\pi_c = P_{int}$. Since $P_{int} = P_{int}^*$, we have:

$$\pi_c^* - P_{int}^* = \pi_c^* - \pi_c = \frac{\mu}{\sigma_c}\frac{L}{m}\left(\phi_c + \frac{1}{\phi_c} - 2\right).$$

From the inequality of the arithmetic and geometric means, we have $\phi_c + 1/\phi_c > 2$. Thus, since $\pi^*/P_{int}^* > 1$, an eigenvalue in this system is positive.

B.10.

Setting $m^* = 0$ and $n = n^* = \bar{H}$, the characteristic equation of the system can be written as

$$\left(\lambda - \bar{m}\frac{\partial \ln(\pi_c/P_{int})}{\partial m}\right)\left(\lambda - \bar{H}\ln(\pi_c^*/P_{int}^*)\right) \times \left(\lambda + \bar{H}\ln(w_H/w)\right)\left(\lambda + \bar{H}\ln(w_H^*/w^*)\right) = 0 \quad (85)$$

The second eigenvalue is negative when $\pi_c^* - P_{int}^* < 0$. Since $\pi_c - P_{int} = 0$, the profit of downstream firms in the south is written as

$$\pi_c^* - P_{int}^* = \pi_c^* - \pi_c = \frac{\mu}{\sigma_c}\frac{L - \bar{H}}{\bar{m}}(1 - \phi_c)\left[\frac{1 - \phi_c}{\phi_c} + \frac{2(1 - \phi_{int})}{(b-1)(1 + \phi_{int})}\right] > 0. \quad (86)$$

Thus, the second eigenvalue is positive.

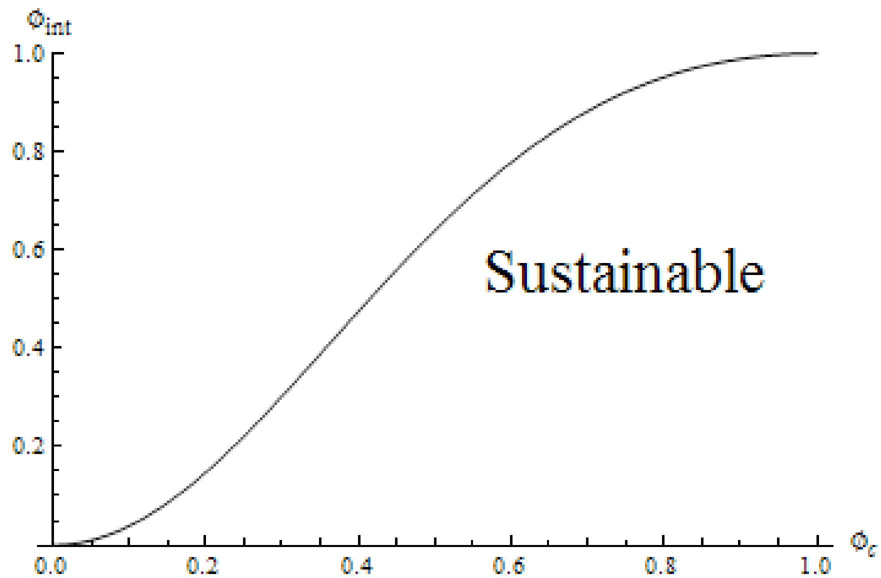

Figure 1: Sustainable condition of the core-periphery structure when entrepreneurs are abundant

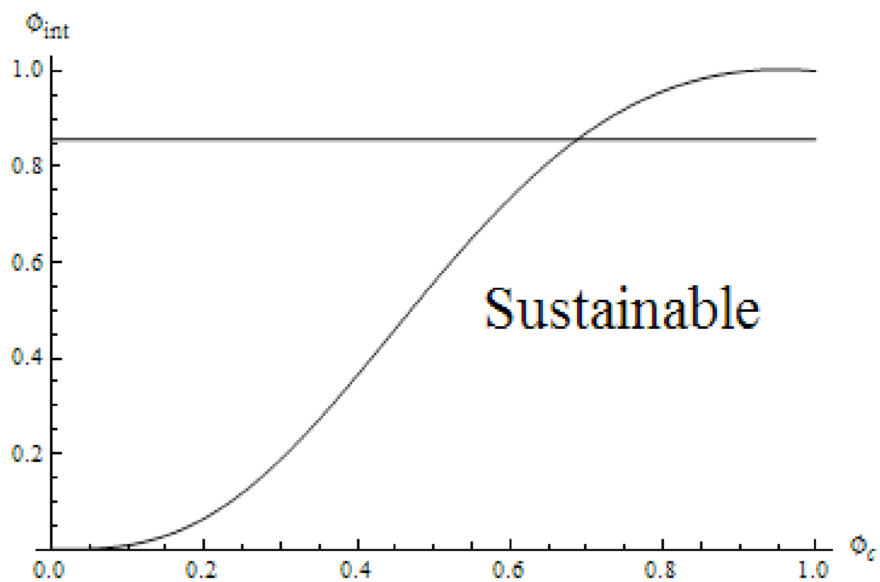

Figure 2: Sustainable condition of the core-periphery structure when entrepreneurs are scarce $b = 3, L = 1, H = 0.6, \sigma_{int} = 4$