

# Structural Propagation of Productivity Shocks: The Case of Korea

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| journal or<br>publication title | IDE Discussion Paper   |
| volume                          | 552  |
| year                            | 2016-02  |
| URL                             | <a href="http://hdl.handle.net/2344/1624">http://hdl.handle.net/2344/1624</a>  |

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**IDE DISCUSSION PAPER No. 552**

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Shocks: The Case of Korea**

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February 2016

**Abstract**

We model the transition of technological structure that is associated with the changes in cost induced by the innovation that occurred, using a system of multi-sector, multi-factor production functions. Structural propagation is quantified by using a system of unit-cost functions compatible with multi-level CES, plain CES, Cobb--Douglas, and Leontief production functions whose parameters we estimate via two timely distant input--output accounts. The economy-wide welfare gain obtainable for an exogenously given innovation will hence be quantified via the technological structure after structural propagation. Welfare gain due to productivity doubling for the medical and health services (public) industry is studied as an example, using the 2000--2005 Korean linked input--output table as the source of data.

**Keywords:** productivity, general equilibrium, structural propagation

**JEL classification:** C67, D57

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# Structural Propagation of Productivity Shocks: The Case of Korea

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## Abstract

We model the transition of technological structure that is associated with the changes in cost induced by the innovation that occurred, using a system of multi-sector, multi-factor production functions. Structural propagation is quantified by using a system of unit-cost functions compatible with multi-level CES, plain CES, Cobb–Douglas, and Leontief production functions whose parameters we estimate via two timely distant input–output accounts. The economy-wide welfare gain obtainable for an exogenously given innovation will hence be quantified via the technological structure after structural propagation. Welfare gain due to productivity doubling for the medical and health services (public) industry is studied as an example, using the 2000–2005 Korean linked input–output table as the source of data.

*Keywords:* Innovation, Productivity, General Equilibrium, Structural Propagation

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## 1. Introduction

In this study, we present a methodology that fully accounts for the feedback effect from introducing new technologies into the system of economy-wide production. In so doing, we take the technological substitutions fully into account. While it is known that technology will not substitute under some standard conditions (hence, technological structure will maintain), as far as the change in the final demand is concerned,<sup>1</sup> this will not be the case when any new technology is actually introduced within an industry; technology can be substituted in accordance with the price changes in the factor inputs, which will be triggered by the introduction of a new technology or innovation in any other industry. As the disposition of potential (alternative) technologies is represented by the curvature (or the elasticity of substitution) of a production function, measurement of elasticities has been central to applied quantitative analyses based on general equilibrium frameworks.

Hence, in terms of purpose and motivation, this study is in the same vein of research as studies of computable general equilibrium (CGE) models. CGE modelers, however, frequently resort to selecting elasticities from the empirical literature on the basis of judgement and assumptions (Wing, 2009). Meanwhile, classic multi-sectoral analyses such as Kuroda et al. (1984) have used sector-wise translog production functions for multiple (KLEM) factor inputs, while Saito and Tokutsu (1989) used multi-level constant elasticity of substitution (CES) production functions, all based upon time-series data.<sup>2</sup> Even recently, production functions for KLEM factor inputs have been estimated by using time-series data (van der Werf, 2008; Okagawa and Ban, 2008; Koesler and Schymura, 2015).

In this study, we estimate the elasticity of substitution for multiple industrial sectors by relying more on cross-sectional data (specifically, linked input–output tables) than on time series. We explore two types of production function for multiple industrial sectors. The first is a multi-level, multi-factor CES whose elasticities (between one factor input and a composite of factor inputs) are measured in a stratified manner for all factor inputs for each sector. The stage elasticities are determined by using the price indices and coefficients of the linked input–output tables along with industry-wise productivity, which we estimate via Törnqvist aggregation. The second is a multi-

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<sup>1</sup>This non-substitution theorem will hold under the conditions of constant returns to scale technology, one-to-one correspondences between commodity and industry, and when the number of primary inputs is one (Georgescu-Roegen, 1951).

<sup>2</sup>A two-factor CES function was first introduced by Arrow et al. (1961). It was later shown that the elasticities were still unique in the case of more than two factors (Uzawa, 1962; McFadden, 1963). A two-level CES production function was first introduced by Sato (1967).

factor (plain) CES whose elasticity of substitution is uniquely determined for each sector. Sector-wise elasticities of plain CES functions are estimated by regressing the log differences of the price ratios (which are obtainable by using price indices estimated from linked input–output tables) against the log differences of the cost shares (which are also obtainable from the coefficients of linked input–output tables). Moreover, sector-wise CES-compatible productivity gain can be estimated at the same time from the constant terms of the regression. The numbers of industries and intermediate inputs (excluding primary factors) are the same so that an equilibrium price can be obtained as the fixed point of a system of unit-cost functions that are dual to the production functions. The structural transition of the input–output structure can then be monitored as gradients for the unit-cost functions.

The remainder of this paper is organized as follows. In the next section, we measure the gain in sector-wise total factor productivity during 2000–2005, using linked input–output tables for Korea (BOK, 2015). In doing this, we aggregate labor and capital inputs so that there is a single primary input in addition to the intermediate inputs. In Section 2, we measure the parameters for the multi-factor (plain) CES, and the multi-level CES production functions via regression and solving simultaneous equations, respectively, based on the same database (linked input–output tables for Korea (BOK, 2015)). In Section 3, we formulate the structural propagation under the system of multi-sector, multi-factor production functions, and demonstrate structural propagations triggered by some exogenously given changes in productivity. Section 4 provides concluding remarks.

## 2. Production Functions

### 2.1. Productivity gain

We start with the production function of an industry (the index  $j$  is omitted):

$$y = zf(x_0, x_1, \dots, x_n) = zf(\mathbf{x}). \quad (1)$$

Here, we denote the output of this production by  $y$ , and the  $i$ th input by  $x_i$ . The total factor productivity (TFP), which reflects the technology level of the industry, is denoted by  $z$ . The function  $f(\mathbf{x})$  is assumed to be homogeneous of degree one with respect to the inputs (i.e., constant returns to scale are assumed). Taking the log and

time derivatives, we have

$$\frac{\dot{y}}{y} = \frac{\dot{z}}{z} + \sum_{i=0}^n \left( \frac{\partial f(\mathbf{x})}{\partial x_i} \frac{x_i}{f(\mathbf{x})} \right) \frac{\dot{x}_i}{x_i} \quad (2)$$

The term in parentheses is the cost share. This will be true under the following monetary balance of constant returns to scale production:

$$py = pz f(\mathbf{x}) = \sum_{i=0}^n p_i x_i, \quad pz \frac{\partial f(\mathbf{x})}{\partial x_i} = p_i$$

We may thus describe the cost share of input  $i$ , which we denote by  $\alpha_i$ , as follows:

$$\frac{p_i x_i}{py} = \frac{\partial f(\mathbf{x})}{\partial x_i} \frac{x_i}{f(\mathbf{x})} = \alpha_i. \quad (3)$$

We integrate (2) over two periods  $t = [0, 1]$  in order to obtain productivity growth  $\int_0^1 d \ln z = \Delta \ln z$  as

$$\int_0^1 d \ln y = \int_0^1 d \ln z + \sum_{i=0}^n \int_0^1 \alpha_i d \ln x_i. \quad (4)$$

However, in regard to (2), the right-hand side involves integration by parts, that is,  $\int_0^1 \alpha_i d \ln x_i$ . Assume that the trajectory scenarios for  $\alpha_i$  and  $x_i$  follow

$$\alpha_i(t) = (a_i^1 - a_i^0) \tau(t) + a_i^0, \quad x_i(t) = x_i^0 \exp(\tau(t) \ln x_i^1 / x_i^0),$$

where  $\tau(t)$  is a function of time satisfying  $\tau(0) = 0$  and  $\tau(1) = 1$ , so that  $x_i(0) = x_i^0$ ,  $x_i(1) = x_i^1$ ,  $\alpha_i(0) = a_i^0$ , and  $\alpha_i(1) = a_i^1$ . Note that this will always be true for translog functions whose cost shares are linear with respect to the log of inputs. In this case, the integration reduces to

$$\int_0^1 \alpha_i d \ln x_i = \frac{a_i^1 + a_i^0}{2} \Delta \ln x_i.$$

Thus, (4) is reduced as follows, obtaining productivity growth using Törnqvist aggre-

gation (i.e.,  $\bar{a}_i = (a_i^1 + a_i^0)/2$ ):<sup>3</sup>

$$\Delta \ln z = \Delta \ln y - \sum_{i=0}^n \bar{a}_i \Delta \ln x_i.$$

The above formula can also be described by way of monetary output  $Y = py$  and input  $X_i = p_i x_i$ , such that

$$\Delta \ln z = (\Delta \ln Y - \Delta \ln p) - \sum_{i=0}^n \bar{a}_i (\Delta \ln X_i - \Delta \ln p_i). \quad (5)$$

The productivity gain observed between two periods  $t = 0$  and  $t = 1$  for an industrial sector  $j$  (i.e.,  $z_j^1/z_j^0 = \exp(\Delta \ln z_j)$ ) can then be calculated by way of input–output transactions  $X_{ij}$  and  $Y_j$ , cost share accounts  $a_{ij}$ , and deflators for all commodity prices  $p_i^1/p_i^0 = \exp(\Delta \ln p_i)$ , using (5). For subsequent study, we estimated total factor productivity gain for 350 industrial sectors from the Korean linked input–output tables (coefficients and transactions) and deflators for 2000–2005 (BOK, 2015). Note that we aggregated fixed capital with labor inputs for simplicity, so that there is only one primary factor ( $i = 0$ ). Figure 1 illustrates the estimated values of productivity gain  $z_j^1/z_j^0$  of sector  $j$ .

## 2.2. Multi-level CES production function

The multi-level CES production function of  $n + 1$  factors for an industrial sector (whose index  $j$  is omitted) is

$$y = z \Xi_0$$

$$\Xi_i = \left( \delta_i^{\frac{1}{\sigma_i}} x_i^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \delta_i)^{\frac{1}{\sigma_i}} \Xi_{i+1}^{\frac{\sigma_i-1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i-1}}, \quad i = 0, 1, \dots, n-1, \quad (6)$$

where  $\Xi_{i+1}$  denotes the composite factor of the  $i + 1$ th and subsequent factors. The last composite factor must coincide with the last input factor, that is,  $\Xi_n = x_n$ . Also, we denote by  $\delta_i$  the share parameter for the  $i$ th factor, and by  $\sigma_i$  the elasticity of substitution between the  $i$ th factor and the  $i + 1$ th composite factors.

<sup>3</sup>Star and Hall (1976) showed that Törnqvist aggregation is a robust approximation with respect to trajectory scenarios. Törnqvist aggregation gives the exact productivity index for translog functions (Diewert, 1976).



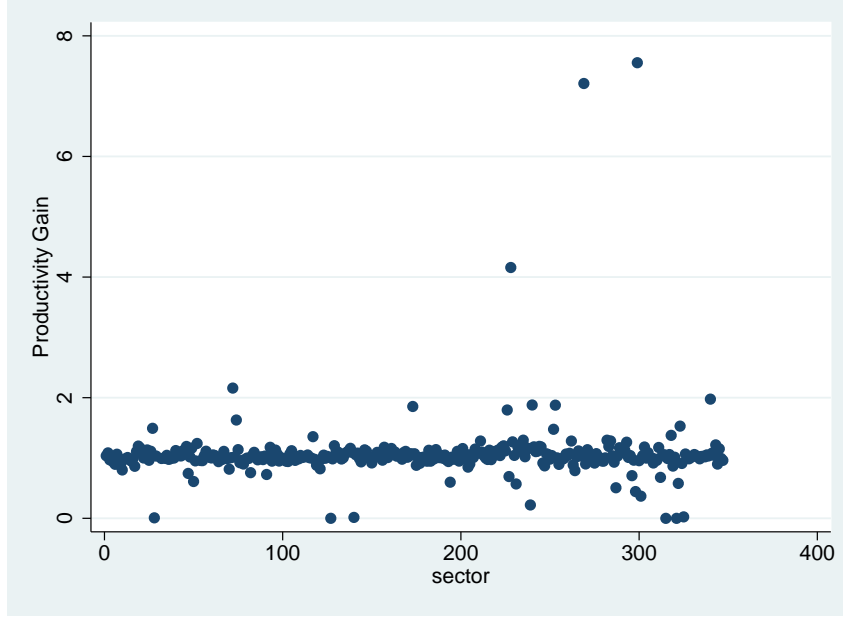


Figure 1: Estimates of TFP gain ( $z_j^1/z_j^0$ ) for various industrial sectors, based on the 2000–2005 linked input–output tables of Korea (BOK, 2015). Notable sectors with large numbers are postal services (7.55), residential building construction (7.21), and household audio equipment (4.15).

The unit-cost function focusing on the baseline nest level ( $i = 0$ ) of (6) is

$$c = \frac{1}{z} \left( \delta_0 p_0^{1-\sigma_0} + (1 - \delta_0) \Phi_1^{1-\sigma_0} \right)^{\frac{1}{1-\sigma_0}}, \quad (7)$$

where  $p_0$  and  $\Phi_1$  denote prices for  $x_0$  and  $\Xi_1$ , respectively.

By applying Shephard's Lemma to (7), the cost share of the 0th input  $a_0$  is derived as

$$a_0 = \frac{\partial c}{\partial p_0} \frac{p_0}{c} = \delta_0 (zc/p_0)^{\sigma_0-1} = \delta_0 (\Phi_0/p_0)^{\sigma_0-1}, \quad (8)$$

where we used  $\Phi_0 \equiv zc$ , or the baseline unit cost.<sup>4</sup> Suppose cost shares ( $a_i$ ), relative prices ( $p_i$ ), and the relative productivity ( $z$ ) for two periods are available via a linked input–output table for all sectors. By taking the log of (8) at two periods (where time periods ( $t = 0, 1$ ) are indexed via superscripts with parenthesis) while assuming the

<sup>4</sup>We call  $\Phi_0$  the baseline unit cost because it reflects the unit cost without productivity gain.

parameters are stable between the two periods, we have

$$\begin{aligned}\ln a_0^{(0)} &= \ln \delta_0 + (\sigma_0 - 1) \ln \Phi_0^{(0)} / p_0^{(0)} \\ \ln a_0^{(1)} &= \ln \delta_0 + (\sigma_0 - 1) \ln \Phi_0^{(1)} / p_0^{(1)}.\end{aligned}$$

By subtracting and reorganizing terms we obtain

$$\sigma_0 = \frac{\Delta \ln a_0 / p_0 + \Delta \ln \Phi_0}{\Delta \ln \Phi_0 / p_0} = \frac{\ln a_0^{(1)} - \ln a_0^{(0)}}{\ln \Phi_0^{(1)} / p_0^{(1)} - \ln \Phi_0^{(0)} / p_0^{(0)}} + 1 \quad (9)$$

$$\delta_0 = a_0 p_0^{\sigma_0 - 1} \Phi_0^{1 - \sigma_0} = a_0^{(0)} \left( \Phi_0^{(0)} / p_0^{(0)} \right)^{1 - \sigma_0} = a_0^{(1)} \left( \Phi_0^{(1)} / p_0^{(1)} \right)^{1 - \sigma_0}. \quad (10)$$

We thus see that the 0th level parameters are derivable by values available from a linked input–output table. Further, by substituting the parameters (9) and (10) into (7) we obtain

$$\Phi_1 = \left( \frac{\Phi_0^{1 - \sigma_0} - \delta_0 p_0^{1 - \sigma_0}}{1 - \delta_0} \right)^{\frac{1}{1 - \sigma_0}}.$$

Next, consider the unit-cost function of the 1st level composite:

$$\Phi_1 = \left( \delta_1 p_1^{1 - \sigma_1} + (1 - \delta_1) \Phi_2^{1 - \sigma_1} \right)^{\frac{1}{1 - \sigma_1}}. \quad (11)$$

From (7) and (11), we have

$$\frac{\partial c}{\partial \Phi_1} = (1 - \delta_0) c^{\sigma_0} z^{\sigma_0 - 1} \Phi_1^{-\sigma_0}, \quad \frac{\partial \Phi_1}{\partial p_1} = \delta_1 \Phi_1^{\sigma_1} p_1^{-\sigma_1}, \quad (12)$$

so by (12) the cost share for the 1st (nesting-level) input factor becomes

$$a_1 = \frac{\partial c}{\partial p_1} \frac{p_1}{c} = \frac{\partial c}{\partial \Phi_1} \frac{\partial \Phi_1}{\partial p_1} \frac{p_1}{c} = \delta_1 (1 - \delta_0) \Phi_0^{\sigma_0 - 1} \Phi_1^{\sigma_1 - \sigma_0} p_1^{1 - \sigma_1}. \quad (13)$$

Here again by taking the log and subtracting the two observations on (13), we obtain the parameters

$$\sigma_1 = \frac{\Delta \ln a_1 / p_1 + \Delta \ln \Phi_0^{1 - \sigma_0} + \Delta \ln \Phi_1^{\sigma_0}}{\Delta \ln \Phi_1 / p_1} \quad (14)$$

$$\delta_1 = \frac{a_1 p_1^{\sigma_1 - 1} \Phi_0^{1 - \sigma_0} \Phi_1^{\sigma_0 - \sigma_1}}{1 - \delta_0}. \quad (15)$$

We may then substitute (14) and (15) into (11) to obtain  $\Phi_2$  as

$$\Phi_2 = \left( \frac{\Phi_1^{1-\sigma_1} - \delta_1 p_1^{1-\sigma_1}}{1 - \delta_1} \right)^{\frac{1}{1-\sigma_1}}.$$

Further, let us decompose the 2nd composite price into the 2nd factor input and the remaining (3rd) composite as

$$\Phi_2 = \left( \delta_2 p_2^{1-\sigma_2} + (1 - \delta_2) \Phi_3^{1-\sigma_2} \right)^{\frac{1}{1-\sigma_2}}. \quad (16)$$

From (11) and (16), we have

$$\frac{\partial \Phi_1}{\partial \Phi_2} = (1 - \delta_1) \Phi_1^{\sigma_1} \Phi_2^{-\sigma_1}, \quad \frac{\partial \Phi_2}{\partial p_2} = \delta_2 \Phi_2^{\sigma_2} p_2^{-\sigma_2}, \quad (17)$$

so by (12) and (17) the cost share for the 2nd (nesting-level) input factor becomes

$$\begin{aligned} a_2 &= \frac{\partial c}{\partial p_2} \frac{p_2}{c} = \frac{\partial c}{\partial \Phi_1} \frac{\partial \Phi_1}{\partial \Phi_2} \frac{\partial \Phi_2}{\partial p_2} \frac{p_2}{c} \\ &= \delta_2 (1 - \delta_1) (1 - \delta_0) \Phi_0^{\sigma_0-1} \Phi_1^{\sigma_1-\sigma_0} \Phi_2^{\sigma_2-\sigma_1} p_2^{1-\sigma_2}. \end{aligned} \quad (18)$$

By taking the log and subtracting the two observations on (18), we obtain the parameters

$$\sigma_2 = \frac{\Delta \ln a_2 / p_2 + \Delta \ln \Phi_0^{1-\sigma_0} + \Delta \ln \Phi_1^{\sigma_0-\sigma_1} + \Delta \ln \Phi_2^{\sigma_1}}{\Delta \ln \Phi_2 / p_2} \quad (19)$$

$$\delta_2 = \frac{a_2 p_2^{\sigma_2-1} \Phi_0^{1-\sigma_0} \Phi_1^{\sigma_0-\sigma_1} \Phi_2^{\sigma_1-\sigma_2}}{(1 - \delta_0)(1 - \delta_1)}. \quad (20)$$

We may then substitute (19) and (20) into (16) to obtain  $\Phi_3$ :

$$\Phi_3 = \left( \frac{\Phi_2^{1-\sigma_2} - \delta_2 p_2^{1-\sigma_2}}{1 - \delta_2} \right)^{\frac{1}{1-\sigma_2}}.$$

We can execute this procedure for  $i = 1, \dots, n$  until it reaches the last input factor.

The generic formula for obtaining parameters is

$$\sigma_i = \frac{\Delta \ln a_i / p_i + \Delta \ln \Phi_0^{1-\sigma_0} + \sum_{k=1}^{i-1} \Delta \ln \Phi_k^{\sigma_{k-1}-\sigma_k} + \Delta \ln \Phi_i^{\sigma_{i-1}}}{\Delta \ln \Phi_i / p_i} \quad (21)$$

$$\delta_i = \frac{a_i p_i^{\sigma_i-1} \Phi_0^{1-\sigma_0} \prod_{k=1}^i \Phi_k^{\sigma_{k-1}-\sigma_k}}{\prod_{k=1}^i (1 - \delta_{k-1})}, \quad (22)$$

where the initial parameter values are given by (9) and (10). Furthermore, the generic composite price is

$$\Phi_i = \left( \frac{\Phi_{i-1}^{1-\sigma_{i-1}} - \delta_{i-1} p_{i-1}^{1-\sigma_{i-1}}}{1 - \delta_{i-1}} \right)^{\frac{1}{1-\sigma_{i-1}}}. \quad (23)$$

Note that the initial value is given as  $\Phi_0 = zc$ , while the last composite price is that of the  $n$ th input factor, so that  $\Phi_n = p_n$ .

### 2.3. Plain CES production function

A plain CES production function of an industrial sector (the index  $j$  is omitted) is of the form

$$y = z f(\mathbf{x}) = z \left( \delta_0^{\frac{1}{\sigma}} x_0^{\frac{\sigma-1}{\sigma}} + \delta_1^{\frac{1}{\sigma}} x_1^{\frac{\sigma-1}{\sigma}} + \dots + \delta_n^{\frac{1}{\sigma}} x_n^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad (24)$$

where the share parameters ( $\delta_i > 0$ ,  $\sum_i \delta_i = 1$ ) and the elasticity of substitution  $\sigma \geq 0$  are subject to estimation. Because we assume that the productivity gain is available, we set the benchmark ( $t = 0$ ) absolute productivity  $z^0 = 1$  and the ex-post ( $t = 1$ ) absolute productivity  $z^1 = \exp(\Delta \ln z)$  in regard to (5).

The CES unit-cost function compatible with the production function (24) is

$$h(p_0, \mathbf{p}; z) = \frac{1}{z} (\delta_0 p_0^{1-\sigma} + \delta_1 p_1^{1-\sigma} + \dots + \delta_n p_n^{1-\sigma})^{1/(1-\sigma)}. \quad (25)$$

The cost share of the  $i$ th input  $\alpha_i$  can be found, in regard to Shephard's Lemma, by differentiating this unit-cost function. That is,

$$\alpha_i = \frac{\partial h(p_0, \mathbf{p}; z)}{\partial p_i} \frac{p_i}{p} = \delta_i (z p / p_i)^{\sigma-1}. \quad (26)$$

We suppose that the cost shares can be monitored for two periods  $t = 0, 1$ . That is,

$$\alpha_i^0 = \delta_i (z^0 p^0 / p_i^0)^{\sigma-1}, \quad \alpha_i^1 = \delta_i (z^1 p^1 / p_i^1)^{\sigma-1}. \quad (27)$$

Naturally, the parameters  $\delta_i$  and  $\sigma$  are assumed to be constant over time, but there is only a small chance that these identities are simultaneously true. We therefore find the parameters that best fit the two observations. We first rewrite (27) to describe the share parameter  $\delta_i$  as a function of  $\sigma$  that is consistent with observations for the two periods. That is,

$$\delta_i(\sigma; t = 0) \equiv \alpha_i^0 (z^0 p^0 / p_i^0)^{1-\sigma}, \quad \delta_i(\sigma; t = 1) \equiv \alpha_i^1 (z^1 p^1 / p_i^1)^{1-\sigma}.$$

These parameters are constant, so we search for the value of  $\sigma$  for which these two parameters are as close as possible. In other words, we set

$$\sigma = \arg \max_{\sigma \geq 0} D(\delta(\sigma; t = 0), \delta(\sigma; t = 1)), \quad (28)$$

where  $D(\mathbf{r}, \mathbf{s})$  is some distance function between vectors  $\mathbf{r}$  and  $\mathbf{s}$ . In this study, we employ the following squared sum of log-deviations:

$$D(\mathbf{r}, \mathbf{s}) = \sum_i (\ln r_i - \ln s_i)^2.$$

The rationale for using this distance metric is as follows. By taking the log of the cost shares equality (26), we have

$$\ln \alpha_i = \ln \delta_i + (\sigma - 1) \ln (zp / p_i).$$

Hence, we may consider estimating  $(\sigma - 1)$  via regression through the origin, using two time-distant observations of the variables as

$$\Delta \ln \alpha_i = (\sigma - 1) \Delta \ln (zp / p_i) + \Delta \ln \delta_i, \quad (29)$$

assuming that  $\Delta \ln \delta_i$  (which is supposed to be null for every  $i$ ) is normally distributed with mean zero. That is, the solution for (28) is essentially the same as the estimate via regression through the origin (29).<sup>5</sup>

Figure 2 shows the estimated elasticities with corresponding  $P$ -values for 350 industrial sectors, using the Korean linked input–output tables for 2000 and 2005 (BOK, 2015). Of the 350 sectors, 168 had  $P$ -values over 10%, for which we accepted the null hypothesis (so we set  $\sigma - 1 = 0$ ). As a result, no sector was estimated to be Leontief ( $\sigma = 0$ ), while one sector (regenerated fiber yarn) was estimated to be sub-Cobb–

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<sup>5</sup>The only difference is that all negative estimates for  $\sigma$  via (29) are zeroed in the case of (28).

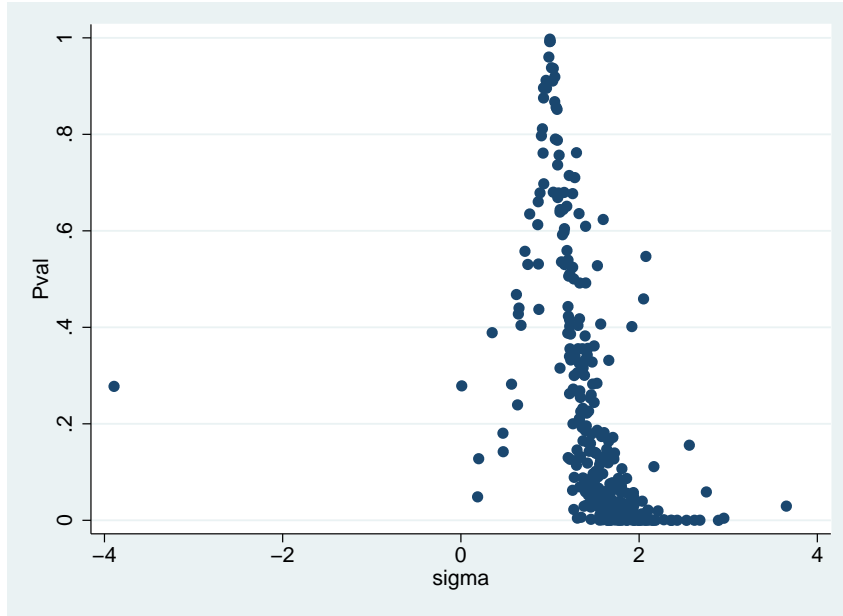


Figure 2:  $P$ -values vs. estimates for  $\sigma$

Douglas ( $\sigma = 0.188 < 1$ ). Otherwise, 168 sectors were set to be Cobb–Douglas, and the remaining 181 sectors were estimated to be meta-Cobb–Douglas ( $\sigma > 1$ ). The accepted elasticities are shown in Figure 3. The sector-wise estimates are shown in Tables 2–8.

Further, for the subsequent analysis of structural propagation, we calibrated the sector-wise CES parameters  $\delta_{ij}$  to agree with the latest technological structure (i.e., that revealed by the 2005 input–output coefficients) under the estimated marginal elasticity of substitution  $\sigma_j$ , while resetting the relative productivity gain  $z_j$  to unity. In other words, we set the parameters according to the latter equilibrium price  $p_j$  and cost shares  $a_{ij}$  (or input–output coefficients) for the reference period so that they satisfy the identity

$$\delta_{ij} = a_{ij} (p_j / p_i)^{1-\sigma_j} . \quad (30)$$

Note that because CES comprehends both Cobb–Douglas ( $\sigma_j = 1$ ) and Leontief ( $\sigma_j = 0$ ) functions with regard to the elasticities,  $\delta_{ij}$  equals the monetary input–output coefficient ( $a_{ij}$ ) for Cobb–Douglas functions, and the physical input–output coefficient

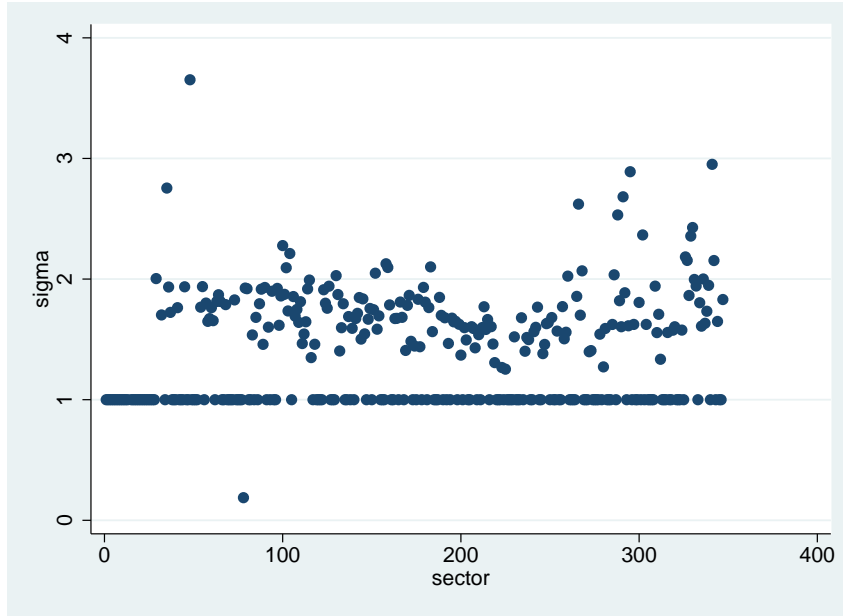


Figure 3: Estimates of CES marginal elasticity of substitution for various industrial sectors ( $\sigma_j$ ), based on the 2000–2005 linked input–output table for Korea (BOK, 2015).

( $\xi_{ij} = a_{ij} (p_j / p_i)$ ) for Leontief functions, in light of (30).<sup>6</sup>

### 3. Propagation Analysis

#### 3.1. Technological structure

The unit-cost function for a multi-factor CES production function for an industrial sector (index  $j$  omitted) compatible with (24) is

$$h(p_0, p_1, \dots, p_n; z) = \frac{1}{z} (\delta_0 p_0^{1-\sigma} + \delta_1 p_1^{1-\sigma} + \dots + \delta_n p_n^{1-\sigma})^{1/(1-\sigma)}.$$

We abbreviate the system of the above unit-cost functions as

$$\mathbf{h}(p_0, \mathbf{p}; \mathbf{z}) = (h_1(p_0, \mathbf{p}; z_1), \dots, h_n(p_0, \mathbf{p}; z_n)). \quad (31)$$

<sup>6</sup>This is an alternative statement of the result obtained by Klein (1952–1953).

Applying Shephard's Lemma on  $\mathbf{h}(p_0, \mathbf{p}; \mathbf{z})$ , we have

$$\begin{bmatrix} \frac{\partial h_1(p_0, \mathbf{p}; z_1)}{\partial p_0} & \frac{\partial h_2(p_0, \mathbf{p}; z_2)}{\partial p_0} & \dots & \frac{\partial h_n(p_0, \mathbf{p}; z_n)}{\partial p_0} \\ \frac{\partial h_1(p_0, \mathbf{p}; z_1)}{\partial p_1} & \frac{\partial h_2(p_0, \mathbf{p}; z_2)}{\partial p_1} & \dots & \frac{\partial h_n(p_0, \mathbf{p}; z_n)}{\partial p_1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_1(p_0, \mathbf{p}; z_1)}{\partial p_n} & \frac{\partial h_2(p_0, \mathbf{p}; z_2)}{\partial p_n} & \dots & \frac{\partial h_n(p_0, \mathbf{p}; z_n)}{\partial p_n} \end{bmatrix} = \begin{bmatrix} \nabla_0 \mathbf{h}(p_0, \mathbf{p}; \mathbf{z}) \\ \nabla \mathbf{h}(p_0, \mathbf{p}; \mathbf{z}) \end{bmatrix} \quad (32)$$

Note that  $\nabla_0 \mathbf{h}(p_0, \mathbf{p}; \mathbf{z})$  is the ex-post vector of physical primary input coefficients, and  $\nabla \mathbf{h}(p_0, \mathbf{p}; \mathbf{z})$  is the ex-post matrix of physical input–output coefficients, which we otherwise call the technological structure. Moreover, note that according to (32), innovation (as represented by the productivity gain  $\mathbf{z}$ ) has the influence of changing the technological structure. Structural propagation designates this influence in particular.

### 3.2. Structural propagation

For obvious reasons, the ex-post equilibrium price under given  $\mathbf{z}$  is needed, to examine the ex-post technological structure of (32). Because the equilibrium price will coincide with the unit cost under perfect competition, we have the identity

$$\mathbf{p} = \mathbf{h}(p_0, \mathbf{p}; \mathbf{z}). \quad (33)$$

Let  $\boldsymbol{\pi}(\mathbf{z}) = (\pi_1(\mathbf{z}), \dots, \pi_n(\mathbf{z}))$  be the solution for (33), given the numéraire price  $p_0$ . The ex-post propagated equilibrium technological structure is the technological structure (32) evaluated at this equilibrium solution as

$$\boldsymbol{\xi}_0(\mathbf{z}) \equiv \nabla_0 \mathbf{h}(p_0, \mathbf{p}; \mathbf{z})|_{\mathbf{p}=\boldsymbol{\pi}(\mathbf{z})}, \quad \boldsymbol{\Xi}(\mathbf{z}) \equiv \nabla \mathbf{h}(p_0, \mathbf{p}; \mathbf{z})|_{\mathbf{p}=\boldsymbol{\pi}(\mathbf{z})}. \quad (34)$$

Also, note that ex-post element-wise physical input–output coefficients can be derived for CES production functions as

$$\xi_{ij}(\mathbf{z}) = \frac{\partial h_j(p_0, \mathbf{p}; z_j)}{\partial p_i} \Big|_{\mathbf{p}=\boldsymbol{\pi}(\mathbf{z})} = \delta_{ij} z_j^{\sigma_j - 1} \left( \frac{\pi_j(\mathbf{z})}{\pi_i(\mathbf{z})} \right)^{\sigma_j} = a_{ij}(\mathbf{z}) \frac{\pi_j(\mathbf{z})}{\pi_i(\mathbf{z})}. \quad (35)$$

We may then use (34) to perform ex-post input–output analysis, for example as

$$\mathbf{V}(\mathbf{z}) = p_0 \boldsymbol{\xi}_0(\mathbf{z}) \left( [\mathbf{I} - \boldsymbol{\Xi}(\mathbf{z})]^{-1} \bar{\mathbf{d}}' \right) = \mathbf{a}_0(\mathbf{z}) \left( [\mathbf{I} - \mathbf{A}(\mathbf{z})]^{-1} \langle \boldsymbol{\pi}(\mathbf{z}) \rangle \bar{\mathbf{d}}' \right), \quad (36)$$

where  $\mathbf{V}(\mathbf{z}) = (V_1(\mathbf{z}), \dots, V_n(\mathbf{z}))$  denotes the sector-wise primary factor (in monetary terms) required for the economy to consume a fixed (vector) amount of final



demand, which we denote by  $\bar{\mathbf{d}} = (\bar{d}_1, \dots, \bar{d}_n)$ . Note that the second identity is due to the third identity for (35), and that angle brackets indicate diagonalization.

So the question is how to solve (33). Although we may have an analytical solution for specific cases, such as  $\delta = 1$  (Cobb–Douglas) and  $\delta = 0$  (Leontief), which we present in the Appendix, there is no general analytical solution. Yet, we can still use the recursive methodology, since the system of unit-cost functions (31) is strictly concave with respect to the entries  $\mathbf{p}$ . In other words, we may apply (33) recursively, iteratively feeding the output back as input, to eventually reach the equilibrium solution. That is,

$$\mathbf{p}^{t+1} = \mathbf{h}(p_0, \mathbf{p}^t; \mathbf{z}), \quad \lim_{t \rightarrow \infty} \mathbf{p}^t = \boldsymbol{\pi}(\mathbf{z}), \quad (37)$$

where  $\mathbf{p}^t$  denotes the price vector at the  $t$ th iteration.

Below we present the results obtained for calculating  $\mathbf{V}(\mathbf{z})$ , where we used  $\mathbf{z} = \mathbf{z}_{\text{MH}}$ , or a doubling of “Medical and Health Services (Public),” the 327th sector’s productivity ( $\mathbf{z}_{327} = (1, \dots, 1, z_{327}, 1, \dots, 1)$ , where  $z_{327} = 2$ ), as the trigger of structural propagation. We have obtained an ex-post equilibrium price via (37) with 20 iterations.<sup>7</sup> Figures 6, 4, 5, and 7 respectively show the primary input saved,  $\Delta \mathbf{V} = \mathbf{V}(\mathbf{1}) - \mathbf{V}(\mathbf{z}_{327})$ , for CES, Cobb–Douglas, and Leontief productions. Naturally, we used the estimated sector-wise elasticity of substitution (Figure 3) for CES productions while setting all the elasticities to unity for Cobb–Douglas and zero for Leontief productions. The sum of the saved primary factor,  $\Delta \mathbf{V} \mathbf{1}'$ , is displayed in Table 1. The kurtosis, which measures the degree of polarization of the sector-wise distribution of the savings  $\Delta \mathbf{V}$ , is also displayed.

Table 1: Saved primary input by productivity doubling of Medical and Health Services (Public) (327th sector) in different functional forms (unit: Million KRW)

|                                 | Leontief | Cobb-Douglas | CES     | msCES   |
|---------------------------------|----------|--------------|---------|---------|
| $\Delta \mathbf{V} \mathbf{1}'$ | 871,089  | 891,347      | 900,227 | 638,842 |
| kurtosis                        | (328)    | (324)        | (322)   | (331)   |

As seen from the numbers in Table 1, the magnitude of propagation is relatively larger for Cobb–Douglas productions than for Leontief productions, whereas the sector-wise distribution is more polarized for Leontief productions than for Cobb–Douglas productions. It is possible that inflexibility of technology (zero elasticity) can consoli-

<sup>7</sup>Note that the final differential (difference in values between the 19th and 20th iterations) was negligibly small.

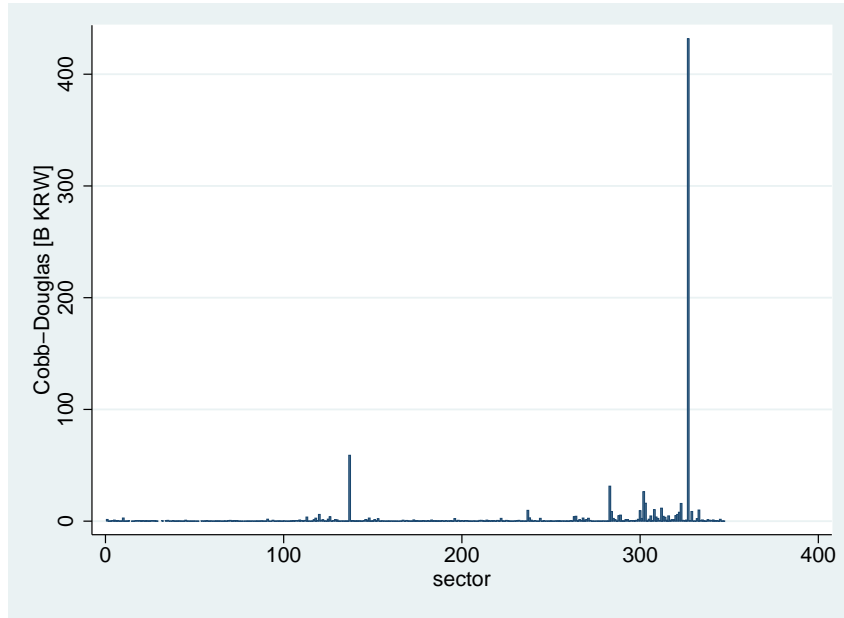


Figure 4: Propagation of the 327th sector productivity doubling under Cobb-Douglas production.

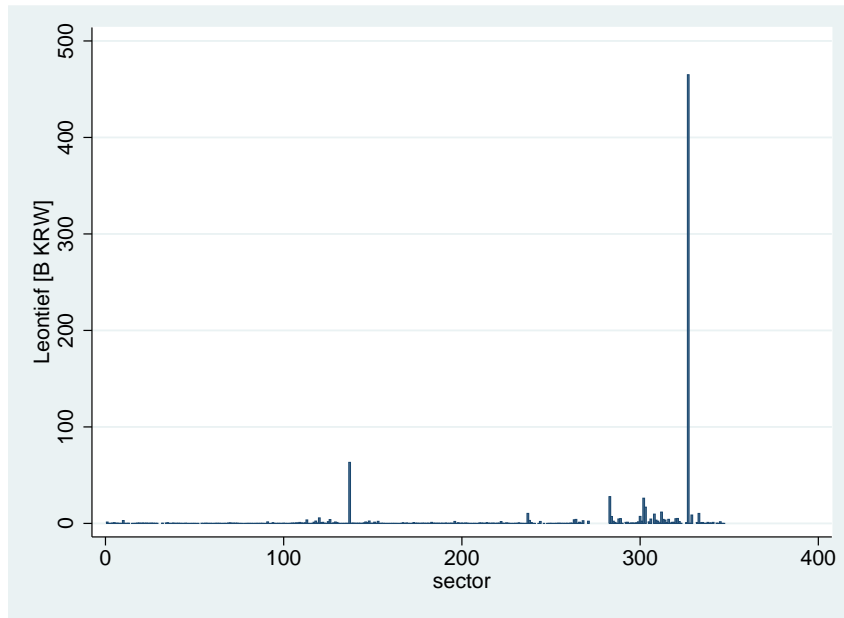


Figure 5: Propagation of the 327th sector productivity doubling under Leontief production.

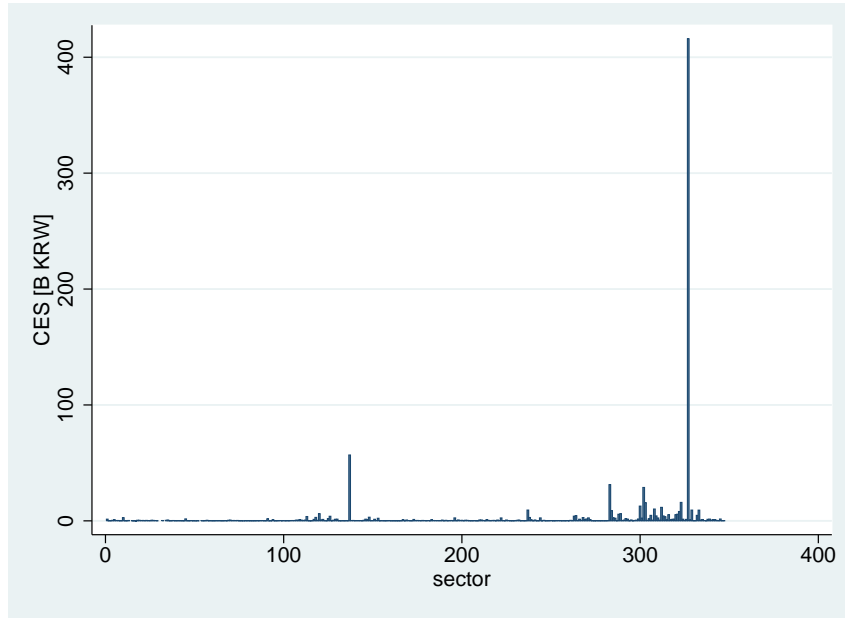


Figure 6: Propagation of the 327th sector productivity doubling under CES production.

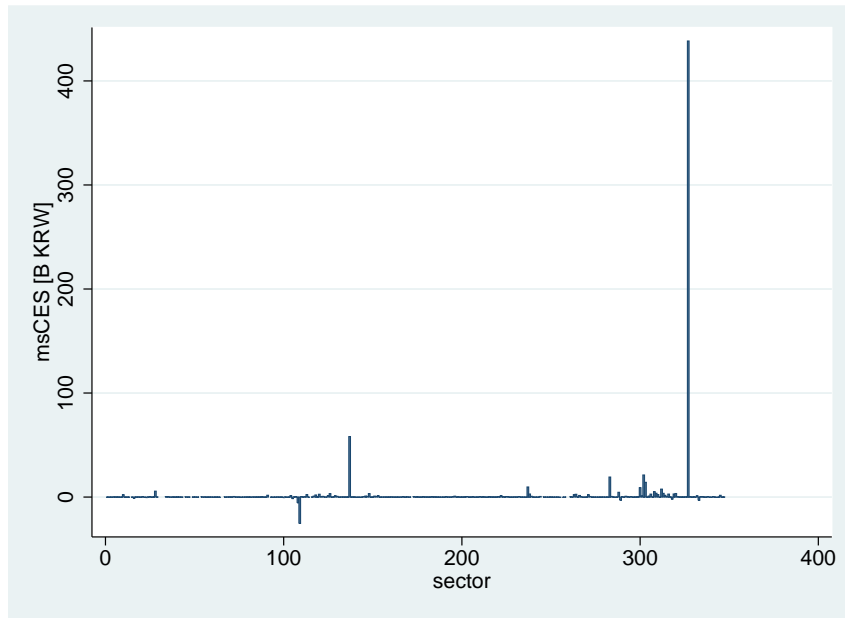


Figure 7: Propagation of the 327th sector productivity doubling under multi-stage CES production.

date the potential propagation effects while flexibility of technology (non-zero elasticity) can do the opposite. Our estimates on CES production indicate that the propagation effects of Cobb–Douglas production, both in terms of magnitude and distribution, lie in-between those for Leontief and CES productions. This result is closely related to our estimates on the elasticities, whose sector-wide average was 1.39, which is meta-Cobb–Douglas, on average.

#### **4. Concluding Remarks**

To date, input–output analysis has been extensively used for assessing the costs and benefits of new goods and new innovations. These studies implicitly rely upon the non-substitution theorem, which allows investigators to study effects under a fixed technological structure, while restricting the subjects of analysis to transformations within the final demand. Nevertheless, substitution of technology will prevail in any industry when a new technology or innovation is introduced into any component (industry) of the economy. Stronger influence is typically foreseeable for intermediate-industry technologies, as they have much stronger and wider feedback within the economy-wide system of production.

In order to take the full set of technology substitution possibilities into account, we proposed in this study a methodology to measure the sector-wise elasticity of substitution for CES production functions, instead of using uniform a priori elasticities of substitution (such as zeros and ones) in modeling economy-wide, multi-sector, multi-factor production systems. A recursive method in the dual (unit-cost functions) was used to evaluate influences on the general equilibrium technological substitutions and eventually on the social costs and benefits (so-called structural propagation) initiated by the introduction of new technology or innovation, which we treat as a gain in productivity.

We have found that more elastic production functions (here, CES production functions) have more significant and wider propagation effects, whereas those for inelastic production functions (here, Leontief production functions) were relatively smaller and more polarized; effects for the Cobb–Douglas production functions were in-between. In the end, the reliability of this analytical framework depends on the measurement of sector-wise technological elasticities, which we obtained in this study as the maximizer of the correlation between the two observation-consistent share parameters. Naturally, different metrics (e.g., Euclidean distances or cosine similarity) can be tested for vector similarity evaluation. Applications and extensions of structural propagation analysis are potentially immense, including internationalization, dynamicalization, quality

consideration, and structural viability assessment, which are all left to future investigations.

### **Acknowledgements**

An earlier version of this study was presented at the PAPAIOS Conference 2015 at Meiji University. We would like to thank Jiro Nemoto, Taiji Hagiwara, Mitsuo Yamada, Kiyoshi Fujikawa, and all participants at the session for their valuable and helpful comments.

## Appendix

### *Cobb–Douglas Production*

We present the ex-post and benchmark Cobb–Douglas unit-cost functions for the  $j$ th industry. That is,

$$\pi_j(\mathbf{z}) = \frac{1}{z_j} \prod_{i=0}^n \left( \frac{\pi_i(\mathbf{z})}{a_{ij}} \right)^{a_{ij}}, \quad \bar{p}_j = \prod_{i=0}^n \left( \frac{\bar{p}_i}{a_{ij}} \right)^{a_{ij}}. \quad (38)$$

Here,  $a_{ij}$  denotes the  $j$ th industry's output elasticity for the  $i$ th input, which is assumed to be constant under Cobb–Douglas production. Note that  $a_{ij}$  is also identical to the benchmark cost share of  $i$ th input for the  $j$ th industry's output (or the benchmark monetary input–output coefficient). Also, note that  $\bar{p}_i$  denotes benchmark (i.e.,  $\pi_i(\mathbf{1}) = \bar{p}_i$ ) equilibrium price.

By taking the log and subtraction on (38), we obtain

$$\ln \pi_j(\mathbf{z}) - \ln \bar{p}_j = \sum_{i=0}^n a_{ij} (\ln \pi_j(\mathbf{z}) - \ln \bar{p}_j - \ln z_i). \quad (39)$$

Rewriting (39) for an  $n \times n$  multiple-industry setting, we have

$$\ln \boldsymbol{\pi}(\mathbf{z}) - \ln \bar{\mathbf{p}} = [\ln \boldsymbol{\pi}(\mathbf{z}) - \ln \bar{\mathbf{p}} - \ln \mathbf{z}] \mathbf{A}, \quad (40)$$

where we abbreviate, for example,  $\ln \boldsymbol{\pi} = (\ln \pi_1, \dots, \ln \pi_n)$ . Then, we can solve (40) for  $\boldsymbol{\pi}(\mathbf{z})$  to obtain the analytical solution to (33). That is,

$$\boldsymbol{\pi}(\mathbf{z}) = \bar{\mathbf{p}} \left\langle \exp \left( -(\ln \mathbf{z}) [\mathbf{I} - \mathbf{A}]^{-1} \right) \right\rangle. \quad (41)$$

Furthermore, the following identities must hold for  $\sigma_j = 1$  and  $\mathbf{z} = \mathbf{1}$  in regard to (35):

$$\delta_{ij} = a_{ij}(\mathbf{z}), \quad \delta_{ij} = a_{ij}(\mathbf{1}) = a_{ij}. \quad (42)$$

Thus, we see that  $a_{ij}(\mathbf{z})$  will remain unchanged. In other words, we may substitute ex-post input–output coefficients with those of the benchmark, that is,

$$\mathbf{a}_0(\mathbf{z}) = \mathbf{a}_0, \quad \mathbf{A}(\mathbf{z}) = \mathbf{A}. \quad (43)$$

Hence, for Cobb–Douglas production, (36) can be evaluated as follows:

$$\mathbf{V}(\mathbf{z}) = \mathbf{a}_0 \left\langle [\mathbf{I} - \mathbf{A}]^{-1} \left\langle \exp \left( -(\ln \mathbf{z}) [\mathbf{I} - \mathbf{A}]^{-1} \right) \right\rangle \langle \bar{\mathbf{p}} \rangle \bar{\mathbf{d}}' \right\rangle. \quad (44)$$

#### *Leontief Production*

The ex-post equilibrium monetary balance for the  $j$ th industry is

$$y_j \pi_j(\mathbf{z}) = \pi_0(\mathbf{z}) x_{0j} + \pi_1(\mathbf{z}) x_{1j} + \cdots + \pi_n(\mathbf{z}) x_{nj}.$$

Rearranging this formula for further investigation gives

$$\begin{aligned} \pi_j(\mathbf{z}) &= \pi_0(\mathbf{z}) \frac{x_{0j}}{y_j} + \pi_1(\mathbf{z}) \frac{x_{1j}}{y_j} + \cdots + \pi_n(\mathbf{z}) \frac{x_{nj}}{y_j} \\ &= \pi_0(\mathbf{z}) \xi_{0j}(\mathbf{z}) + \pi_1(\mathbf{z}) \xi_{1j}(\mathbf{z}) + \cdots + \pi_n(\mathbf{z}) \xi_{nj}(\mathbf{z}) \\ &= \pi_0(\mathbf{z}) \frac{\xi_{0j}}{z_0} + \pi_1(\mathbf{z}) \frac{\xi_{1j}}{z_1} + \cdots + \pi_n(\mathbf{z}) \frac{\xi_{nj}}{z_n}. \end{aligned} \quad (45)$$

Note that the last identity can be derived by applying  $\sigma_j = 0$  and  $\mathbf{z} = \mathbf{1}$  in (35), giving

$$\xi_{ij}(\mathbf{z}) = \delta_{ij} z_j^{-1}, \quad \xi_{ij}(\mathbf{1}) = \xi_{ij} = \delta_{ij}.$$

Thus, (45) can be reduced to

$$\boldsymbol{\pi}(\mathbf{z}) \langle \mathbf{z} \rangle = \boldsymbol{\xi}_0 + \boldsymbol{\pi}(\mathbf{z}) \boldsymbol{\Xi} = \mathbf{a}_0 \bar{\mathbf{p}} + \boldsymbol{\pi}(\mathbf{z}) \langle \bar{\mathbf{p}} \rangle^{-1} \mathbf{A} \langle \bar{\mathbf{p}} \rangle, \quad (46)$$

where we normalized prices using  $\pi_0 = \bar{p}_0 = 1$ . For the second identity, we used  $\xi_{ij} = a_{ij} \bar{p}_j / \bar{p}_i$ . Now, (46) can be solved for  $\boldsymbol{\pi}(\mathbf{z})$  as follows:

$$\boldsymbol{\pi}(\mathbf{z}) = \mathbf{a}_0 [\mathbf{z} - \mathbf{A}]^{-1} \langle \bar{\mathbf{p}} \rangle. \quad (47)$$

Hence, for Leontief production, (36) can be evaluated as

$$\mathbf{V}(\mathbf{z}) = \mathbf{a}_0 \left\langle [\mathbf{z} - \mathbf{A}]^{-1} \langle \bar{\mathbf{p}} \rangle \bar{\mathbf{d}}' \right\rangle. \quad (48)$$

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Table 2: Estimated elasticities for all sectors ( $i = 1 \dots 50$ )

| no. | sector   | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|--|----------------|-------|-----------|---------|----------|------|
| 1   | Unmilled rice                                      | 0.645          | 0.445 | -0.797    | 0.428   | 1.000    | 78   |
| 2   | Barley   | 1.399          | 0.778 | 0.514     | 0.609   | 1.000    | 59   |
| 3   | Wheat  | 0.351          | 0.740 | -0.877    | 0.389   | 1.000    | 25   |
| 4   | Misc. cereals                                      | 0.010          | 0.903 | -1.096    | 0.279   | 1.000    | 46   |
| 5   | Vegetables   | 1.326          | 0.686 | 0.475     | 0.636   | 1.000    | 101  |
| 6   | Fruits   | 1.660          | 0.676 | 0.976     | 0.332   | 1.000    | 90   |
| 7   | Pulses   | 1.532          | 0.837 | 0.636     | 0.528   | 1.000    | 49   |
| 8   | Potatoes   | 1.278          | 0.745 | 0.374     | 0.710   | 1.000    | 46   |
| 9   | Oleaginous crops                                   | 1.254          | 0.607 | 0.419     | 0.677   | 1.000    | 46   |
| 10  | Cultivated medicinal herbs                         | 1.297          | 0.976 | 0.304     | 0.762   | 1.000    | 58   |
| 11  | Other edible crops                                 | 2.166          | 0.720 | 1.619     | 0.111   | 1.000    | 54   |
| 12  | Cotton and hemp                                    | 0.928          | 0.456 | -0.159    | 0.875   | 1.000    | 22   |
| 13  | Horticultural specialities                         | 1.807          | 0.496 | 1.626     | 0.107   | 1.000    | 101  |
| 14  | Natural rubber                                     |                |       |           |         | 1.000    |      |
| 15  | Seeds and seedlings                                | 0.200          | 0.521 | -1.537    | 0.128   | 1.000    | 95   |
| 16  | Other Inedible crops                               | 2.564          | 1.050 | 1.490     | 0.156   | 1.000    | 16   |
| 17  | Dairy farming                                      | 0.568          | 0.400 | -1.080    | 0.282   | 1.000    | 117  |
| 18  | Beef cattle  | 1.342          | 0.225 | 1.515     | 0.132   | 1.000    | 119  |
| 19  | Pigs   | 1.272          | 0.261 | 1.041     | 0.300   | 1.000    | 120  |
| 20  | Poultry and birds                                  | 1.372          | 0.369 | 1.007     | 0.316   | 1.000    | 122  |
| 21  | Other animals                                      | 1.224          | 0.241 | 0.928     | 0.356   | 1.000    | 102  |
| 22  | Operation of timber tracts                         | 0.475          | 0.355 | -1.480    | 0.142   | 1.000    | 92   |
| 23  | Raw timber   | 0.751          | 0.394 | -0.632    | 0.530   | 1.000    | 46   |
| 24  | Edible forest products                             | 0.915          | 0.356 | -0.239    | 0.811   | 1.000    | 74   |
| 25  | Misc. forest products                              | 1.469          | 0.346 | 1.357     | 0.179   | 1.000    | 68   |
| 26  | Fishing  | 1.307          | 0.299 | 1.027     | 0.306   | 1.000    | 160  |
| 27  | Aquaculture  | 1.147          | 0.319 | 0.463     | 0.644   | 1.000    | 126  |
| 28  | Agriculture, forestry and fishing related services | 1.037          | 0.464 | 0.080     | 0.937   | 1.000    | 131  |
| 29  | Anthracite   | 2.004          | 0.350 | 2.872     | 0.005   | 2.004    | 128  |
| 30  | Bituminous coal                                    |                |       |           |         | 1.000    |      |
| 31  | Crude petroleum and Natural gas                    |                |       |           |         | 1.000    |      |
| 32  | Iron ores  | 1.703          | 0.372 | 1.887     | 0.063   | 1.703    | 78   |
| 33  | Copper ores  |                |       |           |         | 1.000    |      |
| 34  | Lead and zinc ores                                 | 2.076          | 1.700 | 0.633     | 0.547   | 1.000    | 7    |
| 35  | Misc. non-ferrous metal ores                       | 2.755          | 0.907 | 1.936     | 0.059   | 2.755    | 48   |
| 36  | Sand and gravel                                    | 1.934          | 0.289 | 3.236     | 0.002   | 1.934    | 109  |
| 37  | Crushed and broken stone and Other bulk stones     | 1.724          | 0.346 | 2.092     | 0.039   | 1.724    | 116  |
| 38  | Limestone  | 1.557          | 0.352 | 1.585     | 0.116   | 1.000    | 122  |
| 39  | Materials for ceramics                             | 1.495          | 0.423 | 1.170     | 0.244   | 1.000    | 113  |
| 40  | Crude salt   | 1.527          | 0.489 | 1.078     | 0.284   | 1.000    | 91   |
| 41  | Misc. non-metallic minerals                        | 1.763          | 0.302 | 2.525     | 0.013   | 1.763    | 104  |
| 42  | Slaughtering and meat processing                   | 1.393          | 0.404 | 0.974     | 0.332   | 1.000    | 101  |
| 43  | Poultry slaughtering and processing                | 1.569          | 0.683 | 0.833     | 0.407   | 1.000    | 91   |
| 44  | Prepared meat products                             | 1.633          | 0.422 | 1.498     | 0.136   | 1.000    | 138  |
| 45  | Dairy products                                     | 1.936          | 0.489 | 1.916     | 0.057   | 1.936    | 140  |
| 46  | Canned seafoods                                    | 1.597          | 1.213 | 0.492     | 0.624   | 1.000    | 106  |
| 47  | Frozen fish and seafoods                           | 2.050          | 1.412 | 0.743     | 0.459   | 1.000    | 98   |
| 48  | Salted, dried and smoked seafoods                  | 3.652          | 1.199 | 2.212     | 0.029   | 3.652    | 94   |
| 49  | Misc. processed seafoods                           | 1.919          | 1.091 | 0.842     | 0.402   | 1.000    | 109  |
| 50  | Polished rice                                      | 0.955          | 0.401 | -0.111    | 0.912   | 1.000    | 92   |

Table 3: Estimated elasticities for all sectors ( $i = 51 \dots 100$ )

| no. | sector  | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|---|----------------|-------|-----------|---------|----------|------|
| 51  | Polished barley   | 1.216          | 0.588 | 0.367     | 0.715   | 1.000    | 69   |
| 52  | Flour and cereal preparations                             | 0.772          | 0.478 | -0.476    | 0.635   | 1.000    | 98   |
| 53  | Raw sugar   |                |       |           |         | 1.000    |      |
| 54  | Refined sugar   | 1.767          | 0.443 | 1.730     | 0.087   | 1.767    | 97   |
| 55  | Starches  | 1.938          | 0.472 | 1.987     | 0.050   | 1.938    | 98   |
| 56  | Glucose, glucose syrup and maltose                        | 1.607          | 0.452 | 1.344     | 0.182   | 1.000    | 106  |
| 57  | Bakery and confectionery products                         | 1.801          | 0.442 | 1.814     | 0.071   | 1.801    | 170  |
| 58  | Noodles   | 1.650          | 0.266 | 2.444     | 0.016   | 1.650    | 131  |
| 59  | Seasonings  | 1.672          | 0.376 | 1.789     | 0.076   | 1.672    | 149  |
| 60  | Soy sauce ad bean paste                                   | 1.763          | 0.284 | 2.688     | 0.008   | 1.763    | 123  |
| 61  | Animal and marine fats and oils                           | 1.655          | 0.340 | 1.927     | 0.057   | 1.655    | 103  |
| 62  | Vegetable fats and oils, and processed edible refined oil | 1.495          | 0.541 | 0.916     | 0.362   | 1.000    | 123  |
| 63  | Canned or cured fruits and vegetables                     | 1.814          | 0.384 | 2.122     | 0.036   | 1.814    | 135  |
| 64  | Coffee and tea  | 1.870          | 0.343 | 2.540     | 0.012   | 1.870    | 125  |
| 65  | Ginseng products  | 1.815          | 0.347 | 2.349     | 0.021   | 1.815    | 100  |
| 66  | Malt and yeast  | 1.706          | 0.512 | 1.378     | 0.172   | 1.000    | 86   |
| 67  | Bean curd and Misc. foodstuffs                            | 1.451          | 0.394 | 1.146     | 0.254   | 1.000    | 158  |
| 68  | Ethyl alcohol for beverages                               | 1.788          | 0.267 | 2.956     | 0.004   | 1.788    | 104  |
| 69  | Blended and distilled sojoo                               | 1.652          | 0.415 | 1.570     | 0.119   | 1.000    | 117  |
| 70  | Beer  | 0.652          | 0.449 | -0.775    | 0.440   | 1.000    | 106  |
| 71  | Other liquors   | 1.717          | 0.467 | 1.535     | 0.127   | 1.000    | 124  |
| 72  | Soft drinks and Manufactured ice                          | 1.054          | 0.533 | 0.102     | 0.919   | 1.000    | 137  |
| 73  | Prepared livestock feeds                                  | 1.827          | 0.325 | 2.549     | 0.012   | 1.827    | 150  |
| 74  | Tobacco products  | 0.904          | 0.373 | -0.258    | 0.797   | 1.000    | 98   |
| 75  | Woolen yarn   | 1.578          | 0.422 | 1.369     | 0.174   | 1.000    | 109  |
| 76  | Cotton yarn   | 1.053          | 0.317 | 0.167     | 0.868   | 1.000    | 123  |
| 77  | Silk and hempen yarn                                      | 0.961          | 0.299 | -0.132    | 0.896   | 1.000    | 82   |
| 78  | Regenerated fiber yarn                                    | 0.188          | 0.406 | -2.002    | 0.049   | 0.188    | 82   |
| 79  | Synthetic fiber yarn                                      | 1.926          | 0.433 | 2.136     | 0.035   | 1.926    | 120  |
| 80  | Thread and other fiber yarns                              | 1.920          | 0.287 | 3.205     | 0.002   | 1.920    | 110  |
| 81  | Woolen fabrics  | 1.433          | 0.356 | 1.217     | 0.226   | 1.000    | 110  |
| 82  | Cotton fabrics  | 1.159          | 0.385 | 0.414     | 0.680   | 1.000    | 123  |
| 83  | Silk and hempen fabrics                                   | 1.537          | 0.289 | 1.860     | 0.066   | 1.537    | 106  |
| 84  | Regenerated fiber fabrics                                 | 1.164          | 0.261 | 0.630     | 0.530   | 1.000    | 100  |
| 85  | Synthetic fiber fabrics                                   | 1.683          | 0.326 | 2.094     | 0.038   | 1.683    | 124  |
| 86  | Other fiber fabrics                                       | 1.473          | 0.482 | 0.982     | 0.328   | 1.000    | 116  |
| 87  | Knitted fabrics   | 1.796          | 0.268 | 2.972     | 0.004   | 1.796    | 107  |
| 88  | Fiber bleaching and dyeing                                | 1.916          | 0.449 | 2.041     | 0.044   | 1.916    | 115  |
| 89  | Knitted wearing apparels                                  | 1.459          | 0.230 | 1.998     | 0.048   | 1.459    | 124  |
| 90  | Knitted clothing accessories                              | 1.930          | 0.402 | 2.316     | 0.022   | 1.930    | 112  |
| 91  | Textile wearing apparels and Clothing accessories         | 1.069          | 0.378 | 0.182     | 0.856   | 1.000    | 137  |
| 92  | Leather wearing apparels                                  | 1.601          | 0.306 | 1.964     | 0.052   | 1.601    | 104  |
| 93  | Fur and Fur wearing apparels                              | 1.345          | 0.284 | 1.218     | 0.226   | 1.000    | 121  |
| 94  | Textile products and Misc. textile products               | 1.899          | 0.358 | 2.509     | 0.013   | 1.899    | 154  |
| 95  | Cordage, rope, and fishing nets                           | 1.425          | 0.459 | 0.926     | 0.356   | 1.000    | 111  |
| 96  | Leather   | 1.368          | 0.307 | 1.200     | 0.232   | 1.000    | 125  |
| 97  | Luggage and handbags                                      | 1.923          | 0.338 | 2.729     | 0.007   | 1.923    | 114  |
| 98  | Footwear  | 1.617          | 0.258 | 2.396     | 0.018   | 1.617    | 127  |
| 99  | Other leather products                                    | 1.860          | 0.497 | 1.732     | 0.087   | 1.860    | 87   |
| 100 | Lumber  | 2.278          | 0.384 | 3.331     | 0.001   | 2.278    | 101  |

Table 4: Estimated elasticities for all sectors ( $i = 101 \dots 150$ )

| no. | sector  | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|---|----------------|-------|-----------|---------|----------|------|
| 101 | Plywood   | 1.874          | 0.321 | 2.718     | 0.008   | 1.874    | 118  |
| 102 | Reconstituted and densified wood                                      | 2.094          | 0.463 | 2.364     | 0.020   | 2.094    | 113  |
| 103 | Wooden products for construction                                      | 1.736          | 0.312 | 2.360     | 0.020   | 1.736    | 110  |
| 104 | Wooden containers and Other wooden products                           | 2.212          | 0.512 | 2.366     | 0.020   | 2.212    | 120  |
| 105 | Pulp  | 1.233          | 0.240 | 0.974     | 0.332   | 1.000    | 108  |
| 106 | Newsprint   | 1.855          | 0.366 | 2.338     | 0.021   | 1.855    | 115  |
| 107 | Printing paper  | 1.692          | 0.377 | 1.837     | 0.068   | 1.692    | 138  |
| 108 | Other raw paper and paperboard  | 1.749          | 0.252 | 2.974     | 0.003   | 1.749    | 146  |
| 109 | Corrugated paper and solid fiber boxes                                | 1.641          | 0.284 | 2.253     | 0.026   | 1.641    | 115  |
| 110 | Paper containers  | 1.812          | 0.296 | 2.742     | 0.007   | 1.812    | 128  |
| 111 | Stationery paper and office paper                                     | 1.466          | 0.256 | 1.817     | 0.072   | 1.466    | 121  |
| 112 | Other paper products  | 1.545          | 0.277 | 1.969     | 0.051   | 1.545    | 156  |
| 113 | Printing  | 1.646          | 0.170 | 3.791     | 0.000   | 1.646    | 139  |
| 114 | Reproduction of recorded media  | 1.919          | 0.267 | 3.435     | 0.001   | 1.919    | 132  |
| 115 | Coal briquettes   | 1.992          | 0.176 | 5.649     | 0.000   | 1.992    | 74   |
| 116 | Coke and other coal products  | 1.349          | 0.126 | 2.780     | 0.006   | 1.349    | 119  |
| 117 | Naphtha   | 0.870          | 0.208 | -0.628    | 0.531   | 1.000    | 117  |
| 118 | Gasoline and Jet oil  | 1.460          | 0.146 | 3.139     | 0.002   | 1.460    | 123  |
| 119 | Kerosene  | 1.267          | 0.394 | 0.676     | 0.500   | 1.000    | 122  |
| 120 | Light oil   | 1.207          | 0.257 | 0.804     | 0.423   | 1.000    | 122  |
| 121 | Heavy oil   | 1.191          | 0.326 | 0.586     | 0.559   | 1.000    | 121  |
| 122 | Liquefied petroleum gas   | 1.394          | 0.449 | 0.877     | 0.382   | 1.000    | 121  |
| 123 | Lubricants  | 1.912          | 0.370 | 2.462     | 0.015   | 1.912    | 127  |
| 124 | Misc. petroleum refinery products                                     | 1.800          | 0.417 | 1.920     | 0.057   | 1.800    | 123  |
| 125 | Petrochemical basic products  | 1.758          | 0.239 | 3.174     | 0.002   | 1.758    | 121  |
| 126 | Petrochemical intermediate products and Other basic organic chemicals | 1.941          | 0.273 | 3.442     | 0.001   | 1.941    | 159  |
| 127 | Coal chemicals  | 0.997          | 0.314 | -0.010    | 0.992   | 1.000    | 105  |
| 128 | Industrial gases  | 1.724          | 0.487 | 1.487     | 0.140   | 1.000    | 120  |
| 129 | Basic inorganic chemicals   | 1.162          | 0.313 | 0.519     | 0.604   | 1.000    | 157  |
| 130 | Synthetic resins  | 2.029          | 0.410 | 2.506     | 0.013   | 2.029    | 151  |
| 131 | Synthetic rubber  | 1.872          | 0.451 | 1.934     | 0.056   | 1.872    | 116  |
| 132 | Regenerated cellulose fibers  | 1.404          | 0.225 | 1.795     | 0.076   | 1.404    | 95   |
| 133 | Synthetic fibers  | 1.597          | 0.274 | 2.177     | 0.031   | 1.597    | 124  |
| 134 | Nitrogen compounds  | 1.795          | 0.266 | 2.986     | 0.003   | 1.795    | 110  |
| 135 | Fertilizers   | 1.653          | 0.425 | 1.538     | 0.126   | 1.000    | 138  |
| 136 | Pesticides and other agricultural chemicals                           | 1.363          | 0.392 | 0.927     | 0.356   | 1.000    | 130  |
| 137 | Medicaments   | 1.689          | 0.279 | 2.467     | 0.015   | 1.689    | 171  |
| 138 | Cosmetics and dentifrices   | 1.510          | 0.344 | 1.483     | 0.140   | 1.000    | 161  |
| 139 | Soap and detergents   | 1.592          | 0.354 | 1.672     | 0.097   | 1.592    | 147  |
| 140 | Dyes, pigments, and tanning materials                                 | 0.987          | 0.254 | -0.050    | 0.960   | 1.000    | 141  |
| 141 | Paints, varnishes, and allied products                                | 1.673          | 0.337 | 1.999     | 0.047   | 1.673    | 151  |
| 142 | Printing ink  | 1.716          | 0.288 | 2.489     | 0.014   | 1.716    | 123  |
| 143 | Adhesives, gelatin and sealants                                       | 1.847          | 0.291 | 2.908     | 0.004   | 1.847    | 139  |
| 144 | Explosives and fireworks products                                     | 1.502          | 0.300 | 1.675     | 0.096   | 1.502    | 135  |
| 145 | Recording media and Photographic chemical products                    | 1.836          | 0.315 | 2.654     | 0.009   | 1.836    | 138  |
| 146 | Misc. chemical products   | 1.546          | 0.296 | 1.847     | 0.067   | 1.546    | 168  |
| 147 | Primary plastic products  | 1.637          | 0.437 | 1.456     | 0.147   | 1.000    | 151  |
| 148 | Industrial plastic products   | 1.667          | 0.319 | 2.095     | 0.038   | 1.667    | 163  |
| 149 | Household articles of plastic material                                | 1.756          | 0.316 | 2.395     | 0.018   | 1.756    | 120  |
| 150 | Tires and tubes   | 1.412          | 0.336 | 1.226     | 0.222   | 1.000    | 140  |

Table 5: Estimated elasticities for all sectors ( $i = 151 \dots 200$ )

| no. | sector   | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|--|----------------|-------|-----------|---------|----------|------|
| 151 | Rubber products  | 1.746          | 0.257 | 2.900     | 0.004   | 1.746    | 150  |
| 152 | Sheet glass and primary glass products                                     | 2.049          | 0.359 | 2.924     | 0.004   | 2.049    | 125  |
| 153 | Industrial glass products  | 1.585          | 0.175 | 3.346     | 0.001   | 1.585    | 165  |
| 154 | Household glass products and others  | 1.695          | 0.268 | 2.594     | 0.011   | 1.695    | 132  |
| 155 | Pottery  | 1.440          | 0.299 | 1.472     | 0.143   | 1.000    | 151  |
| 156 | Refractory ceramic products  | 1.361          | 0.276 | 1.310     | 0.192   | 1.000    | 142  |
| 157 | Clay products for construction   | 1.341          | 0.298 | 1.144     | 0.255   | 1.000    | 136  |
| 158 | Cement   | 2.127          | 0.336 | 3.359     | 0.001   | 2.127    | 150  |
| 159 | Ready mixed concrete   | 2.096          | 0.288 | 3.806     | 0.000   | 2.096    | 128  |
| 160 | Concrete blocks, bricks, and other concrete products                       | 1.786          | 0.278 | 2.823     | 0.005   | 1.786    | 140  |
| 161 | Lime, gypsum, and plaster products   | 1.530          | 0.399 | 1.328     | 0.187   | 1.000    | 130  |
| 162 | Cut stone and stone products   | 1.255          | 0.399 | 0.638     | 0.525   | 1.000    | 130  |
| 163 | Asbestos and mineral wool products   | 1.673          | 0.331 | 2.032     | 0.044   | 1.673    | 141  |
| 164 | Abrasives  | 1.674          | 0.338 | 1.994     | 0.048   | 1.674    | 138  |
| 165 | Asphalts   | 1.659          | 0.470 | 1.400     | 0.164   | 1.000    | 121  |
| 166 | Misc. nonmetallic minerals products  | 1.810          | 0.359 | 2.258     | 0.026   | 1.810    | 136  |
| 167 | Pig iron   | 1.683          | 0.147 | 4.647     | 0.000   | 1.683    | 134  |
| 168 | Ferrous alloys   | 1.335          | 0.301 | 1.113     | 0.268   | 1.000    | 108  |
| 169 | Steel ingots and semifinished products                                     | 1.409          | 0.188 | 2.178     | 0.031   | 1.409    | 140  |
| 170 | Steel rods and bars  | 1.782          | 0.210 | 3.717     | 0.000   | 1.782    | 124  |
| 171 | Section steel  | 1.865          | 0.176 | 4.926     | 0.000   | 1.865    | 117  |
| 172 | Rails and wires  | 1.484          | 0.289 | 1.675     | 0.096   | 1.484    | 127  |
| 173 | Hot rolled steel plates and sheets   | 1.060          | 0.226 | 0.267     | 0.790   | 1.000    | 135  |
| 174 | Steel pipe and tubes, except foundry iron pipe and tubes                   | 1.445          | 0.235 | 1.894     | 0.060   | 1.445    | 138  |
| 175 | Cold rolled steel sheet, strip, and bars                                   | 0.868          | 0.300 | -0.440    | 0.660   | 1.000    | 143  |
| 176 | Iron foundries and foundry iron pipe and tubes                             | 1.833          | 0.263 | 3.160     | 0.002   | 1.833    | 148  |
| 177 | Forgings   | 1.439          | 0.236 | 1.857     | 0.066   | 1.439    | 118  |
| 178 | Coated steel plates  | 1.222          | 0.336 | 0.660     | 0.511   | 1.000    | 140  |
| 179 | Misc. primary iron and steel products                                      | 1.931          | 0.288 | 3.236     | 0.002   | 1.931    | 113  |
| 180 | Copper ingots  | 1.809          | 0.234 | 3.454     | 0.001   | 1.809    | 120  |
| 181 | Aluminium ingots   | 0.719          | 0.477 | -0.588    | 0.558   | 1.000    | 120  |
| 182 | Lead and zinc ingots   | 1.764          | 0.252 | 3.031     | 0.003   | 1.764    | 132  |
| 183 | Gold and silver ingots   | 2.101          | 0.470 | 2.341     | 0.021   | 2.101    | 108  |
| 184 | Other nonferrous metal ingots  | 1.564          | 0.256 | 2.205     | 0.029   | 1.564    | 117  |
| 185 | Primary copper products  | 1.455          | 0.321 | 1.416     | 0.159   | 1.000    | 130  |
| 186 | Primary aluminium products   | 1.387          | 0.372 | 1.039     | 0.301   | 1.000    | 140  |
| 187 | Other nonferrous metal casting and forgings, and primary nonferrous metals | 1.479          | 0.444 | 1.080     | 0.282   | 1.000    | 125  |
| 188 | Metal products for construction  | 1.848          | 0.341 | 2.485     | 0.014   | 1.848    | 130  |
| 189 | Metal products for structure   | 1.699          | 0.394 | 1.776     | 0.078   | 1.699    | 146  |
| 190 | Metal tanks and reservoirs for equipment                                   | 1.327          | 0.332 | 0.987     | 0.326   | 1.000    | 125  |
| 191 | Metal cans, barrels, and drums   | 1.679          | 0.374 | 1.815     | 0.072   | 1.679    | 128  |
| 192 | Handtools  | 1.102          | 0.330 | 0.310     | 0.757   | 1.000    | 141  |
| 193 | Bolts, nuts, screws, rivets, and washers                                   | 1.466          | 0.234 | 1.990     | 0.049   | 1.466    | 135  |
| 194 | Fabricated wire products   | 1.096          | 0.231 | 0.415     | 0.679   | 1.000    | 144  |
| 195 | Fastening metal products   | 1.676          | 0.315 | 2.149     | 0.033   | 1.676    | 133  |
| 196 | Treatment and coating of metals and Misc. fabricated metal products        | 1.646          | 0.285 | 2.263     | 0.025   | 1.646    | 167  |
| 197 | Internal combustion engines and turbines                                   | 1.650          | 0.219 | 2.962     | 0.004   | 1.650    | 152  |
| 198 | Parts of general-purposed machinery and equipment                          | 1.398          | 0.299 | 1.329     | 0.186   | 1.000    | 154  |
| 199 | Conveyors and conveying equipment  | 1.625          | 0.323 | 1.938     | 0.054   | 1.625    | 161  |
| 200 | Air-conditioning equipment and industrial refrigeration equipment          | 1.370          | 0.215 | 1.717     | 0.088   | 1.370    | 159  |

Table 6: Estimated elasticities for all sectors ( $i = 201 \dots 250$ )

| no. | sector  | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|---|----------------|-------|-----------|---------|----------|------|
| 201 | Boiler, Heating apparatus and cooking appliances                            | 1.513          | 0.312 | 1.647     | 0.102   | 1.000    | 160  |
| 202 | Pumps and compressors   | 1.597          | 0.291 | 2.051     | 0.042   | 1.597    | 154  |
| 203 | Misc. machinery and equipment of general purpose                            | 1.495          | 0.263 | 1.880     | 0.062   | 1.495    | 171  |
| 204 | Metal cutting type machine tools  | 1.230          | 0.265 | 0.869     | 0.386   | 1.000    | 157  |
| 205 | Metal forming machine tools   | 1.261          | 0.236 | 1.102     | 0.272   | 1.000    | 153  |
| 206 | Agricultural implements and machinery                                       | 1.605          | 0.230 | 2.630     | 0.009   | 1.605    | 151  |
| 207 | Construction and mining machinery   | 1.591          | 0.276 | 2.137     | 0.034   | 1.591    | 152  |
| 208 | Food processing machinery   | 1.430          | 0.220 | 1.957     | 0.052   | 1.430    | 139  |
| 209 | Textile machinery   | 1.327          | 0.214 | 1.525     | 0.129   | 1.000    | 161  |
| 210 | Metal molds and industrial patterns   | 1.539          | 0.316 | 1.706     | 0.090   | 1.539    | 148  |
| 211 | Misc. machinery and equipment of special purpose                            | 0.863          | 0.271 | -0.507    | 0.613   | 1.000    | 178  |
| 212 | Motors and generators   | 1.598          | 0.214 | 2.790     | 0.006   | 1.598    | 157  |
| 213 | Electric transformers   | 1.771          | 0.223 | 3.458     | 0.001   | 1.771    | 146  |
| 214 | Capacitors and rectifiers, Electric transmission and distribution equipment | 1.583          | 0.222 | 2.629     | 0.009   | 1.583    | 163  |
| 215 | Insulated wires and cables  | 1.665          | 0.188 | 3.527     | 0.001   | 1.665    | 165  |
| 216 | Batteries   | 1.220          | 0.195 | 1.125     | 0.263   | 1.000    | 147  |
| 217 | Electric lamps and electric lighting fixtures                               | 1.606          | 0.276 | 2.196     | 0.030   | 1.606    | 156  |
| 218 | Misc. electric equipment and supplies                                       | 1.462          | 0.258 | 1.792     | 0.075   | 1.462    | 151  |
| 219 | Electron tubes  | 1.307          | 0.107 | 2.876     | 0.005   | 1.307    | 155  |
| 220 | Digital display   | 1.037          | 0.090 | 0.413     | 0.680   | 1.000    | 155  |
| 221 | Semiconductor devices   | 1.226          | 0.147 | 1.536     | 0.126   | 1.000    | 158  |
| 222 | Integrated circuits   | 1.112          | 0.111 | 1.007     | 0.315   | 1.000    | 163  |
| 223 | Electric resistors and storage batteries                                    | 1.267          | 0.116 | 2.305     | 0.022   | 1.267    | 152  |
| 224 | Electric coils, transformers  | 1.331          | 0.262 | 1.260     | 0.210   | 1.000    | 138  |
| 225 | Printed circuit boards  | 1.253          | 0.135 | 1.878     | 0.062   | 1.253    | 156  |
| 226 | Misc. electronic components   | 1.087          | 0.204 | 0.428     | 0.669   | 1.000    | 166  |
| 227 | Television  | 0.875          | 0.161 | -0.779    | 0.437   | 1.000    | 146  |
| 228 | Electric household audio equipment  | 1.086          | 0.254 | 0.337     | 0.737   | 1.000    | 147  |
| 229 | Other audio and visual equipment  | 1.219          | 0.228 | 0.958     | 0.340   | 1.000    | 160  |
| 230 | Line telecommunication apparatuses  | 1.521          | 0.244 | 2.139     | 0.034   | 1.521    | 157  |
| 231 | Wireless telecommunication and broadcasting apparatuses                     | 0.930          | 0.179 | -0.389    | 0.698   | 1.000    | 159  |
| 232 | Computer and peripheral equipment   | 1.204          | 0.134 | 1.522     | 0.130   | 1.000    | 162  |
| 233 | Office machines and devices   | 1.330          | 0.262 | 1.257     | 0.211   | 1.000    | 150  |
| 234 | Household refrigerators   | 1.679          | 0.241 | 2.816     | 0.006   | 1.679    | 148  |
| 235 | Household laundry equipment   | 1.258          | 0.200 | 1.287     | 0.200   | 1.000    | 141  |
| 236 | Other household electrical appliances                                       | 1.402          | 0.232 | 1.735     | 0.085   | 1.402    | 156  |
| 237 | Medical instruments and supplies  | 1.516          | 0.270 | 1.910     | 0.058   | 1.516    | 163  |
| 238 | Regulators and Measuring and analytical instruments                         | 1.498          | 0.279 | 1.786     | 0.076   | 1.498    | 163  |
| 239 | Photographic and optical instruments  | 0.888          | 0.270 | -0.415    | 0.679   | 1.000    | 161  |
| 240 | Watches and clocks  | 1.220          | 0.269 | 0.817     | 0.415   | 1.000    | 143  |
| 241 | Passenger automobiles   | 1.564          | 0.248 | 2.272     | 0.025   | 1.564    | 151  |
| 242 | Buses and vans  | 1.600          | 0.215 | 2.797     | 0.006   | 1.600    | 148  |
| 243 | Trucks and Motor vehicles with special equipment                            | 1.767          | 0.216 | 3.555     | 0.001   | 1.767    | 150  |
| 244 | Motor vehicle engines, chassis, bodies and parts                            | 1.210          | 0.315 | 0.666     | 0.506   | 1.000    | 184  |
| 245 | Trailers and containers   | 1.358          | 0.293 | 1.220     | 0.225   | 1.000    | 131  |
| 246 | Steel ships   | 1.382          | 0.174 | 2.198     | 0.029   | 1.382    | 177  |
| 247 | Other ships   | 1.458          | 0.206 | 2.225     | 0.027   | 1.458    | 162  |
| 248 | Ship repairing and ship parts   | 1.631          | 0.209 | 3.019     | 0.003   | 1.631    | 147  |
| 249 | Railroad vehicles and parts   | 1.638          | 0.205 | 3.113     | 0.002   | 1.638    | 153  |
| 250 | Aircraft and parts  | 1.280          | 0.293 | 0.956     | 0.341   | 1.000    | 155  |

Table 7: Estimated elasticities for all sectors ( $i = 251 \dots 300$ )

| no. | sector   | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|--|----------------|-------|-----------|---------|----------|------|
| 251 | Motorcycles and parts  | 1.681          | 0.321 | 2.120     | 0.036   | 1.681    | 144  |
| 252 | Bicycles and parts and misc. transportation equipment              | 1.202          | 0.262 | 0.769     | 0.443   | 1.000    | 128  |
| 253 | Wood furniture   | 1.112          | 0.239 | 0.470     | 0.639   | 1.000    | 161  |
| 254 | Metal furniture  | 1.569          | 0.224 | 2.544     | 0.012   | 1.569    | 142  |
| 255 | Other furniture  | 1.083          | 0.307 | 0.270     | 0.788   | 1.000    | 162  |
| 256 | Toys and games   | 1.199          | 0.230 | 0.866     | 0.388   | 1.000    | 157  |
| 257 | Sporting and athletic goods  | 1.772          | 0.369 | 2.089     | 0.038   | 1.772    | 155  |
| 258 | Musical instruments  | 1.505          | 0.244 | 2.066     | 0.041   | 1.505    | 151  |
| 259 | Pens, pencils, and other artists' materials                        | 1.559          | 0.217 | 2.575     | 0.011   | 1.559    | 141  |
| 260 | Jewelry and plated ware  | 2.024          | 0.262 | 3.904     | 0.000   | 2.024    | 120  |
| 261 | Misc. manufacturing products                                       | 1.417          | 0.297 | 1.403     | 0.162   | 1.000    | 192  |
| 262 | Hydroelectric power generation                                     | 1.158          | 0.298 | 0.529     | 0.598   | 1.000    | 109  |
| 263 | Fire power generation  | 0.636          | 0.308 | -1.183    | 0.239   | 1.000    | 119  |
| 264 | Nuclear power generation   | 1.644          | 0.424 | 1.519     | 0.131   | 1.000    | 122  |
| 265 | Other generation   | 1.857          | 0.241 | 3.558     | 0.001   | 1.857    | 94   |
| 266 | Manufactured gas supply  | 2.621          | 0.417 | 3.891     | 0.000   | 2.621    | 109  |
| 267 | Steam and hot water supply   | 1.701          | 0.315 | 2.226     | 0.028   | 1.701    | 101  |
| 268 | Water supply   | 2.068          | 0.201 | 5.302     | 0.000   | 2.068    | 120  |
| 269 | Residential building construction                                  | 1.030          | 0.265 | 0.113     | 0.910   | 1.000    | 174  |
| 270 | Non-residential building construction                              | 1.129          | 0.208 | 0.620     | 0.536   | 1.000    | 178  |
| 271 | Building repairs   | 1.140          | 0.261 | 0.537     | 0.592   | 1.000    | 164  |
| 272 | Road construction  | 1.398          | 0.209 | 1.900     | 0.059   | 1.398    | 175  |
| 273 | Railroad construction  | 1.407          | 0.214 | 1.902     | 0.059   | 1.407    | 166  |
| 274 | Breakwater, pier, and harbor construction                          | 1.114          | 0.246 | 0.463     | 0.644   | 1.000    | 156  |
| 275 | Airport construction   | 1.163          | 0.262 | 0.623     | 0.534   | 1.000    | 154  |
| 276 | Dam, levee, and flood control project construction                 | 1.408          | 0.314 | 1.299     | 0.196   | 1.000    | 158  |
| 277 | Water main line and drainage project construction                  | 1.297          | 0.187 | 1.588     | 0.114   | 1.000    | 165  |
| 278 | Land clearing and reclamation, and irrigation project construction | 1.543          | 0.262 | 2.072     | 0.040   | 1.543    | 163  |
| 279 | Land leveling and athletic field construction                      | 1.303          | 0.208 | 1.461     | 0.146   | 1.000    | 169  |
| 280 | Electric power plant construction                                  | 1.272          | 0.159 | 1.710     | 0.089   | 1.272    | 167  |
| 281 | Communications line construction                                   | 1.591          | 0.240 | 2.460     | 0.015   | 1.591    | 155  |
| 282 | Misc. construction   | 0.472          | 0.393 | -1.345    | 0.181   | 1.000    | 170  |
| 283 | Wholesale and Retail trade   | 0.675          | 0.389 | -0.837    | 0.404   | 1.000    | 145  |
| 284 | Restaurants  | 1.188          | 0.416 | 0.453     | 0.651   | 1.000    | 177  |
| 285 | Accommodation  | 1.624          | 0.276 | 2.259     | 0.026   | 1.624    | 128  |
| 286 | Railroad passenger transport                                       | 2.036          | 0.498 | 2.078     | 0.040   | 2.036    | 131  |
| 287 | Railroad freight transport   | 0.922          | 0.255 | -0.305    | 0.761   | 1.000    | 117  |
| 288 | Road passenger transport   | 2.532          | 0.264 | 5.802     | 0.000   | 2.532    | 127  |
| 289 | Road freight transport   | 1.821          | 0.433 | 1.896     | 0.060   | 1.821    | 127  |
| 290 | Coastal and inland water transport                                 | 1.605          | 0.280 | 2.164     | 0.032   | 1.605    | 130  |
| 291 | Oceangoing transport   | 2.682          | 0.450 | 3.735     | 0.000   | 2.682    | 136  |
| 292 | Air transport  | 1.887          | 0.285 | 3.107     | 0.002   | 1.887    | 153  |
| 293 | Supporting land transport activities                               | 1.420          | 0.268 | 1.571     | 0.119   | 1.000    | 122  |
| 294 | Supporting water transport activities                              | 1.613          | 0.247 | 2.478     | 0.015   | 1.613    | 121  |
| 295 | Supporting air transport activities                                | 2.890          | 0.249 | 7.592     | 0.000   | 2.890    | 104  |
| 296 | Cargo handling   | 1.202          | 0.329 | 0.616     | 0.539   | 1.000    | 118  |
| 297 | Warehousing and storage  | 1.624          | 0.329 | 1.900     | 0.060   | 1.624    | 126  |
| 298 | Other services incidental to transportation                        | 1.080          | 0.426 | 0.187     | 0.852   | 1.000    | 117  |
| 299 | Postal services  | 0.927          | 0.556 | -0.131    | 0.896   | 1.000    | 112  |
| 300 | Telecommunications   | 1.806          | 0.285 | 2.827     | 0.006   | 1.806    | 119  |

Table 8: Estimated elasticities for all sectors ( $i = 301 \dots 350$ )

| no. | sector  | $\hat{\sigma}$ | s.e.  | $t$ value | P value | $\sigma$ | obs. |
|-----|---|----------------|-------|-----------|---------|----------|------|
| 301 | Broadcasting  | 0.965          | 0.306 | -0.114    | 0.909   | 1.000    | 119  |
| 302 | Central bank and banking institutions, Non-bank depository institutions | 2.366          | 0.265 | 5.154     | 0.000   | 2.366    | 116  |
| 303 | Other financial brokerage institutions                                  | 1.463          | 0.409 | 1.132     | 0.260   | 1.000    | 104  |
| 304 | Life insurance  | 1.624          | 0.318 | 1.960     | 0.053   | 1.624    | 102  |
| 305 | Non-life insurance  | 1.499          | 0.336 | 1.484     | 0.141   | 1.000    | 103  |
| 306 | Services auxiliary to finance and insurance                             | 1.585          | 0.375 | 1.561     | 0.121   | 1.000    | 105  |
| 307 | Owner-occupied housing  | -3.892         | 4.017 | -1.218    | 0.278   | 1.000    | 5    |
| 308 | Renting and subdividing of real estate                                  | 1.521          | 0.349 | 1.492     | 0.138   | 1.000    | 119  |
| 309 | Services related to real estate   | 1.942          | 0.429 | 2.193     | 0.031   | 1.942    | 87   |
| 310 | Research institutes(public)   | 1.556          | 0.250 | 2.224     | 0.027   | 1.556    | 178  |
| 311 | Research institutes(private, non-profit, commercial)                    | 1.708          | 0.265 | 2.677     | 0.008   | 1.708    | 148  |
| 312 | Research and experiment in enterprise                                   | 1.335          | 0.184 | 1.825     | 0.069   | 1.335    | 221  |
| 313 | Legal and accounting services   | 1.333          | 0.482 | 0.690     | 0.492   | 1.000    | 83   |
| 314 | Market research and management consultancy                              | 1.371          | 0.265 | 1.400     | 0.165   | 1.000    | 91   |
| 315 | Advertising services  | 0.999          | 0.341 | -0.003    | 0.997   | 1.000    | 121  |
| 316 | Architectural engineering services                                      | 1.557          | 0.154 | 3.609     | 0.000   | 1.557    | 139  |
| 317 | Computer softwares development and supply                               | 1.331          | 0.407 | 0.813     | 0.418   | 1.000    | 111  |
| 318 | Computer related services   | 1.315          | 0.340 | 0.928     | 0.356   | 1.000    | 107  |
| 319 | Renting of machinery and goods  | 1.576          | 0.291 | 1.981     | 0.050   | 1.576    | 129  |
| 320 | Cleaning and disinfection services                                      | 1.605          | 0.305 | 1.981     | 0.050   | 1.605    | 100  |
| 321 | Misc. business services   | 1.003          | 0.355 | 0.009     | 0.993   | 1.000    | 125  |
| 322 | Public government   | 0.623          | 0.518 | -0.727    | 0.468   | 1.000    | 201  |
| 323 | Local government  | 1.401          | 0.582 | 0.688     | 0.492   | 1.000    | 210  |
| 324 | Education (public)  | 1.577          | 0.212 | 2.724     | 0.007   | 1.577    | 165  |
| 325 | Education (private, non-profit)   | 1.017          | 0.218 | 0.078     | 0.938   | 1.000    | 144  |
| 326 | Education (commercial)  | 2.184          | 0.290 | 4.076     | 0.000   | 2.184    | 123  |
| 327 | Medical and health services(public)                                     | 2.155          | 0.225 | 5.140     | 0.000   | 2.155    | 134  |
| 328 | Medical and health services(Private, non-profit)                        | 1.864          | 0.282 | 3.060     | 0.003   | 1.864    | 137  |
| 329 | Medical and health services (commercial)                                | 2.357          | 0.321 | 4.223     | 0.000   | 2.357    | 156  |
| 330 | Social work activities(public)  | 2.427          | 0.361 | 3.959     | 0.000   | 2.427    | 117  |
| 331 | Social work activities(other)   | 1.997          | 0.328 | 3.042     | 0.003   | 1.997    | 133  |
| 332 | Sanitary services(public)   | 1.942          | 0.303 | 3.106     | 0.002   | 1.942    | 126  |
| 333 | Sanitary services(commercial)   | 1.418          | 0.439 | 0.953     | 0.343   | 1.000    | 125  |
| 334 | Newspapers  | 1.804          | 0.225 | 3.566     | 0.001   | 1.804    | 114  |
| 335 | Publishing  | 1.610          | 0.190 | 3.216     | 0.002   | 1.610    | 120  |
| 336 | Library, museum and similar recreation related services(public)         | 2.001          | 0.265 | 3.781     | 0.000   | 2.001    | 129  |
| 337 | Library, museum and similar recreation related services(other)          | 1.634          | 0.290 | 2.186     | 0.031   | 1.634    | 131  |
| 338 | Motion picture, Theatrical producers, bands, and entertainers           | 1.733          | 0.209 | 3.509     | 0.001   | 1.733    | 147  |
| 339 | Sports organizations and sports facility operation                      | 1.948          | 0.173 | 5.492     | 0.000   | 1.948    | 140  |
| 340 | Misc. amusement and recreation services                                 | 1.221          | 0.263 | 0.840     | 0.402   | 1.000    | 149  |
| 341 | Business and professional organizations                                 | 2.951          | 0.667 | 2.924     | 0.004   | 2.951    | 91   |
| 342 | Other membership organizations  | 2.154          | 0.324 | 3.560     | 0.001   | 2.154    | 110  |
| 343 | Motor repair services   | 1.305          | 0.274 | 1.114     | 0.267   | 1.000    | 140  |
| 344 | Other personal repair services  | 1.649          | 0.169 | 3.831     | 0.000   | 1.649    | 143  |
| 345 | Laundry and cleaning services   | 1.314          | 0.374 | 0.839     | 0.404   | 1.000    | 87   |
| 346 | Barber and beauty shops   | 1.567          | 0.364 | 1.557     | 0.123   | 1.000    | 89   |
| 347 | Personal services   | 1.831          | 0.281 | 2.956     | 0.004   | 1.831    | 120  |
| 348 | Office supplies   |                |       |           |         | 1.000    |      |
| 349 | Business consumption expenditures                                       |                |       |           |         | 1.000    |      |
| 350 | Nonclassifiable activities  |                |       |           |         | 1.000    |      |